Hecht 2.29:

By factoring the given expression, we obtain:

$$\psi(x, t) = Ae^{-ab^2[x+(c/b)t]^2}.$$ 

This is of the form

$$\psi(x, t) = f(x \mp vt)$$

for the waveform function

$$f(u) = Ae^{-ab^2u^2},$$

and we can read off the answer to be

$$v = -\frac{c}{b}.$$ 

(That is, a speed of $c/b$ in the negative-$x$ direction. Quoting the speed as either $c/b$ or $-c/b$ would be accepted in this problem, but it’s important to understand what the “$\mp$” implies about the direction of propagation.)

For the next part, we evaluate:

$$\left(\frac{\partial \psi}{\partial t}\right)_x = Ae^{a(bx+ct)^2}[-2a(bx + ct)](c);$$

$$\left(\frac{\partial \psi}{\partial x}\right)_t = Ae^{a(bx+ct)^2}[-2a(bx + ct)](b).$$

Thus,

$$\pm v = -\left(\frac{\partial \psi/\partial t}{\partial \psi/\partial x}\right)_x = (-)\frac{c}{b}.$$ 

This problem was the Quiz #1 problem, with the arbitrary constants replaced according to $a \rightarrow b$, $b \rightarrow c$, and $c \rightarrow a$, and with the exponential function replaced by a sin function. The quiz answer is thus $v = (-)a/c$ (accepted either with or without the “$-$” sign), and the exponential in the partial derivatives is replaced by a cos function.

Hecht 2.32:

(a):

$$\psi(y, t) = e^{-(a^2y^2+b^2t^2-2abyt)}$$

$$= e^{-a^2[y-(b/a)t]^2}.$$
Thus this is a traveling wave, with a speed of \( v = \frac{b}{a} \), propagating in the positive-\( y \) direction. The profile is a Gaussian centered on \( y = \left( \frac{b}{a} \right) t \).

(b):

\[
\psi(z, t) = A \sin(az^2 - bt^2) = A \sin[a(z - (b/a)^{1/2}t)(z + (b/a)^{1/2}t)] .
\]

Thus this is not a traveling wave.

(c):

\[
\psi(x, t) = A \sin 2\pi \left( \frac{x}{a} + \frac{t}{b} \right)^2 = A \sin \left[ \left( \frac{2\pi}{a^2} \right) \left( x + \frac{a}{b} t \right)^2 \right] .
\]

Thus this is a traveling wave, with a speed of \( v = \frac{a}{b} \), propagating in the negative-\( x \) direction. Although the form is somewhat reminiscent of a sine-wave, the squaring of the argument alters it. The resulting waveform is centered on \( x = -(a/b)t \), and oscillates with amplitude \( \pm A \), with a “wavelength” that gets shorter the farther away you get from the center of symmetry.

(d):

\[
\psi(x, t) = A \cos^2 2\pi(t - x) = A \cos^2 [(-2\pi)(x - t)] = A \cos^2 2\pi(x - t) .
\]

Thus this is a traveling wave, propagating with a speed \( v = 1 \) in the positive-\( x \) direction. The form is a cosine-squared wave, with a “wavelength” of \( 1/2 \). (Without the square, the wavelength would be 1, but squaring makes the negative bits match the positive bits.)

Hecht 2.36:

(Solution in back of textbook.)