Car A is traveling north as shown at 45.0 mi/hr. Car B is traveling in the direction given at 35.0 mi/hr. The mass of A is 2250 kg, and B 3250 kg. They collide in a completely inelastic collision.

(a) Find the velocity (magnitude and direction) just after the collision.
(b) If the coefficient of kinetic friction is 0.55, how far will the wreckage slide?

\[ x \text{ components} \]
\[ m_A v_{b_x} \cos 45^\circ = (m_A + m_B) v_x \]
\[ v_x = \frac{m_A v_{b_x} \cos 45^\circ}{m_A + m_B} = 6.539 \text{ m/s} \]

\[ y \text{ components} \]
\[ m_A v_{b_y} - m_B v_{b_x} \sin 45^\circ = (m_A + m_B) v_y \]
\[ v_y = \frac{m_A v_{b_y} - m_B v_{b_x} \sin 45^\circ}{m_A + m_B} = 1.692 \text{ m/s} \]

\[ v = \sqrt{v_x^2 + v_y^2} = 6.754 \text{ m/s} \]
\[ \Theta = \tan^{-1} \frac{v_y}{v_x} = 14.51^\circ \]

\[ v = 6.75 \text{ m/s at } 14.5^\circ \text{N of E} \]

b) Kinematics
\[ a = \frac{F}{m} = \frac{m u}{m} = \frac{m(2g)}{m} = 2g \]
\[ v^2 - 2a\Delta x \Rightarrow \Delta x = \frac{v^2}{2ug} \]

Energy
\[ \frac{1}{2} (m_A + m_B) v^2 - \mu m N \Delta x = c \]
\[ \frac{1}{2} (m_A + m_B) v^2 - \mu (m_A + m_B) g \Delta x = 0 \]
\[ \Delta x = \frac{v^2}{2ug} = 4.232 \text{m} \]

\[ \Delta x = 4.232 \text{m} \]

Common Errors —

1) About 1/4 the class wrote the y component of momentum as
\[ m_A v_A + m_B v_B \sin 45^\circ = (m_A + m_B) v_y \]
\[ \uparrow \text{ 016} \]
This would correspond to

\[ \theta = 45^\circ \]

This error -4 pts

2) Conservation of Kinetic Energy in a)

\[ \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} (m_A + m_B) v^2 \]

Wrong 0/18 pts

3) A strange conservation of Energy in 5)

\[ \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} (m_A + m_B) v^2 + \int f_k \cdot d \]

Wrong 0/7 pts

4) Treating conservation of momentum as a scalar quantity (part a) \(\rightarrow 4/18\)

5) Algebra errors, calculator errors pts off due to seriousness usually -2 to -5
Mass 1 rests on a frictionless inclined plane, supported by a Hooke's law spring of constant \( k = 111.2 \) N/m. Mass 2 slides down the plane and collides with 1 and sticks to 1. If the maximum compression of the spring is 35.0 cm from its length just before impact, calculate the distance 2 moves from rest before colliding with 1.

\[ M_1 = 1.25 \text{ kg} \]
\[ M_2 = 0.75 \text{ kg} \]

(i) As \( m_2 \) slides down from the top, gravitational potential energy is transformed into kinetic energy [no friction!]:

\[ m_2 g \sin 30^\circ \cdot d = \frac{m_2}{2} v^2 \]

\[ \Rightarrow \quad \frac{v^2}{g} = d \quad (1) \]

(ii) The subsequent collision with \( m_1 \) is inelastic, i.e., part of the kinetic energy \( \frac{1}{2} m_2 v^2 \) is dissipated. We assume the collision to be instantaneous and hence momentum conservation at the moment of impact applies:

\[ (m_1 + m_2) v' = m_2 v \quad (2) \]

where \( v' \) is the speed of \( (m_1 + m_2) \) immediately after the collision.
(iii) The compression \((\Delta x)_1\) of the spring before the collision is given by the balance of forces:

\[
m_1 g \sin \theta = \frac{m_1 g}{2} = k (\Delta x)_1
\]

