THIRD MIDTERM

Name (print) ___________________________ Name (signed) ___________________________

Discussion Instructor (circle one): Hamed Hari Molina Nott Paul Reeve Zhang

Discussion Section # __________________________

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES:
Use the conversion constants and data given on the front page.

(a) Calculate \( \vec{A} \cdot \vec{B} \), if

\[
\begin{align*}
A &= 5.0i + 6.0j - 9.5k \\
B &= -3.0i - 7.0j - 4.5k
\end{align*}
\]

\[
\vec{A} \cdot \vec{B} = -15 - 42 + 42.75 = -14.3
\]

(b) A force of 75.0 N acts through a distance of 45.0 m. Calculate the work done in ft-lbs.

\[
W = \mathbf{F} \cdot \mathbf{S} = 75 \times 45 (\text{J}) = 75 \times 45 \times \frac{3.28}{0.224} \times \frac{0.224}{10} \cdot 1 \cdot 2.49 \times 10^3 \text{ ft-lbs}
\]

(c) Assume a spring with a force law given by \( F = -kx^3 \). If \( k = 250 \text{ N/m}^3 \), calculate the work done to compress the spring from \( x = 0 \) to \( x = 1.25 \text{ cm} \) in joules.

\[
W = \int_0^{1.25 \times 10^{-2}} kx^3 dx = \frac{1}{4} \times 250 \times (1.25 \times 10^{-2})^4 = 1.53 \times 10^{-6} \text{ J}
\]

(d) Calculate the maximum safe speed for a car traveling around an unbanked curve of radius 400 ft if the friction coefficients are \( \mu_s = 0.75 \) and \( \mu_k = 0.55 \). (The answer should be in ft/s.)

\[
\begin{align*}
f &= \mu_k N = \frac{m v^2}{R} \\
J &= \left( \frac{m v}{m} \right)^2 = \left( \frac{\mu_k \text{mg} R}{m} \right)^2 = \mu_k g R = 98.3 \text{ ft/s}
\end{align*}
\]

(e) A car goes around an unbanked curve at 40.0 mi/hr. The curve has a radius of 500 ft. At what angle to the vertical does a weight suspended on a string hang in the car?

\[
\begin{align*}
\tan \theta &= \frac{v^2}{R} = (40 \times \frac{5280}{3600})^2 = \frac{1}{32.2 \times 500} = 0.214 \\
\theta &= 1.21^\circ
\end{align*}
\]

-1 SF
-1 units

Right if off ±1 on last digit.
A block of mass \( m = 0.175 \text{ kg} \), is launched with an initial velocity \( v_0 = 1.37 \text{ m/s} \) down the incline. The coefficients of friction are \( \mu_g = 0.65 \) and \( \mu_k = 0.55 \). Use the work-energy theorem to calculate how far down the incline the block slides before stopping. The plane is as long as needed.

Use correct \( \mu \). \( \circled{5} \)

**FBD + Force Diagram** \( \circled{5} \)

Friction work

**Friction work** \( \circled{6} \)

Gravitational work

**Gravity work** \( \circled{6} \)

\[ \sum F_y = 0 \quad N - mg \cos \theta = 0 \quad N = mg \cos \theta \]

\[ f_k = \mu_k N = \mu_k mg \cos \theta \]

\[ \sum F_x = (-ma_x) = f_k - mg \sin \theta \]

\[ \sum F_x = -ma_x = (\mu_k mg \cos \theta - mg \sin \theta) \]

\[ \text{Work done} = +ma_x \Delta x = -(\mu_k mg \cos \theta - mg \sin \theta) \Delta x \]

\[ \text{Work done} = \text{Change in kinetic energy} = \Theta \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 \]

\[ V_f = 0 \]

\[ -\frac{1}{2} m V_0^2 = -(\mu_k mg \cos \theta - mg \sin \theta) \Delta x \]

\[ \Delta x = \frac{1}{2} \frac{m V_0^2}{[\mu_k mg \cos \theta - mg \sin \theta]} = 0.261 \text{ m} \]

\[ V_0 = 1.37 \text{ m/s} \]

\[ m = 0.175 \text{ kg} \]

\[ \mu_k = 0.55 \]
In this system the table is frictionless. The coefficients of friction between block 1 and block 2 are \( \mu_2 = 0.67 \) and \( \mu_k = 0.62 \). Calculate the maximum value of \( m_3 \) such that 1 does not slide with respect to 2. For full credit you must have free body diagrams for each object.

