A block, of mass 1.75 kg, slides down the plane shown. The coefficients of friction are $\mu_S = 0.470$, $\mu_K = 0.415$. The block starts at rest and slides 1.25 m before striking a spring of spring constant $k = 130$ N/m. Calculate the amount by which the spring is compressed when the block is brought to a stop.

**Initial Energy = Final Energy + Work**

$$mg(1.25 + x)\sin35^\circ = \frac{1}{2}kx^2 + \mu_k mg\cos35^\circ (1.25 + x)$$

$$\Rightarrow (20 \text{ pts.})$$

$$x^2 + \frac{2mg(\mu_k\cos35^\circ - \sin35^\circ)}{k}x + \frac{2mg(\mu_k\cos35^\circ - \sin35^\circ)}{k} = 0$$

$$x^2 + \frac{2 \times 1.75 \times 9.80 (0.415 \cos35^\circ - \sin35^\circ)}{130}x + \frac{2 \times 1.75 \times 9.80 (0.415 \cos35^\circ - \sin35^\circ)}{130} = 0$$

$$x^2 - 0.0616x - 0.0771 = 0$$

$$x = \frac{-0.0616 \pm \sqrt{0.0616^2 + 4 \times 0.0771}}{2}$$

$$= 0.310 \text{ (m)}$$

$$10 \text{ (pts.)}$$

(The negative root is ignored.)
THIRD EXAM

SHOW YOUR WORK!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A constant force, \( F = 40.0 \text{ N} \), is applied to the massless string. A block starts at rest. Calculate:

(a) The kinetic energy of the block after it has travelled 1.75 m up the plane.
(b) The work done by friction while the block is traveling 1.75 m up the plane.
(c) The work done by gravity on the block while traveling 1.75 m up the plane.

\[ \mu_s = 0.65 \]
\[ \mu_k = 0.60 \]
\[ m = 3.25 \text{ kg} \]
\[ \theta = 25.0^\circ \]

\[ \mu_s \]

\[ \mu_k \]

\[ m \]

\[ \theta \]

\[ a = \frac{F}{m} - g \left( \sin \theta + \mu_k \cos \theta \right) \]

\[ v^2 - v_0^2 = 2ax \]

\[ \frac{m}{2} v^2 = ma x = \left[ F - mg(\sin \theta + \mu_k \cos \theta) \right] x \]

\[ F = 40.0 \text{ N}, m = 3.25 \text{ kg}, \mu_k = 0.60, \theta = 25.0^\circ \text{ and } g = 9.80 \text{ m/s}^2 \]

\[ m v^2 = \left[ 40.0 - 3.25 \times 9.80 \left( \sin 25.0^\circ + 0.60 \times \cos 25.0^\circ \right) \right] \times 1.75 \text{ m} \]

\[ = 16.13 \text{ J} \approx 16.1 \times 10^{-1} \text{ J} \]

\[ W_{friction} = -(\mu_k mg \cos \theta)x \]

\[ = -0.6 \times 3.25 \times 9.80 \times \frac{1}{2} \times 25.0^\circ \times 1.75 \text{ m} = -30.34 \text{ J} \]

\[ \approx -3.03 \times 10^{-1} \text{ J} \]

\[ W_{gravity} = -(mg \sin \theta)x \]

\[ = -3.25 \times 9.80 \times \frac{1}{2} \times 25.0^\circ \times 1.75 \text{ m} = -23.56 \text{ J} \]

\[ \approx -2.36 \times 10^{-1} \text{ J} \]

\[ 0.50 \]
A massless Hooke's Law spring has an unstretched length of 2.25 m. When a 10.0 kg mass is placed on it, and slowly lowered until the mass is at rest, the spring is squeezed to 2.00 m length. The same 10.0 kg mass is dropped from a height of 6.25 m above the spring.