With \(m_1 = 1.25\) kg and \(k = 111.2\) N/m, we find

\[
(\Delta x)_1 = 5.508 \text{ cm } (\pm 2) \quad (3)
\]

(iv) After the collision, \((m_1+m_2)\) compresses the spring another distance \((\Delta x)_2 = 35.0\) cm. The maximal compression relative the relaxed length of the spring is

\[
(\Delta x)_{\text{max}} = (\Delta x)_1 + (\Delta x)_2 = 40.508 \quad (4)
\]

(v) After the collision we have energy conservation:

\[
\frac{m_1 + m_2}{2} (V')^2 + (m_1 + m_2) \frac{g}{2} (\Delta x)_2 + k (\Delta x)_1^2
\]

\[
= \frac{1}{2} \frac{(\Delta x)_{\text{max}}^2}{2}
\]

This equation is solved numerically for \((V')^2\):

\[
(V')^2 = 5.528 \left( \frac{m}{s} \right)^2
\]

\[
\Rightarrow \quad V^2 = 39.28 \left( \frac{m}{s} \right)^2 \quad \text{eq. (2)}
\]

We substitute this value for \(v^2\) into eq. (1):

\[
d = 4.01 \text{ m}
\]
Car A is traveling east at 45.0 mi/hr. Car B is traveling north at 55.0 mi/hr. The masses of the cars are $M_A = 2200 \text{ kg}$, $M_B = 3750 \text{ kg}$. They collide and the wreckage sticks together.

(a) Calculate the direction the wreckage moves (angle $\theta$ in the diagram).

(b) If the coefficient of friction is taken to be 0.65, how far does the wreckage slide (in feet)?

From momentum conservation law:

$$m_A \vec{v}_A + m_B \vec{v}_B = (m_A + m_B) \vec{V}$$

or

$$(m_A v_A, m_B v_B) = (m_A + m_B) V_x, (m_A + m_B) V_y$$

$$V_x = V \cos \theta = \frac{m_A v_A}{m_A + m_B}, \quad V_y = V \sin \theta = \frac{m_B v_B}{m_A + m_B}$$

If $M_A = 2.2 \times 10^3 \text{ kg}$, $M_B = 3.75 \times 10^3 \text{ kg}$

$v_A = 45.0 \text{ mi/hr}$, $v_B = 55.0 \text{ mi/hr}$

$$\tan \theta = \frac{3.75 \times 10^3 \text{ kg} \times 55.0 \text{ mi/hr}}{2.2 \times 10^3 \text{ kg} \times 45.0 \text{ mi/hr}} = \frac{2.0833}{2.2} = 0.9412$$

$$\theta = \tan^{-1} (0.9412) = 46.35^\circ \approx 46.2^\circ$$

$$V = \sqrt{\frac{2.2 \times 10^3 \text{ kg} \times 45.0 \text{ mi/hr}}{2.2 \times 10^3 \text{ kg} \times 45.0 \text{ mi/hr}}} \sqrt{1 + (0.9412)^2} = \frac{2.2 \times 45.0 \text{ mi/hr}}{82.25} \sqrt{1 + (0.9412)^2} = 38.4 \text{ m/s}$$

From energy conservation law:

$$\frac{V^2}{2} = M(M_A + M_B)g \alpha$$

$$\alpha = \frac{M(M_A + M_B)g V^2}{2 M A B} = \frac{(38.4 \text{ m/s})^2 x (5280 \text{ ft/m})}{2 x 0.65 x 9.8 \text{ m/s}^2 x (60 x 60 \text{ ft/m})^2 x 3.281 \text{ ft/m}} = 76.06 = 76.1 \text{ ft}$$
# students taking test = 308

## FOURTH EXAM

**Graded by:** LINDA J. LAKNER

**Name (Print):**

**Name (Signed):**

\[ \theta = 18.1 \]

\[ \sigma = 6.0 \]

**Part a):** 15 pts. (4 points are left over for math and sig fig errors providing the other 11 points show student has understanding of how to do problem.)