\[
\begin{align*}
&\text{block 1} \\
&\text{block 2} \\
&\text{block 3}
\end{align*}
\]

\[
\begin{align*}
&f = M_1 a \\
&T - f = M_2 a \\
&M_3 g - T = M_3 a
\end{align*}
\]

\[
\begin{align*}
N_1 &= M_1 g \\
N_2 &= M_2 g \\
N_3 &= M_3 g
\end{align*}
\]

\[
\begin{align*}
\text{Maximizing } M_3 \text{ maximizes } f \text{ so } f = M_1 a \text{ must be maximized} \\
&f = f_{\text{max}} = M_3 N_1
\end{align*}
\]

\[
\begin{align*}
&f = M_3 N_1 = M_3 M_1 g = 0.67 \times 1.35 \times 9.8 = 8.86 \text{ N} \\
&a = \frac{f}{M_1} = 6.57 \frac{M_3}{M_1} \\
&T = M_2 a + f = 23.31 \text{ N} \\
&T = M_3 g - M_3 a = M_3 (g - a) \\
&\quad M_3 = \frac{T}{g - a} \\
&M_3 = \frac{23.31}{9.8 - 6.57} = 7.21 \text{ kg}
\end{align*}
\]
The cone shown rotates about the vertical axis. A block of mass \( m = 1.25 \text{ kg} \), is at a point 2.00 m up the cone as shown. The coefficients of friction between cone and block are \( \mu_k = 0.55 \) and \( \mu_s = 0.62 \).

(a) Calculate the minimum value of the tangential velocity such that the block does not slide down the cone.

(b) Calculate the normal force for this velocity.

The easiest way to do the problem is to choose vertical and horizontal axes so that the horizontal axis is along the direction of the centripetal acceleration.

\[ a = \frac{v^2}{r} \]

1. Vertical components balance: \( N \cos \theta - \mu_k N \sin \theta = mg \) \( \Rightarrow N = \frac{mg}{\cos \theta - \mu_k \sin \theta} \)

2. \( a = \frac{v^2}{r} \) \( \Rightarrow v = \sqrt{rg \frac{\sin \theta - \mu_k \cos \theta}{\cos \theta - \mu_k \sin \theta}} \approx 1.471 \text{ m/sec} \) \( (17 \text{ ft/s}) \)

b) from (a), \( N = \frac{mg}{\cos \theta - \mu_k \sin \theta} = 10.52 \text{ N} \) after plugging in numbers \( (8 \text{ pt}) \)

**Common Errors**

1. Many people set \( d \) up on the incline, but didn’t break the required acceleration into components. This lost \( 18 \text{ pts} \), all other things being equal.

2. Incorrectly solving for \( r \).

Physics 301  
Winter Quarter 1993  
February 19, 1993  
George Williams

Name (print)  MARK REEVE  Name (signed)  Solutions  
Discussion Instructor (circle): Condella  DiCarlo  Ganesan  Hollier  Reeve

Report all numbers to three significant figures!  
Use the conversion constants and data given on the front page.

5 pts each. -1 s.f.  -1 unit

(a) Calculate $\vec{A} \cdot \vec{B}$ if

$$\vec{A} = -3.00\hat{i} + 4.00\hat{j} + 7.25\hat{k}$$

and

$$\vec{B} = +5.25\hat{i} - 4.00\hat{j} - 2.00\hat{k}$$

$$\vec{A} \cdot \vec{B} = (-3)(5.25) + (4)(-4) + (7.25)(-2) = -46.25 = -46.3$$

-2 for incorrect sign

(b) A car goes around a curve that is not banked at 65.0 mi/hr. The curve has a radius of curvature of 720 ft. At what angle to the vertical does a weight suspended on a string hang in the car?

