Ch2 P27

1. \( v_0 = 35.0 \text{ mi/h} \) \( \Delta x = 40.0 \text{ ft} \) \( v = 0 \)

2. \( v_0 = 70.0 \text{ mi/h} \) \( \Delta x = ? \text{ ft} \) \( v = 0 \)

\[
\begin{align*}
\text{I1} & \\
& v^2 = v_0^2 + 2a \Delta x \\
\Rightarrow & \quad a = \frac{v^2 - v_0^2}{2 \Delta x} = -\frac{v_0^2}{2 \Delta x} \quad \text{if} \quad v = 0
\end{align*}
\]

Note that \( \Delta x \) in ft while \( v_0 \) is in mi/h

\[
\begin{align*}
v_0 & = \left( 35.0 \text{ mi/h} \right) \left( \frac{1.47 \text{ ft/s}}{1 \text{ mi/h}} \right) = 51.45 \text{ ft/s}
\end{align*}
\]

\[
\begin{align*}
\text{II} & \\
& a = -\frac{v_0^2}{2 \Delta x} = -\frac{(51.45)^2}{2 \left( 40.0 \right)} = -33.09
\end{align*}
\]

\[
\begin{align*}
\text{II} & \\
& \text{solve for} \ \Delta x \\
& v^2 = v_0^2 + 2a \Delta x \\
\Rightarrow & \quad \Delta x = -\frac{v_0^2}{2a}
\end{align*}
\]

\( a \) is the same as for car I1; \( v_0 = 102.9 \text{ ft/s} \)

\[
\begin{align*}
& \Delta x = -\frac{(102.9)^2}{2 \times (-33.09)} = 160 \text{ ft}
\end{align*}
\]

Note that if initial velocity is doubled, i.e.,

\[
35 \text{ mi/h} \rightarrow 70 \text{ mi/h}, \quad \text{then stopping distance is quadrupled}
\]

This follows from above: \( \Delta x \propto v_0^2 \)
\[ v_0 = 0 \quad a = 10.0 \, \text{m/s}^2 \quad \Delta x = 400 \, \text{m} \]

a) how long did it take

b) what is the final speed

Easier to start with part b

\[ v^2 = v_0^2 + 2a \Delta x \]

\[ v^2 = 2a \Delta x \quad \text{since} \quad v_0 = 0 \]

\[ v = \sqrt{2a \Delta x} \]

\[ v = \left( 2 \times 10.0 \, \text{m/s}^2 \times 400 \, \text{m} \right)^{1/2} = \sqrt{89.4 \, \text{m/s}} \quad \text{(ans. 8.6)} \]

for part a we can now use

\[ v = v_0 + at \]

\[ v = at \]

\[ t = \frac{v}{a} = \frac{89.4 \, \text{m/s}}{10 \, \text{m/s}^2} = 8.94 \, \text{s} \quad \text{(ans to a)} \]
\( \text{Solution} \quad \text{Physcs 1100} \quad \text{HW#4} \)

1. \( v_0 = 0 \quad a = 2.77 \text{ m/s}^2 \quad t = 15.0 \text{ s} \)
2. \( a = 0 \quad t = 2.05 \text{ min} \quad v = ? \)
3. \( a = -\ 9.47 \text{ m/s}^2 \quad t = 4.39 \text{ s} \)

Find (a) Total displacement:

\[ \Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3 \]

1. \( \Delta x_1 = \frac{v_0^2}{2} + \frac{1}{2} at^2 = \frac{1}{2} at^2 \quad \text{since } v_0 = 0 \text{ for } \text{stage } 1 \]

\[ \Delta x_1 = \frac{1}{2} \left( 2.77 \text{ m/s}^2 \right) \left( 15.0 \text{ s} \right)^2 = 311.6 \text{ m} \]

2. \( \Delta x_2 = \frac{v_0^2}{2} + \frac{1}{2} at^2 = v_0 t \quad \text{since } a = 0 \text{ for } \text{stage } 2 \)
   
   but what is \( v_0 \)?; \( v_0 \) is the velocity at the end of stage 1.
   
   Then, from (1)

\[ v = v_0 + at = at \]

\[ v = 2.77 \text{ m/s}^2 \times 15 \text{ s} = 41.55 \text{ m/s} \]

For stage 2 \( v_0 = 41.55 \text{ m/s} \) and

\[ \Delta x_2 = \frac{v_0}{2} t = \frac{41.55 \text{ m/s}}{5} \times 2.05 \text{ min} \times 60 \text{ s} = 511.0 \text{ m} \]

3. \( \Delta x_3 = \frac{v_0}{2} t + \frac{1}{2} at^2 \quad (v_0 \text{ same as for stage 2}) \]

\[ = \left( \frac{41.55 \text{ m}}{5} \right) \left( 4.39 \text{ s} \right) + \frac{1}{2} \left( -9.47 \text{ m/s}^2 \right) \left( 4.39 \text{ s} \right)^2 = 91.2 \text{ m} \]

\[ \Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3 \]

\[ = \left( 311.6 + 511.0 + 91.2 \right) \text{ m} \]

\[ \Delta x = 5.51 \times 10^3 \text{ m} \]
average velocities:

