1. (a) In Fig. 10.2 the peak occurs at about 280 MeV/c lab energy for a pion. The c.m. energy is
\[(p_\pi + p_\rho)^2 = W^2\]
where \(p_\pi\) and \(p_\rho\) are four-momenta.
Expanding the square gives
\[W^2 = p_\pi^2 + 2p_\pi \cdot p_\rho + p_\rho^2 = m_\pi^2 c^4 + m_\rho^2 c^4 + 2 p_\pi \cdot p_\rho\]
The dot product is Lorentz invariant so we can evaluate it in the lab frame.

\[p_\pi \cdot p_\rho = E_\pi m_\rho c^2\]
and \[E_\pi = \sqrt{m_\pi^2 c^4 + p_\pi^2 c^2} = \sqrt{139^2 + 280^2} = 313 \text{ MeV}\]
so
\[W = \sqrt{139^2 + 938^2 + 2 \cdot 313 \cdot 938} = 1219 \text{ MeV}\]

In Fig. 11.9a the beam is a photon. The peak occurs when its lab energy is about 320 MeV. We use the same procedure to get
\[W = \sqrt{0^2 + 938^2 + 2 \cdot 320 \cdot 938} = 1216 \text{ MeV}\]

These results are closer than we would expect given the error in reading the graphs.

(b) 3°K corresponds to a photon energy \(kT \approx 8.6 \cdot 10^{-5} \text{ eV/K} \cdot 3K\)
\[E_\gamma \approx 2.5 \cdot 10^{-4} \text{ eV}\]
For a head-on collision of a proton and photon
\[p_p = (E_p, E_p) \quad p_\gamma = (E_\gamma, -E_\gamma)\]
\[1219^2 = (p_p + p_\gamma)^2 = m_p^2 c^4 + 2 p_p \cdot p_\gamma + m_\gamma^2 c^4 + 4 E_p E_\gamma\]
\[E_p = \frac{1219^2 - 938^2}{4 \cdot 2.5 \cdot 10^{-4}} \text{ eV} = 0.6 \cdot 10^{-2} \text{ eV}\]
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Assume a neutrino mass $m_{\nu}$.

The relativistic energy-mass relation gives

$$E_{\gamma}/m_{\nu} = \gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \text{where} \quad \beta = v/c.$$

Now $\beta$ is very close to 1 so we can approximate

$$\left(\frac{m_{\nu}^2}{E}\right)^2 \approx 2\left(1-\beta^2\right) = 2\frac{\Delta v}{c} \quad \text{where} \quad \Delta v = c-v.$$

The arrival time $t = 160,000$ y and the distance $L = 160,000$ l.y. $= tc$.

The variation in arrival times is $\Delta t \approx 2$ sec.

So the variation $\Delta v/c = L/tc - L/(t+\Delta t)c \approx \frac{\Delta t}{t} \frac{L}{c} = \frac{\Delta t}{t} \frac{L}{c}$

Note that we are comparing arrival times of very fast neutrinos — speed of light with average neutrinos — stragglers.

So we get $(E \approx 8\text{ MeV})$

$$\left(\frac{m_{\nu}^2}{E}\right)^2 \approx 2\frac{\Delta t}{t}$$

$$m_{\nu} < \sqrt{\frac{2\cdot2\text{ sec}}{160,000\text{ y}}} \cdot \frac{1}{3\cdot10^7\text{ sec/y}} \times 8\text{ MeV}$$

$$m_{\nu} \approx 7\text{ eV}$$