1. Find the expressions for the index of refraction \( n_{\text{cladding}} \) of the cladding needed in an optical slab waveguide, given the index of the core is \( n_{\text{core}} \) and the ray of maximum incident angle at the edge of the core is \( \theta_{\text{max}} \), measured relative to the direction perpendicular to the face of the slab waveguide.

\[
\sin \theta_{\text{max}} = n_{\text{core}} \sin \theta_{\text{c,max}}
\]

\[
\frac{\sin \theta_{\text{c,max}}}{n_{\text{core}}} = \frac{\sin \theta_{\text{max}}}{n_1}
\]

For internal reflection to occur,

\[
\frac{n_{\text{core}} \sin \theta_{\text{c,max}}}{n_1} = n_{\text{cladding}}
\]

\( n_{\text{cladding}} = n_{\text{core}} \sin (\theta_{\text{c,max}} - \theta_{\text{max}}) \)

\( \theta_{\text{c,max}} = \sin^{-1} \left( \frac{n_1}{n_{\text{core}}} \sin \theta_{\text{max}} \right) \)

2. A ray passes straight through a region of vacuum of length \( L_1 \) and a region of water (of index \( n_{\text{water}} \)) of length \( L_2 \).

A. What is the optical path length this ray has taken?

B. If the wavelength of light in vacuum is \( \lambda_1 \), over how many wavelengths has the ray traveled after passing through both regions? (Leave your answers in terms of \( L_1, L_2, \lambda_1, \) and \( n_{\text{water}} \))

\[
\text{A. } \text{OPL} = \sum n_i L_i
\]

\[
\text{OPL} = (1)L_1 + n_{\text{water}} L_2
\]

\[
\text{B. } N_1 = \text{# of wavelengths in region 1} = \frac{L_1}{\lambda_1}
\]

\[
N_2 = \text{# of wavelengths in region 2} = \frac{L_2}{\lambda_1}
\]
3. A stamp collector examines a 1 mm feature on a stamp using a magnifying glass (at maximum magnification) with a focal length of 50 mm. Assume that the stamp collector has good eye sight with a near point at 25 cm.

A. What is the maximum magnifying power of this system?

\[ M_{\text{Max}} = \frac{d_o}{f} + 1 = \frac{25 \text{ cm}}{5 \text{ cm}} + 1 = 5 + 1 = 6 \]

B. Magnifying power = \[ \frac{\text{image size with magnifier}}{\text{image size at near point (d)} \text{ without magnifier}} = 6 \]

Image size at 100 cm will be 4 times smaller than at \( d_o = 25 \text{ cm} \).

Feature will appear: \( 6 \times 4 = 24 \times \) larger than at 100 cm without magnifier.

4. Find the image of the object by ray tracing. Indicate whether the object and image are real or virtual.

Positive lens
5. Draw two rays that define the exit pupil in the following system. The two lenses are thin.

6. A near-sighted sailor needs glasses. He has a far point at 5 meters from his eyes without correction. (treat the eye as if it is composed of a single lens surrounded by air with a 2 cm distance to the retina)

A. Calculate the focal length of the uncorrected relaxed eye lens.

B. Design a thin glass (n=1.5) lens that will correct his vision. Make one surface of the lens planar. You may neglect the distance between the lens and the eye. Identify the focal length and shape of the lens (name, radii of curvature, magnitude and sign).

C. To minimize spherical aberration of the corrective lens, how should the lens be oriented (which surface should be closest to the eye).

D. In one sentence, describe the optical reason why a near-sighted person cannot bring distant objects into focus by accommodation.