$$\tan \theta = \frac{v^2}{r g}$$

$$\theta = \tan^{-1} \left( \frac{65 \text{ mi/hr}}{720 \text{ ft}} \right) = 21.4^\circ$$

(c) Convert 7,200 J to ft pounds.

$$7200 \text{ J} \left( \frac{3.78 \text{ ft-lb}}{1 \text{ Nm}} \right) \left( \frac{1 \text{ N}}{1 \text{ m}} \right) = 531 \times 10^3 \text{ ft-lb}$$

(d) If a spring has the force law $F = -kx - bx^3$, calculate the work to stretch it from $x_1$ to $x_2$, where neither $x_1$ nor $x_2$ are the unstretched length. (The unstretched portion is $x = 0$.)

$$W = \int_{x_1}^{x_2} F \, dx = \int_{x_1}^{x_2} (-kx - bx^3) \, dx$$

$$W = \frac{k}{2} (x_2^2 - x_1^2) + \frac{b}{4} (x_2^4 - x_1^4)$$

The wrong sign lost 3 points. Many people misunderstood.

(e) A mass of $m = 4.70 \text{ kg}$ is accelerated by a horizontal force of 27.0 N on a horizontal, frictionless surface. How much work is done by the force in 2.00 seconds if the mass starts from rest?

$$a = \frac{F}{m} \Delta x = \frac{1}{2} \frac{m}{L^2} \Delta v^2 = \frac{1}{2} (\frac{F}{m}) L^2$$

$$W = \frac{1}{2} \left( \frac{27 \text{ N}}{4.70 \text{ kg}} \right)^2 (2.00 \text{ s}) = 31.0 \text{ J}$$
THIRD MIDTERM

Name (print) ___________________________ Name (signed) ___________________________

Discussion Instructor (circle): Condella  DiCarlo  Ganesan  Hollier  Reeve

Discussion Section # _________

SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A mass \( m \) (\( m = 3.30 \) kg) is fastened to a pivot by a massless rod of length \( L = 0.750 \) m. The system rotates in the vertical plane at constant speed. The tangential speed of the mass is 4.25 m/s.

(a) Calculate the tension in the rod at the bottom of the circle.
(b) Calculate the force the rod exerts on the mass at the top. What is the direction of that force (indicate clearly).

\[
\begin{align*}
\text{a)} & \quad & T &= \text{force due to } r = L \\
\text{b)} & \quad & \frac{mg}{\sin \theta} &= \frac{T - mg}{m} = a_c \\
& \quad & a_c &= \frac{v^2}{r} \\
& \quad & r &= L \\
\Rightarrow & \quad & T - mg &= \frac{mv^2}{r} \\
& \quad & T &= mg + \frac{mv^2}{r} = 112 \text{ N} \\
\end{align*}
\]

11 pts

11 pts

3 pts

\( a_c \) always

-1 s.g., ft/s²

-1 u.m., ft/s
A block of mass \(m = 1.80 \text{ kg}\) is pulled at constant speed down the plane as shown. The coefficients of friction are: \(\mu_s = 0.75\) and \(\mu_k = 0.55\).

(a) Calculate the numerical value of \(P\).
(b) Find the work done by \(P\) to move the block 2.50 m along the plane.
(c) Determine the work done by gravity when the block moves 2.50 m along the plane.
(d) Calculate the work done by friction when the block moves 2.50 m along the plane.

\[
P \cos 30^\circ + mg \sin 20^\circ - f_k = 0
\]

\[
N + P \sin 30^\circ = mg \cos 20^\circ
\]

\[
f_k = \mu_k N
\]

\[
P(\cos 30^\circ + \mu_k \sin 30^\circ) = \mu_k mg \cos 20^\circ - mg \sin 20^\circ
\]

\[
P = \frac{\mu_k mg \cos 20^\circ - mg \sin 20^\circ}{\cos 30^\circ + \mu_k \sin 30^\circ}
\]

\[
P = 2.70 N
\]
A block of mass \( m \) sits on a conical platform. The angle \( \theta \) is small enough that the block does not slide when the platform is at rest. The platform is rotated about its vertical axis of symmetry. In terms of \( \theta \), \( m \), \( g \), \( \mu \), and \( L \), as needed, calculate the maximum tangential speed of the block such that it does not slide with respect to the platform. An accurate and labeled free body diagram and separate force diagram are needed for full credit. Show clearly the second law equations obtained from the diagrams. BE NEAT.