**Part b):** 10 pts. (2 points serve the same function as the \( \frac{4}{5} \) in part a)

**REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!**

**Use the conversion constants and data given on the front page.**

A bullet of mass 28 grams is fired into a block of mass 1.35 kg. The velocity of the bullet is 275 m/s and it stops inside the block. The block \( M \) is part of a pendulum of pendulum of length \( L = 2.00 \text{ m} \), as shown.

(a) Calculate the maximum value of the angle \( \theta \) that the pendulum achieves.

(b) Calculate the tension in the string when \( \theta = \theta_{\text{max}}/2 \).

\[ \text{Before Collision} \ (\text{sticky collision = conservation of momentum only}) \]

\[ \begin{align*}
V_a &= \frac{m_b V_b}{m_b + M} \\
&= \frac{m_b V_b}{m_b + M}
\end{align*} \]

\[ \text{As Pendulum Block with Bullet in it (all other forces are negligible = conservation of mechanical energy)} \]

\[ \frac{1}{2} M v_a^2 + 0 = 0 + M g h_{\text{max}} \rightarrow h_{\text{max}} = \frac{v_a^2}{2g} \]

\[ \cos \theta_{\text{max}} = 1 - \frac{v_a^2}{2g} - 1 = \left( \frac{m_b V_b}{M} \right)^2 - \frac{1}{2g} \]

\[ \theta_{\text{max}} = \cos^{-1} \left[ 1 - \left( \frac{m_b V_b}{M} \right)^2 \frac{1}{2g} \right] \]

\[ \theta_{\text{max}} = 78.3^\circ \]

**Common Errors:**

\[ \text{(Part a)} \] Tried to do problem, without breaking up into 3 parts. Must have a collision part and a pendulum swinging part. Why? Because without knowing how much energy was dissipated in collision (heat, noise, friction, etc.) mechanical energy is not conserved, so must use collision approximation.

\[ m_b v_b^2 \neq M v_a^2 + M h_{\text{max}} \]

\[ \text{Also, in a sticky (inelastic) collision only momentum is conserved, not kinetic energy too.} \]

**Part b)** Must realize \( \theta_{\text{max}} \) in direction of \( \vec{F} \) is not pointing in horizontal direction as this would mean the pendulum is swinging in a horizontal instead of a vertical circle. Also \( V_a \) was incorrectly solved for: 1) \( V_a \neq V_{\text{max}}^2 \)

\[ \frac{1}{2} M v_a^2 \neq M g h_{\text{max}} \]

\[ \theta = 0 \]

\[ \vec{a} = \vec{F} \]

\[ T = \frac{1}{2} \left[ \frac{1}{2} (m_b v^2) + 3g \cos \theta - \frac{2g}{2} \right] \]

\[ T = 25.9 \text{ N} \rightarrow \]

\[ \vec{a} = \vec{F} \]

\[ \text{Since this would mean } M_r \text{ is } \vec{F} \text{ in space (an unlikely possibility! Also } V_a \text{ was incorrectly solved for: 1) } V_a \neq V_{\text{max}}^2 \]

\[ 2) \frac{1}{2} M v_a^2 \neq M g h_{\text{max}} \]

\[ 3) h_c \neq h_{\text{max}} \]
PROBLEM 2

(a) Find the y coordinate of the center of mass of the object shown. The object is a sheet of metal of density \( \rho \) and thickness \( t \).

(b) Find the mass of this object.