(a) What is the maximum value of the compression of the spring? (Numerical answer)

(b) What is the velocity of the block after the spring has been compressed 0.25 m. (Numerical answer).

\[ E_A = E_B \]
\[ m \ddot{x} = -kx \]
\[ m = 10 \text{kg} \]
\[ F = mg \]
\[ mg = -kx \]
\[ \frac{mg}{x} = k \]

\[ 10x = \frac{9.8}{(9.8/x)} = k \]
\[ -k(0.25) \]
\[ k = 98 \text{N/m} \]
\[ k = 392 \text{N/m} \]

\[ \frac{mg}{k} = \frac{9.8}{392} = 0.025 \]

\[ x_1 = 0.025 \]

\[ x_2 = \frac{12.5 \text{mg} - \sqrt{12.5 \text{mg}^2 - 4 \cdot 0.5 \cdot 12.5}}{2 \cdot 0.5} \]

\[ x_2 = \frac{12.5 \text{mg} + \sqrt{12.5 \text{mg}^2 - 4 \cdot 0.5 \cdot 12.5}}{2 \cdot 0.5} \]

\[ X_1 = 0.025 \pm 1.785 \]

\[ X_2 = 0.045 \text{m} \]

\[ X_2 = \pm 0.045 \text{m} \]

\[ 0.025 \pm 1.785 \]

\[ X_1 = 2.04 \text{m} \]

\[ X_2 = 0.59 \text{m} \text{ (extraneous)} \]

\[ +10 \text{ for } X_1 \]
Problem 3 continued

(b) Find \( v_{\text{block}} \) when \( x = 0.25 \text{ m} \)

\[ E_{A'} = E_{B'} \]

\( m g \cdot H' = \frac{1}{2} m v_B^2 + \frac{1}{2} k x^2 \)

\[ H' = 6.25 + x \]

\[ x = 0.25 \text{ m} \text{ (given in statement c of problem)} \]

\[ . \quad H' = 6.5 \text{ m} \]

\[ m g (6.5) = \frac{1}{2} m v_B^2 + \frac{1}{2} k (0.25)^2 \]

From part (a):

\[ m = 10 \text{ kg} \]

\[ k = 392 \text{ N/m} \]

\[ m g = 98 \text{ N} \]

\[ (98 \text{ N})(6.5) = \frac{1}{2} (10) v_B^2 + \frac{1}{2} (392)(0.25)^2 \]

\[ 637 = 5 v_B^2 + 12.25 \]

\[ v_B^2 = \sqrt{\frac{637 - 12.25}{5}} \]

\[ v_B = 11.2 \text{ m/s} \]  

\( \boxed{v_B = 11.2 \text{ m/s}} \)

\( +10 \) for \( v_B \)

\( \boxed{10} \) for \( v_B \)
Problem 3 still continued

An alternate method

A method for finding $k$ is the same as in the first method.

Find velocity at $\Theta$.

\[ \gamma = 0.25m \]
\[ a = g \]
\[ \gamma = 0 \]
\[ \gamma = ? \]

\[ \gamma = g (6.25m) \]
\[ \gamma = \sqrt{(9.8)(6.25)} \]
\[ \gamma = 11.17 \text{ m/s} \]

Now,

\[ E_B = E_\theta \]
\[ \frac{1}{2} m \gamma^2 A^2 + mgh \]
\[ h = x \]
\[ mg = 96N \]
\[ m = 10 \]
\[ k = 392 \]

\[ 5 \gamma^2 A^2 + 98x = 5 \gamma^2 A^2 + 196x^2 \]

but $\gamma_A = c$

\[ 5 \gamma^2 A^2 + 98x = 196x^2 \]

\[ 5(11.17)^2 + 98x = 196x^2 \]

\[ 612.67 + 98x = 196x^2 \]

\[ 3 \cdot 12.6 + 98x = x^2 \]

\[ x^2 - 98x - 3.126 = 0 \]

which is identical to the quadratic equation in the first method.

\[ x = 2.64m \]
Problem 3 again still continued

In alternate method part (b)

b) Find \( v_{\text{block}} \) at point B'

From part (a):

\[ v_B = 11.0 \text{ m/s} \]

In 3 sig. figs., this is 11.1 m/s. This is \textbf{NOT} the answer. Only 11.2 m/s will be accepted as correct.

So:

\[ E_B = E_{B'} \]

\[
\left( \frac{1}{2} m v_B^{-2} \right) + mgy = \frac{1}{2} m v_{B'}^{-2} + \frac{1}{2} k x^2
\]

\[
h = x = 0.25 \text{ m}
\]

\[
m = 10 \text{ kg}
\]

\[
m g = 98 \text{ N}
\]

\[
k = 392 \text{ N/m}
\]

\[
\left\{ \begin{array}{l}
\frac{1}{2} \left( 10 \times 11.2^2 \right) + (98 \times 0.25) = \frac{1}{2} \left( 10 \right) v_{B'}^{-2} + \frac{1}{2} (392 \times 0.25)^2 \\
612.7 + 24.15 = 5 v_{B'}^{-2} + 12.25 v_{B'}^{-1}
\end{array} \right.
\]

\[
\Rightarrow \quad v_{B'}^{-2} = \frac{1}{5} \left( 624.7 v_{B'}^{-1} \right)
\]

\[
\Rightarrow \quad v_{B'}^{-1} = 11.2 \text{ m/s}
\]

\[ +10 \quad \text{for} \quad v_{B'}^{-1} \]
(a) A car weighing 2000 pounds is moving at 30.0 m/hr. Calculate its linear momentum in SI units.