\[
\begin{align*}
\sum F_y &= m\alpha^2 \quad \Rightarrow \quad F_N \cos \theta + F_r \sin \theta - mg = 0 \quad (I) \\
\sum F_x &= m\alpha \quad \Rightarrow \quad F_r \cos \theta - F_r \sin \theta = \frac{m\alpha}{L \cos \theta} \quad (II)
\end{align*}
\]

Substitute \( \mu F_N = F_r \) into \( I \) and \( II \)

\[
\begin{align*}
\sum F_y &= \Rightarrow \quad F_N \cos \theta + \mu F_N \sin \theta - mg = 0 \quad (III) \\
\sum F_x &= \Rightarrow \quad \mu F_N \cos \theta - F_N \sin \theta = \frac{m\alpha}{L \cos \theta} \quad (IV)
\end{align*}
\]
THIRD MIDTERM

Name (print) C. Gundlach Name (signed) Average 16.2/25

Discussion Instructor (circle): Chakhbazian Condella DiCarlo Gundlach Paul Romer Wei

Discussion Section # ________

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!

Use the conversion constants and data given on the front page.

(a) Calculate $\mathbf{A} \cdot \mathbf{B}$ if

$$\mathbf{A} = 9.20\mathbf{i} + 8.70\mathbf{j}$$
$$\mathbf{B} = -3.20\mathbf{i} + 3.70\mathbf{j}$$

$$\mathbf{A} \cdot \mathbf{B} = (9.20)(-3.20) + (8.70)(3.70) = 2.75$$

(b) Convert 62.7 Joules into foot-pounds using the data given.

$$62.7 \text{ Nm} \left( \frac{3.28 \text{ ft}}{1 \text{ m}} \right) \left( \frac{0.231 \text{ lb}}{1 \text{ N}} \right) = 46.3 \text{ ft} \cdot \text{ lb}$$

(c) A non-Hooke's Law spring has the force law $F = kx^2$ (the sign convention is as in Hooke's Law). If $k = 125 \text{ N/m}^2$, calculate the work done to stretch the spring from $x = 0$ to $x = 0.360 \text{ m}$.

$$W = \frac{1}{2} k x^2 = 0.525 \text{ Nm}$$

(d) A car comes to a stop with constant acceleration. If the initial speed is $60.0 \text{ ft/s}$, and the stopping distance $250$ feet, calculate the angle from the vertical for a mass suspended on a string in the car (while it is slowing down). Assume no oscillations, just the steady state.

$$a = g \sin \theta \quad a = \frac{v^2}{2s} \quad \theta = \arctan \left( \frac{\frac{60^2}{2 \cdot 250}}{2 \cdot 125} \right) = 12.6$$

(e) An object weighing 275 pounds is raised vertically $4.20 \text{ m}$. Calculate the work (in Joules) necessary to do this.

$$W = F_s = 275 \text{ lb} \cdot 4.20 \text{ m} \left( \frac{1 \text{ N}}{0.231 \text{ lb}} \right) = 12,000 \text{ Nm} \times 5.13 \times 10^3 \text{ J}$$