Solution:

(a) 
\[
\bar{y} = \frac{\int y \, dm}{\int dm}
\]

where 
\[
dm = \rho t (x_0 - x) \, dy = \rho t (x_0 - \frac{4\sqrt{a}}{a}) \, dy
\]

Thus 
\[
\int y \, dm = \rho t \int_0^{ax_0^4} y (x_0 - \frac{4\sqrt{a}}{a}) \, dy = \frac{\rho at^2 x_0^9}{18}
\]

\[
\int dm = \rho t \int_0^{ax_0^4} (x_0 - \frac{4\sqrt{a}}{a}) \, dy = \frac{a \rho t x_0^5}{5}
\]

Therefore, we find
\[
\bar{y} = \frac{\int y \, dm}{\int dm} = \frac{\frac{\rho at^2 x_0^9}{18}}{\frac{a \rho t x_0^5}{5}} = \frac{5a}{18} x_0^4
\]

(b) 
\[
M_{\text{total}} = \int dm = \rho t \int_0^{ax_0^4} (x_0 - \frac{4\sqrt{a}}{a}) \, dy = \frac{a \rho t x_0^5}{5}
\]

\[
M_{\text{total}} = \int dm = \rho t \int_0^{x_0} y \, dx = \rho t \int_0^{x_0} ax^4 \, dx = \frac{a \rho t x_0^5}{5}
\]
Calculate the x coordinate of the center of mass of the object shown. The object is the lined region. It is a sheet of thickness \( t \), and density \( \rho \).

\[
X_{cm} = \frac{\int x \, dm}{\int dm}
\]

\[
dm = \rho t (B-y) \, dx = \rho t (B-ax^3) \, dx
\]

\[
limit \quad 0 \to (\frac{B}{3})^{\frac{1}{3}}
\]

\[
\int_{0}^{(\frac{B}{3})^{\frac{1}{3}}} \frac{1}{(Bx-ax^4)} \, dx
\]

\[
X_{cm} = \frac{\int_{0}^{(\frac{B}{3})^{\frac{1}{3}}} x \, (Bx-ax^4) \, dx}{\int_{0}^{(\frac{B}{3})^{\frac{1}{3}}} (Bx-ax^4) \, dx}
\]

\[
= \frac{\frac{B^2}{2} - \frac{A^5}{5}}{\frac{B}{3} \cdot \frac{A^4}{4}}
\]

\[
= \frac{\frac{2}{5} \cdot \frac{B^2}{A^2} - \frac{1}{5} \cdot \frac{B^3}{A^3}}{\frac{B}{3} \cdot \frac{A^4}{4}}
\]

\[
= \frac{2}{5} \cdot (\frac{B}{A})^{\frac{1}{3}}
\]
The cross hatched figure shown has a thickness $t$, a uniform density mass $\rho$ and is bounded by the horizontal $x$ axis, the line $x=x_0$ and the curve $y^2=x$.

(a) Calculate the $x$ coordinate of the center of mass of the figure shown. (Express your answer as a number times $x_0$.)

(b) (Harder) Calculate the $y$ coordinate of the center of mass of the same object. (Express your answer in terms of $x_0$.)

\[
\begin{align*}
\bar{x} & = \frac{\int x \, dm}{\int dm} \\
& = \frac{\int_{x_0}^{x_0} x \rho \sqrt{x} \, dx}{\int_{x_0}^{x_0} \rho \sqrt{x} \, dx} \\
& = \frac{\frac{2}{5} x_0^{5/2}}{\frac{2}{3} x_0^{3/2}} \\
& = \frac{3}{5} x_0
\end{align*}
\]

\[
\begin{align*}
\bar{y} & = \frac{\int y \, dm}{\int dm} \\
& = \frac{\int_{y=0}^{y=x_0} y \cdot (x_0-y^2) \, dy}{\int_{y=0}^{y=x_0} dy} \\
& = \frac{\frac{3}{8} \sqrt{x_0}}{\frac{2}{3} x_0^{3/2}} \\
& = \frac{3}{8} \sqrt{x_0}
\end{align*}
\]
A steady stream of water strikes a scale and bounces off as shown. The magnitude of the velocity of the water stream is unchanged. The stream of water flows at 7.50 kg/s, and a speed of 22.3 m/s.

(a) What does the scale read (in N)?