\[ 1.216 \times 10^4 \text{ kg} \cdot \text{m/s} \]

(b) Calculate the x-coordinate of the center of mass of the system shown.

\[ 2.09 \text{ m} \quad 2.12 \]

(c) A golf ball at rest is struck and leaves with a velocity of 110 m/s. If the mass of the ball is 0.075 kg, calculate the impulse applied to the ball.

\[ 8.25 \text{ kg} \cdot \text{m/s} \quad 8.24 \]

(d) A car of mass 1000 kg is traveling west at 15.0 m/s. A second car of mass 1500 kg is traveling east at 20.0 m/s. If they collide head-on and stick together, calculate the kinetic energy lost in the collision.

\[ 3.68 \times 10^5 \text{ J} \quad 3.69 \]

(e) Calculate the kinetic energy, in joules, of the car in (a).

\[ 8.15 \times 10^5 \text{ J} \quad 8.16 \]
FORTH MIDTERM

Name (print) HARI __________________________ Name(signed) ________

Discussion Instructor (circle one): Hamed Hari Molina Nott Paul Reeve Zhang

Discussion Section # ______

SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A sphere m of mass 0.750 kg is attached to a massless rod whose length is 1.45 m. The rod is pivoted at P so that it moves in a vertical plane. When the mass is directly above P it is given an initial velocity of \( v_0 = 2.55 \) m/s.

(a) Calculate the velocity of m when it is directly below P.
(b) Determine the tension in the rod when it is at 30° from the vertical as shown by the dotted lines.

\[ E = m g a + \frac{1}{2} m v_0^2 \]

at the top

\[ E = m g l + \frac{1}{2} m v^2 \]

Conservation of energy:

\[ m g a + \frac{1}{2} m v_0^2 = m g l + \frac{1}{2} m v^2 \]

\[ v^2 = v_0^2 + 2 g l \]

\[ v = \sqrt{v_0^2 + 2 g l} = 7.96 \text{ m/s} \]

What is the velocity at \( \theta = 30° \)?

\[ \frac{1}{2} m v^2 = \frac{1}{2} m u^2 + m g l (1 - \cos 30) \]

\[ \frac{v^2}{2} = \frac{u^2}{2} + g l (1 - \cos 30) \]

\[ \frac{u^2}{2} = \frac{v^2}{2} - g l (1 - \cos 30) \]

\[ u = \sqrt{v^2 - 2 g l (1 - \cos 30)} = 7.92 \text{ m/s} \]

\[ T - m g \cos 30 = \frac{m u^2}{l} \]

\[ T = (m g \cos 30 + \frac{u^2}{l}) = 37.2 \text{ N} \]

\[ T = (m g \cos 30 + \frac{m u^2}{l}) = 37.2 \text{ N} \]

\[ \text{Ans.} \]

\[ 4 \text{ m/s} \]
A mass of \( m = 2.55 \, \text{kg} \) is suspended at rest by a massless spring as shown. When the mass is attached to the unstretched spring it stretches the spring by 1.45 cm. A bullet is fired from below so that it strikes the mass and remains in it. If the mass of the bullet is 23.0 grams and the velocity of the bullet when it enters the block is 285 m/s, calculate the maximum height of the block from \( y = 0 \) in the drawing.

\[
V = \frac{m_b v_b}{m + m_b} = 2.548 \, \text{m/s}
\]

\[
k = \frac{mg}{x} = \frac{(2.55 \, \text{kg})(9.8 \, \text{m/s}^2)}{(0.0145 \, \text{m})} = 1723 \, \text{N/m}
\]

\[
\frac{1}{2}(m + m_b)V^2 + \frac{1}{2}kx^2 = (m + m_b)gy + \frac{1}{2}k(y-x)^2
\]