1. \( \overline{v}_1 = \frac{\Delta x_1}{\Delta t_1} = \frac{311.6 \text{ m}}{15.0 \text{ s}} = \frac{20.1 \text{ m}}{\text{s}} \)

or

\[ \overline{v}_1 = \frac{1}{2} (v_0 + v) \]

since accel. is constant

\[ \overline{v}_1 = \frac{1}{2} (0 + 41.55) = 20.1 \text{ m/s} \]

2. \( \overline{v}_2 = \frac{1}{2} (41.55 + 41.55) = \frac{41.55 \text{ m}}{\text{s}} \)

3. use definition of \( \overline{v} \):

\[ \overline{v}_3 = \frac{\Delta x_3}{\Delta t_3} = \frac{91.2 \text{ m}}{4.39 \text{ s}} = 20.1 \text{ m/s} \]

average velocity over entire trip:

\[ \overline{v} = \frac{\Delta x}{\Delta t} = \frac{\Delta x_1 + \Delta x_2 + \Delta x_3}{\Delta t_1 + \Delta t_2 + \Delta t_3} = \frac{551 \times 10^3 \text{ m}}{144.39 \text{ s}} = 38.7 \text{ m/s} \]

Note: at the end of stage three (i.e. at end of trip)

\[ v = v_2 + at = 41.55 + (-7.47)(4.39) = 0.0 \text{ m/s} \]

if we use the relation:

\[ \overline{v} = \frac{1}{2} (v_0 + v) \]

we get \( \overline{v} = \frac{1}{2} (0 + 0) = 0 \text{ m/s} \) which is wrong!

Reason: The acceleration was NOT constant throughout the trip, so the relation is not applicable. A changed at end of stage 1 and 2
Ch2 P37

1. \( v_0 = 0 \), \( a = 1.5 \text{ m/s}^2 \), \( t = 5.0 \text{ s} \)

2. \( v_0 = v \) at end of 1, \( a = -2.0 \text{ m/s}^2 \), \( t = 3.0 \text{ s} \)
   a) What is \( v \) at end of 2?
   b) How far has it gone?

First find \( v \) at end of 1:

1. \( v = v_0 + at = at = 1.5 \text{ m/s}^2 \times 5.0 \text{ s} = 7.5 \text{ m/s} \)

2. \( v = v_0 + at \)

\[ = 7.5 \text{ m/s} - 2.0 \text{ m/s}^2 \times 3.0 \text{ s} = 1.5 \text{ m/s} \] (Ans. to (a))

Total displacement \( \Delta x = \Delta x_1 + \Delta x_2 \)

\[ \Delta x_1 = \frac{1}{2} (v_0^2 + v) t = \frac{1}{2} (0 + 7.5) (5.0) = 18.75 \text{ m} \]

\[ \Delta x_2 = \frac{1}{2} (v + v) t = \frac{1}{2} (7.5 + 1.5) (3.0) = 13.5 \text{ m} \]

\[ \Delta x = 18.75 + 13.5 = 32 \text{ m} \left(\approx \text{2 sig. dig.}\right) \]
Ch2 P43

\[ v_o = 25.0 \text{m/s} \]
\[ a = g = -9.8 \text{ m/s}^2 \]

a) how high does it rise?

At highest point \( v = 0 \)

\[ v^2 = v_o^2 + 2a \Delta y \]
\[ \Delta y_1 = \frac{-v_o^2}{2a} \]
\[ \Delta y_1 = \frac{-(25.0)^2}{2 \times (-9.8)} = [31.9 \text{ m}] \]

b) \( v = v_o + at \)

\[ t = \frac{-v_o}{a} \]
\[ t = \frac{-25.0}{-9.8} = [2.55 \text{ s}] \]

\[ \Delta y_2 = v_o t + \frac{1}{2} at^2 \]
\[ = 0 + \frac{1}{2} at^2 \]
\[ \Delta y = \frac{1}{2} at^2 \]
\[ \Rightarrow t = \sqrt{\frac{2 \Delta y}{a}} \]

but \( \Delta y_2 = -\Delta y_1 \)

same magnitude "height" but different (opposite) initial/final pts

\[ t = \sqrt{\left( \frac{2 \times (-31.9)}{-9.8} \right)} = [2.55 \text{ s}] \]

\[ v = v_o + at \]

\[ = 0 + (-9.8)(2.55) = [-25.0 \text{ m/s}] \]

Note: \( v \) has same speed as ball was thrown at but opposite sign.
Δy = 7.5 m
v = 0 m/s at highest point
v₀ = ?
α = g = -9.8 m/s²

\[ v₀² = v₀² + 2gΔy \]
\[ 0 = v₀² + 2gΔy \]
\[ -v₀² = 2gΔy \]
\[ v₀² = -2gΔy \]
\[ v₀ = \sqrt{-2gΔy} \]
\[ = \left( -2 \times -9.8 \frac{m}{s²} \times 7.5 m \right) \frac{1}{2} \]
\[ v₀ = 12 m/s \]