(b) A bucket that can hold exactly 6.00 kg of water is placed on the scale. The same hose is directed into this bucket in the same direction. What does the scale read at the instant the bucket is full?

\[ \text{(a) Consider the effect of a small piece of mass } \Delta m: \]

\[ p_{oy} = -\Delta mv \sin \theta \]

\[ p_{fy} = \Delta mv \sin \theta \]

\[ \frac{\Delta p}{\Delta t} = \frac{p_{f} - p_{i}}{\Delta t} = \frac{\Delta mv \sin \theta + \Delta mv \sin \theta}{\Delta t} \]

\[ = \frac{2\Delta mv \sin \theta}{\Delta t} \rightarrow 2 \frac{dm}{dt} \sin \theta \text{ as } \Delta t \to 0 \]

\[ F_{net} = N_{scale} = \frac{dp}{dt} \]

\[ N_{scale} = 2mv \sin \theta = 141 \text{ N} \quad \text{when } \mu = \frac{dm}{dt} \]

\[ \text{(b) } p_{oy} = -\Delta mv \sin \theta \]

\[ p_{fy} = 0 \]

\[ \frac{\Delta p}{\Delta t} = \frac{\Delta mv \sin \theta}{\Delta t} \rightarrow \frac{dm}{dt} \sin \theta \text{ as } \Delta t \to 0 \]

\[ F_{net} = N_{scale} - mg = \frac{dp}{dt} \]

\[ N_{scale} = mg + \mu mv \sin \theta = 129 \text{ N} \]
A mass $m_1$ is attached to the ceiling by a Hooke's law spring with spring constant $k$. It is initially at rest. A second mass, $m_2$, is given an initial velocity $v_0$. It strikes $m_1$ and sticks to it after $m_2$ has moved a distance $h$ vertically. Calculate the maximum vertical distance traveled by $m_2$ after the collision. (Numerical answer.)

1. **Conservation of Energy**
   
   \[-m_1gh + \frac{1}{2}m_2v_0^2 = \frac{1}{2}(m_1+m_2)v'^2\]
   \[v_0 = \sqrt{\frac{v'^2 - 2gh}{100 - 2(9.8)(2.35)}} = 7.34 \text{ m/s} \]

2. **Conservation of Momentum**
   
   \[m_2v = (m_1 + m_2)v' \Rightarrow v' = \frac{m_2v}{m_1 + m_2} = 3.02 \text{ m/s} \]

3. **Conservation of Energy**
   
   \[\frac{1}{2}(m_1+m_2)v'^2 + \frac{1}{2}kx^2 = (m_1+m_2)g(d-x) + \frac{1}{2}k(d-x)^2\]
   
   where $x$ = distance by which spring is initially stretched.
   \[kx = m_1g \Rightarrow x = \frac{m_1g}{k} = 9.8 \times 10^{-2} \text{ m} \]

   \[d \text{ must solve this quadratic in } \frac{d}{m} \text{ which reduces to} \]

   \[d^2 + d \cdot 2\left[\left(\frac{m_1+m_2}{k}\right)g - x\right] - \frac{(m_1+m_2)K}{k}v'^2 = 0 \]
   \[d = -0.137 \pm \sqrt{0.137^2 + 4 \left(\frac{0.155}{0.031}\right)} \]

   \[d = 0.231 \text{ m} \]

This answer comes from taking positive root, assuming $d$ positive from diagram.
THIRD MIDTERM

Name (print) Ludi Baserga Name (signed) L. Baserga

Discussion Instructor (circle one): Baselgia Morrill Reeve Stoops Zhang

Discussion Section #

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) Convert 2.30 horsepower into watts.

\[
\text{2.30 horsepowers} \times \frac{550 \text{ ft-lbs}}{1 \text{ horsepowers}} \times \frac{1 \text{ W}}{3.28 \text{ ft-lbs}} \times \frac{\text{W}}{0.225 \text{ pounds}} = 1.71 \times 10^3 \text{ W}.
\]

(b) Convert 625 foot-pounds into joules. (You will have to work out the conversion factor from the definitions and the ones given.)