(using \( y^2 \) instead of \((y-x)^2\) was a common error)

\[
\Rightarrow \frac{1}{2}(m + m_b)V^2 + \frac{1}{2}kx^2 = (m + m_b)gy + \frac{1}{2}ky^2 - kxy + \frac{1}{2}kx^2
\]

Solve the quadratic equation, retain the solution:

\[
y = 9.83 \times 10^{-2} \, \text{m} = 9.83 \, \text{cm}
\]

An alternative admissible interpretation of the problem is to use \( 0.0145 \, \text{m} \) as the maximal stretch, is

\[
\frac{1}{2}kx^2 = mgx \Rightarrow k = \frac{2mg}{x} = 3.45 \times 10^3 \, \text{N/m}
\]

Then,

\[
y = 7.72 \times 10^{-2} \, \text{m} = 7.72 \, \text{cm}
\]
Calculate the y-coordinate of the center of mass of the object shown. The object is a uniform sheet of thickness \( t \) and density \( \rho \).

\[
\overline{y} = \frac{\int_0^B y (x dy) \rho}{\int_0^B (x dy) \rho}
\]

\[
x = \left( \frac{y}{2} \right)^{\frac{3}{2}}
\]

\[
\overline{y} = \frac{\left( \frac{2}{3} \right) y^{\frac{4}{3}}}{\int_0^B y^{\frac{4}{3}} dy} / \frac{\int_0^B y y^{\frac{3}{2}} dy}{\int_0^B y^{\frac{3}{2}} dy} \approx 2.13
\]

\[
= \left[ \frac{3}{7} y^{\frac{7}{3}} \right]_0^B / \left[ \frac{3}{4} y^{\frac{4}{3}} \right]_0^B
\]

\[
= \frac{4}{7} B
\]
Physics 301  
Winter Quarter 1993  
March 5, 1993  
George Williams

\[ \bar{x} = 20.1 \]
\[ \sigma = 5.02 \]
\[ N = 207 \]

FOURTH MIDTERM

Name (print) **MARK REEVE**  
(signed)  
Name  
—  5\(\sigma\).  
— 1 units

Discussion Instructor (circle): Condella  DiCarlo  Ganesan  Hollier  Reeve

Discussion Section #

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!  
Use the conversion constants and data given on the front page.

(a) Calculate the linear momentum of a 17.0 kg object moving at 15.2 m/s.

\[ p = mv = (17 \text{ kg})(15.2 \text{ m/s}) = 258 \text{ kg m/sec} = \rho \]

(b) A golf ball of mass 0.150 kg is given a velocity of 85.0 m/s, starting at rest. Calculate the impulse applied to the ball.

\[ I = \Delta p = mv_f - mv_i = (0.15 \text{ kg})(85 \text{ m/s}) = 12.75 \text{ kg m/sec} = I \]

(c) Calculate the kinetic energy, in Joules, of a car of mass 1500 kg moving at 60 mi/hr.

\[ KE = \frac{1}{2}mv^2 = \frac{1}{2}(1500 \text{ kg})(\frac{60 \text{ mi}}{3.6 \text{ m/s}})^2 = 5.40 \times 10^5 \text{ J} = KE \]

(d) Convert 875 ft·pounds into the proper SI unit.

\[ (875 \text{ ft·pounds}) \left( \frac{1 \text{ m}}{3.28 \text{ ft}} \right) = 1.186 \times 10^3 \text{ J} \]

(e) Find the center of mass of the system shown. \( m_1 = 2.75 \text{ kg}, \ m_2 = 1.25 \text{ kg}, \ m_3 = 0.75 \text{ kg} \).

\[ \bar{x}_{cm} = \frac{\sum_{i=1}^{N} x_i m_i}{\sum_{i=1}^{N} m_i} = \frac{0(2.75) + (2)(1.25) + 5(-2.5)}{2.75 + 1.25 + 0.75} = 1.316 \text{ m} = \bar{x}_{cm} \]
\[ W_P = P \cos 30 \ (2.5\text{m}) \]
\[ = 2.7 \ \cos 30 \ (2.5\text{m}) \]
\[ W_P = 5.85 \text{ J} \]

\[ W_g = mg \sin 20 \ (2.5\text{m}) \]
\[ = (1.8\text{kg}) (9.8\text{ m/s}^2) \sin 20 \ (2.5\text{m}) \]
\[ W_g = 15.1 \text{ J} \]

\[ W_{fk} = - f_k \ (2.5\text{m}) \]
\[ = - \mu_k (mg \cos 20 - P \sin 30) \ (2.5\text{m}) \]
\[ W_{fk} = -20.9 \text{ J} \]
In a ballistic pendulum experiment a bullet is shot into a mass $M$ and remains inside of it. If the pendulum swings to a maximum angle of $\theta = 17.0^\circ$, calculate the initial velocity of the bullet. $M = 2.50 \text{ kg}$, $m_{\text{bullet}} = 0.12 \text{ kg}$, $L = 1.10 \text{ m}$.