\[ \begin{array}{c}
\text{THIRD MIDTERM} \\
\text{Name (print) C. Gundlach Name (signed) Average 16.2/25} \\
\text{Discussion Instructor (circle): Chakhbazian Condella DiCarlo Gundlach Paul Romer Wei} \\
\text{Discussion Section # ________} \\
\text{REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!} \\
\text{Use the conversion constants and data given on the front page.} \\
\text{(a) Calculate $\mathbf{A} \cdot \mathbf{B}$ if} \\
\mathbf{A} = 9.20\mathbf{i} + 8.70\mathbf{j} \\
\mathbf{B} = -3.20\mathbf{i} + 3.70\mathbf{j} \\
\mathbf{A} \cdot \mathbf{B} = (9.20)(-3.20) + (8.70)(3.70) = 2.75 \\
\text{(b) Convert 62.7 Joules into foot-pounds using the data given.} \\
62.7 \text{ Nm} \left( \frac{3.28 \text{ ft}}{1 \text{ m}} \right) \left( \frac{0.231 \text{ lb}}{1 \text{ N}} \right) = 46.3 \text{ ft} \cdot \text{ lb} \\
\text{(c) A non-Hooke's Law spring has the force law $F = kx^2$ (the sign convention is as in Hooke's Law). If} \\
k = 125 \text{ N/m}^2, \text{ calculate the work done to stretch the spring from } x = 0 \text{ to } x = 0.360 \text{ m.} \\
W = \frac{1}{2} k x^2 = 0.525 \text{ Nm} \\
\text{(d) A car comes to a stop with constant acceleration. If the initial speed is } 60.0 \text{ ft/s}, \text{ and the stopping distance } 250 \text{ feet, calculate the angle from the vertical for a mass suspended on a string in the car (while it is slowing down). Assume no oscillations, just the steady state.} \\
a = g \sin \theta \quad a = \frac{v^2}{2s} \\
\theta = \arctan \left( \frac{\frac{60^2}{2 \cdot 250}}{2 \cdot 125} \right) = 12.6 \\
\text{(e) An object weighing 275 pounds is raised vertically } 4.20 \text{ m. Calculate the work (in Joules) necessary to do this.} \\
W = F_s = 275 \text{ lb} \cdot 4.20 \text{ m} \left( \frac{1 \text{ N}}{0.231 \text{ lb}} \right) = 12,000 \text{ Nm} \times 5.13 \times 10^3 \text{ J} \end{array} \]
THIRD MIDTERM

Name (print) ____________________________  Name (signed) ____________________________

Discussion Instructor (circle): Chakhabzian  Condella  DiCarlo  Gundlach  Paul  Romer  Wei

Discussion Section # _______

SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

The diagram is a cross-section of a banked road. The car is traveling into the paper. The road is very icy, so that the coefficients of friction between tires and road are \( \mu_s = 0.20 \) and \( \mu_k = 0.15 \). The road is a curve with radius 400 ft. Take the width of the road as negligible compared to 400 feet.

(a) Show a clear, labeled free-body diagram for the car.
(b) Show a clear, labeled force diagram for the car.
(c) Calculate the minimum speed for this car such that it does not slide off the road.

\[
\begin{align*}
y: & \quad N \cos \alpha + F_t \sin \alpha - mg = 0 \\
x: & \quad N \sin \alpha - F_t \cos \alpha = m \alpha
\end{align*}
\]

\[Q_x = \frac{v^2}{R}\]

\[F_t = \mu_k \cdot N\]

\[
\begin{align*}
N \cos \alpha + \mu_k N \sin \alpha & = mg \\
N (\sin \alpha - \mu_k \cos \alpha) & = m \alpha
\end{align*}
\]

Max speed \( v = \frac{mg}{\alpha \cos \alpha + \mu_k \sin \alpha} \)

\[
v^2 = \frac{\alpha \cos \alpha + \mu_k \sin \alpha}{mg}
\]

\[v = \sqrt{\frac{g}{\alpha \cos \alpha + \mu_k \sin \alpha}} = \frac{33.460 \cdot 0.20 - 0.20 \cdot 0.20}{\cos 0.0 \circ + 0.2 \cdot 0.2 \cdot 0.1} = \sqrt{33.460 \cdot 0.12} = 94.4 \frac{ft}{sec}
\]

\[v = 13.6 \frac{m}{sec}\]
The block of mass \( m = 2.75 \text{ kg} \) is pushed up the inclined plane at constant speed by the force \( F \), directed as shown. \( \mu_s = 0.600, \mu_k = 0.400 \)