\[
\begin{align*}
(1) \quad 625 \text{ ft-lbs} &= 625 \text{ ft-lbs} \times \frac{1 \text{ W}}{3.28 \text{ ft-lbs}} \times \frac{\text{W}}{0.225 \text{ pounds}} = 847 \text{ J} \\
(2) \quad 625 \text{ ft-lbs} &= 625 \text{ ft-lbs} \times \frac{1 \text{ W}}{3.28 \text{ ft-lbs}} \times \frac{\text{W}}{0.225 \text{ pounds}} \times \frac{1 \text{ kg} \cdot \text{m}^2}{3.600 \times 10^3 \text{ ft}} = 848 \text{ kg} \cdot \text{m}^2 = 848 \text{ J}
\end{align*}
\]

(c) A car is moving at 50.0 mi/hr. If it has a mass of 1750 kg, calculate its linear momentum in SI units (metric).

\[
\begin{align*}
\mathbf{p} &= m \cdot \mathbf{v} = 1750 \text{ kg} \times 22.36 \text{ m/s} = 3.91 \times 10^4 \text{ kg} \cdot \text{m/s}
\end{align*}
\]

(d) A baseball is thrown at 45.0 m/s. The batter hits it directly back towards the pitcher at 65.0 m/s. If the mass of the ball is 0.300 kg, calculate the magnitude of the impulse given to the ball.

\[
\begin{align*}
\mathbf{I} &= \mathbf{p_f} - \mathbf{p_i} \\
&= m (v_f - v_i) \\
&= 0.300 \text{ kg} (45 \text{ m/s} - (-65 \text{ m/s})) \\
&= 33.0 \text{ kg} \cdot \text{m/s}
\end{align*}
\]

(e) A car of mass 2100 kg is traveling at 60.0 mi/hr. The brakes are put on and it skids to a stop in 418 ft. Find the power being dissipated as heat (in watts) after it has skidded 400 ft, assuming constant deceleration.

\[
\begin{align*}
\mathbf{P} &= \mathbf{F} \cdot \mathbf{v} = ma \cdot v \\
&= \frac{\Delta v^2 - v_i^2}{2(y-x)} \\
&= \frac{-v_i^2}{2(y-x)} = \frac{\text{W}}{\text{m}} \quad \text{after } u \quad \text{ ft}
\end{align*}
\]

\[
\begin{align*}
\Delta v &= 0 - 45 \text{ m/s} = 45 \text{ m/s} \\
\frac{v^2}{2} &= \frac{1}{2} \frac{1170 \text{ ft}}{3.28 \text{ ft/m}} \cdot \frac{1 \text{ m}}{3.28 \text{ ft}} \cdot \frac{1 \text{ ft}}{3.28 \text{ ft}} = 26.83 \text{ m/s} \\
x - x_i &= 418 \text{ ft} \\
45 \text{ m/s} &= 129.4 \text{ m}
\end{align*}
\]

\[
\begin{align*}
\mathbf{P} &= \mathbf{F} \cdot \mathbf{v} = ma \cdot v = \frac{(468.5 \text{ ft} \cdot \text{sec})^2}{(9.3 \times 10^3 \text{ ft})} = 530 \text{ W/m}
\end{align*}
\]
Find speed $v_f$ after the car had traveled 400 ft.

\[ v_f^2 = v_i^2 + 2a(x - x_0) \]

- $v_i = 26.83 \text{ m/s}$
- $a = -2.825 \text{ m/s}^2$
- $x - x_0 = 400 \text{ ft} = 400 \text{ ft} \cdot \frac{1 \text{ m}}{3.28 \text{ ft}} = 122.0 \text{ ft}$

\[ \Rightarrow v_f^2 = \sqrt{v_i^2 + 2a(x - x_0)} \]
\[ = \sqrt{(26.83 \text{ m/s})^2 + 2 \cdot (-2.825 \text{ m/s}^2) \cdot 122.0 \text{ ft}} \]
\[ = 5.527 \text{ m/s} \]

Then, $P = ma \cdot v_f^2 = 2100 \text{ kg} \cdot 2.825 \text{ m/s}^2 \cdot 5.527 \text{ m/s}$
\[ = 3.28 \cdot 10^6 \text{ kg m}^2 \text{/s} \]
\[ = 3.28 \cdot 10^6 \text{ Watts} \]
THIRD MIDTERM

A car on a frictionless roller-coaster is released from rest at a height $h$ as shown. At the top of the hump the radius of the curvature of the track is $R$. The height of the top of the hump is shown in the diagram.