**First conserve momentum**

Initial

- $p_i = mv_0 \hat{\mathbf{z}}$
- $p_f = (m+M) v_f \hat{\mathbf{z}}$

Final

- $p_f = p_i$

$mv_0 = (m+M) v_f$

$\Rightarrow v_f = \frac{mv_0}{m+M}$

**Second, conserve energy**

$E_{\text{initial}} = \frac{1}{2}(m+M)v_f^2$

$E_f = \frac{1}{2}Mgh$

Substitute

$0.971 = \frac{v_f^2}{2gh} = \frac{\left(\frac{mv_0}{m+M}\right)^2}{2gh}$

$\Rightarrow \sqrt{\frac{(m+M)^2}{m^2}} 2gh = V_0$

but from trig.

$h = L(1 - \cos\theta)$

$\Rightarrow V_0 = \sqrt{\frac{(m+M)^2}{m^2} 2gL(1 - \cos\theta)} = \boxed{21.2 \text{ m/s}}$
A block of mass $m$ is moved down the plane with a constant force $P$ (constant in magnitude and direction). Initially the block is at rest, and at the bottom it has a velocity of $v = 1.75 \text{ m/s}$. A clear free body diagram and separate force diagram for the block are essential for full credit. Calculate using these numerical values: $\mu_s = 0.75$, $\mu_k = 0.55$ and $m = 1.35 \text{ kg}$

(a) Find the work done by friction on the block.
(b) Calculate the work done by $P$ on the block.

$$E_{\text{top}} + (W_P) = E_{\text{bottom}} + (W_{f,k})$$

$$\Delta V = mg \cos 30 - \frac{Ps \sin 50}{2}$$

Solve for $P$

$$P = \frac{\frac{1}{2}mv^2 - mgh + \mu_k mgd \cos 30}{(d \cos 50 + \mu_k \sin 50)}$$

$$P = 0.425 \text{ N}$$

Work done by friction:

$$W_f = -\mu_k Vd = -\mu_k (mg \cos 30 - Ps \sin 50)$$

$$W_f = -16.535$$
(b) \[ W_p = P \cos 50 \ d \quad W_p = 0.738 \ J \]

\[ \sum F \text{ method} \]

\[ N = mg \cos 30 - P \sin 50 \]

\[ mg \sin 30 + P \cos 50 - \frac{f_i}{g} = ma \]

\[ f_i = M_k N \]

\[ mg \sin 30 + P \cos 50 - M_k mg \cos 30 + M_k P \sin 50 = ma \]

\[ P = ma - mg \sin 30 + \mu mg \cos 30 \]

\[ \cos 50 + M_k \sin 50 \]

To determine \( a \):

\[ V_{\text{bottom}}^2 = V_{\text{top}}^2 + 2ad \]

\[ a = \frac{V_{\text{bottom}}^2}{2d} = 0.567 \text{ m/s}^2 \]

\[ P = 0.425 \ N \]

III method: Use work-energy theorem.

The change in kinetic energy is equal to work done by all the forces.

\[ \Delta K = W_p + W_f + W_g \]

\[ \Delta K = \text{change in kinetic energy} = \frac{1}{2} m V_{\text{bottom}}^2 \]

\[ W_p = \text{work done by } P = P \cos 50 \ d \]
\[ \text{Work done by friction} = -\mu_k d (mg \cos 30 - P \sin 50) \]

\[ \text{Work by gravity} = mg \sin 30 d \]

\[ \frac{1}{2} m \nu_{\text{bottom}}^2 = P d \cos 50 - \mu_k d mg \cos 30 + \mu_k d P \sin 50 + mg \sin 30 d \]

\[ P = \frac{\frac{1}{2} m \nu_{\text{bottom}}^2 + \mu_k d mg \cos 30 - mg d \sin 30}{d (\cos 50 + \mu_k \sin 50)} \]

\[ P = 0.425 N \]