(a) Calculate the magnitude of the force \( F \). Clear free-body and force diagrams are a necessary part of this problem for full credit.
(b) Calculate the work done on the block by gravity when it moves 1.27 m up the plane.
(c) Calculate the work done by friction on the block when it moves 1.27 m up the plane.

\[
\begin{align*}
\sum F_x &= F \cos 40^\circ - f_k - mg \sin 30^\circ = 0 \\
\sum F_y &= N - mg \cos 30^\circ - F \sin 40^\circ = 0 \\
\mu_k &= N \mu_k \\
F &= \frac{mg (\sin 30^\circ + \mu_k \cos 30^\circ)}{\cos 40^\circ - \mu_k \sin 40^\circ} = 44.8 \text{ N}.
\end{align*}
\]

(\( b \)). \[ W = -mg \cdot S \sin \theta = -mg \cdot S \sin 30^\circ = -2.75 \times 9.8 \times 1.27 \times \frac{1}{2} \text{ J} = -17.1 \text{ J} \]

(\( c \)). \[ W = -f_k \cdot S = -\mu_k (mg \cos 30^\circ + F \sin 40^\circ) \cdot S \\
= -0.4 (2.75 \times 9.8 \times \cos 30^\circ + 44.8 \times \sin 40^\circ) \times 1.27 \text{ J} \\
= -26.5 \text{ J} \]
THIRD MIDTERM

Name (print) Guifu Chen Name (signed) Average: 15.4

Discussion Instructor (circle): Chakhbaziian Condella DiCarlo Gundlach Paul Romer Wei

Discussion Section #

SHOW ALL WORK!!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

In this system the block, whose mass is given, is launched with an initial velocity \( v_0 \), as shown. The coefficients of friction are given. The spring has a force constant \( k \). The distance shown is from the initial position (where the block has \( V = 0 \)) to the end of the spring when the spring is neither squeezed nor stretched.

USE ENERGY METHODS FOR BOTH PARTS.

\( V_0 = 4.70 \text{ m/s} \)
\( m = 1.25 \text{ kg} \)
\( \mu_k = 0.40 \)
\( \mu_s = 0.60 \)
\( k = 350 \text{ N/m} \)

(a) Calculate the magnitude of the velocity of the mass just before it first touches the spring.
(b) Calculate the TOTAL distance the mass travels before its velocity first becomes zero.

\[
\begin{align*}
\text{(a)} \quad W_{nc} &= F_f - F_i \quad \Rightarrow \quad -f \mu_k mg \cos 35^\circ = \frac{1}{2} m v_i^2 - \left( \frac{1}{2} m v_0^2 + mg s \sin 35^\circ \right) \\
\Rightarrow \quad v_i^2 &= v_0^2 + 2gs \sin 35^\circ - 2gs \mu_k \cos 35^\circ \\
&= v_0^2 + 2gs \left( \sin 35^\circ - \mu_k \cos 35^\circ \right) \\
&= 4.70^2 + 2 \times 9.8 \times 2.25 \left( \sin 35^\circ - 0.4 \cos 35^\circ \right) \approx 32.94 \\
\text{So} \quad v_i &= \sqrt{32.94} \approx 5.74 \text{ m/s}
\end{align*}
\]

\[
\begin{align*}
\text{(b)} \quad W_{nc} &= F_f - F_i \quad \Rightarrow \quad -\mu_k mg \cos 35^\circ = \frac{1}{2} k \left( \frac{1}{2} m v_i^2 + mg x \sin 35^\circ \right) \\
\Rightarrow \quad \frac{1}{2} k x^2 + mg \left( \mu_k \cos 35^\circ - \sin 35^\circ \right)x - \frac{1}{2} m v_i^2 &= 0 \\
\Rightarrow \quad 17.5 x^2 - 3.01 x - 20.58 &= 0 \\
\Rightarrow \quad x_{1,2} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \Rightarrow \quad x_{1,2} = \left\{ 0.35 \text{ (m)} \quad \text{--- physical}, \right. \\
&\quad \left. \quad -0.33 \text{ (m)} \quad \text{--- unphysical}. \right\}
\end{align*}
\]

Total distance = 2.25 + 0.35 = 2.60 m.