(a) If the apparent weight of a person in the car is $1/3$ of his normal weight at the top of the hump (point B), calculate $h$ in terms of $R$, $g$ and numbers. The person and car are assumed small compared to $R$.

(b) For the same starting conditions, calculate the apparent weight of a $100$ kg person at point C, if the radius of the curvature at C is $2R$.

\[ \text{Energy at } b): \, \text{mgh} = \frac{1}{2} \text{mv}^2 + \text{mg} \cdot 2R \]

Part a) \[ \text{mgh} = \frac{1}{2} \text{mv}^2 + \text{mg} \cdot 3R \quad (+5 \text{ if correct}) \]

\[ \text{mg} - N = \frac{\text{mv}^2}{R} \quad (+5 \text{ if correct}) \]

\[ N = \frac{1}{3} \text{mg} \quad \Rightarrow \text{mv}^2 = (\text{mg} - \frac{1}{3} \text{mg})R = \frac{2}{3} \text{mg} \cdot R \]

\[ \text{mgh} = \frac{1}{2} \left( \frac{2}{3} \text{mg} \cdot R \right) + \text{mg} \cdot 3R \quad \Rightarrow h = \frac{10}{3} R \quad (+5 \text{ if correct}) \]

Part b) \[ \text{mgh} = \frac{1}{2} \text{mv}^2 \quad \text{mv}^2 = 2 \text{mgh} = \frac{20}{3} \text{mg} \cdot R \quad (+3 \text{ if correct}) \]

\[ N - \text{mg} = \frac{\text{mv}^2}{2R} \quad (+3 \text{ if correct}) \quad N = \left( \frac{10}{3} + 1 \right) \text{mg} = \frac{13}{3} \text{mg} = 433 \text{ N} \text{ or } 4.33 \text{ kN} \]

\[ \text{A}_{\text{avg}} = 15.41 \text{ cm} \]
The object shown has a uniform density $\rho$, and a constant thickness $t$ in the $z$ direction. Calculate the $x$-coordinate of the center of mass in terms of $A$, $B$ and numbers, as needed.

$$x_{cm} = \frac{\int x \, dm}{\int dm} \quad \text{where} \quad dm = \rho \, dV$$

$$dV = t \, y \, dz = t(A-Bx^2) \, dx$$

$$\int x \, dm = \int_0^{\frac{A}{B}} x \rho t(A-Bx^2) \, dx$$

$$= \rho t \int_0^{\frac{A}{B}} (A-Bx^2) \, dx = \rho t \left[ \frac{Ax^2}{2} - \frac{Bx^3}{4} \right]_0^{\frac{A}{B}}$$

$$= \rho t \left[ \frac{A^2}{2B} - \frac{A^2}{4B} \right] = \rho t \frac{A^2}{B} \cdot \frac{1}{4}$$

$$\int dm = \int_0^{\frac{A}{B}} \rho t(A-Bx^2) \, dx = \rho t \left[ A - \frac{Bx^2}{3} \right]_0^{\frac{A}{B}}$$

$$= \rho t \frac{A^2}{B^3} \cdot \frac{2}{3}$$

$$x_{cm} = \frac{\rho t \frac{A^2}{B} \cdot \frac{1}{4}}{\rho t \frac{A^{3/2}}{B^{3/2}} \cdot \frac{2}{3}} = \frac{3}{8} \left( \frac{A}{B} \right)^{3/2}$$
THIRD MIDTERM

Name (print)  Mark Reeve  Name (signed)  Mark Reeve

Discussion Instructor (circle one): Baselgia  Morrill  Reeve  Stoops  Zhang

Discussion Section #

SHOW ALL WORK!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

When mass A is placed on a long Hooke's law spring and allowed to come to rest, the spring is squeezed 21.0 cm from its equilibrium length. A mass B is given an initial velocity $v_o = 6.20$ m/s downwards, at a point 3.75 m above A. When B strikes A they stick together. Use energy methods where appropriate.

(a) How much kinetic energy is lost in the collision of A and B (numerical answer)?
(b) Calculate the maximum displacement of the spring from the point labeled $y = 0$ after the collision (numerical answer).

$m_A = 4.12$ kg
$m_B = 1.75$ kg
$v_o = 6.20$ m/s
$y_o = 0.210$ m

Now conserve momentum across the inelastic collision:

$$m_Av_{Ax} = (m_A + m_B)v_{Ax}$$

where $v_{Ax}$: velocity of A and B immediately after the collision

$$v_{Ax} = \frac{m_Av_{Ax}}{m_A + m_B} = \frac{4.12 \times (1.75)}{4.12 + 1.75} = 3.15$ m/s

Now calculate the kinetic energy lost:

$$\Delta KE = KE_{after} - KE_{before} = \frac{1}{2}(m_A + m_B)v_{Ax}^2 - \frac{1}{2}m_Bv_o^2$$

$$= -68.7$ Joules

b) After the collision, energy is conserved. There are two main ways to write the energy equation. Let $y_A$ be the additional distance the spring is compressed. Define $E_k$ as the kinetic energy at the bottom of the motion.

$$(m_A + m_B)g(0) + \frac{1}{2}k(0)^2 + \frac{1}{2}(m_A + m_B)v_{Ax}^2 = \frac{1}{2}k(y_A + 0)^2$$

$$0.81$$

First find $k$: $A$ The forces must balance, since $W_{mag}$ is at rest initially.

$$k = \frac{4.12 \times 9.8}{0.210} = 192.27 N/m$$

Use this to find $E_k$:

$$E_k = \frac{1}{2}k(2y_A + 0)^2$$
Problem 4

Rewrite the first energy equation from previous page.

\[
(m_a + m_b)g Ay + \frac{1}{2} k x^2 + \frac{1}{2} (m_a + m_b) v_{0b}^2 = \frac{1}{2} k (y_0 + Ay)^2 = \frac{1}{2} k (y_0^2 + 2y_0 Ay + Ay^2)
\]

\[
\Rightarrow \quad Ay^2 \left( \frac{1}{2} k \right) + Ay \left( k y_0 - g(m_a + m_b) \right) - \frac{1}{2} (m_a + m_b) v_{0b}^2 = 0
\]

\[
\Rightarrow \quad 96.14 Ay^2 - 17.15 Ay - 24.20 = 0
\]

Use \( Ay = \frac{\pm \sqrt{b^2 - 4ac}}{2a} \) and chose the \( + \) radical since as I have defined gravitational potential energy, \( Ay > 0 \)

3 pts

\[ Ay = 0.647 \text{ m} \]

(-2 pts. if you chose the radical inconsistently)

Common Errors

1) Not finding \( V_{0b} \), the velocity of \( B \) just before impact.

2) Trying to use energy to find \( k \), i.e. \( k y_0 = \frac{1}{2} k y_0^2 \).

This is very close, even \( k \) will not be Shirley in this case. In the statement of the problem it assumes that the mass is allowed to come to rest. If \( A \) came to rest, none of the energy is dissipated (unless the elastic term is done quite suddenly in which case \( A \) can occur also). The above equality equation is correct for the maximum displacement of the spring, assuming both were dropped from same altitude on to \( A \). That is not the situation here.

3) Ignoring the initial potential energy of the compressed spring. This approach would be valid only if the energy were linear in the displacement.