An automobile battery has an emf of 12.6 V and an internal resistance of 0.0660 Ω. The headlights together present equivalent resistance 5.80 Ω (assumed constant).

(a) What is the potential difference across the headlight bulbs when they are the only load on the battery?

\[ V = 12.5 \text{ V} \]

(b) What is the potential difference across the headlight bulbs when the starter motor is operated, taking an additional 35.0 A from the battery?

\[ V = 10.2 \text{ V} \]

\[ R = \text{internal resistance of battery} \]

\[ V_{\text{headlights}} = IR = 12.5 \text{ V} \]

We treat the resistance of the battery as a separate resistor, just like all the resistance of the wires is bunched into \( R \).

\[ 12.6 \text{ V} - IR = IR \quad \Rightarrow \quad I = \frac{12.6 \text{ V}}{R + R} = 2.148 \text{ A} \]

The battery now powers two parallel circuits.

Junction: \[ I_b = I_{\text{head}} + 35.0 \text{ A} \]

Loop: \[ 12.6 \text{ V} - I_b R - I_{\text{head}} R = 0 \]
HW 7

Substitute using junction rule

\[ 12.6V - (I_{head} + 3.50A)r - I_{head}R = 0 \]

Now solve for \( I_{head} \)

\[ I_{head} = \frac{12.6V - (3.50A)r}{r + R} \]

\[ V_{head} = I_{head}R = 10.2V \]

Needed \( I_{head} \) to calculate \( V = I_{head}R \)

There are two unknowns, \( I_{head} \) and \( I_{batt} \), so need two equations, junction \& loop.
A 6 V battery supplies current to the circuit shown in Figure P28.14. When the double-throw switch S is open, as shown in the figure, the current in the battery is 1.00 mA. When the switch is closed in position 1, the current in the battery is 1.20 mA. When the switch is closed in position 2, the current in the battery is 1.80 mA. Find the resistances $R_1$, $R_2$, and $R_3$.

There are three cases, and in each case we can find the equivalent resistance and use it in $V=IR_{eq}$. We could use loop rules and junction rules, but that's more algebra.

**Case 1:**

\[ 6.00 \text{ V} = 1.00 \text{ mA} \times (R_1 + R_2 + R_3) \]

\[ \Rightarrow R_1 + R_2 + R_3 = \frac{6.00 \text{ V}}{1.00 \times 10^{-3} \text{ A}} = 6.00 \times 10^3 \Omega \]
case 2: $R_2$ & $R_3$ are in parallel
so $R_{eq} = \frac{R_2}{2}$ → the parallel combination is only half its original
then the rest is in series.

2. so, $R_1 + \frac{R_2}{2} + R_3 = \frac{6.00V}{1.203-3A} \approx 5.0033 \Omega$

3. case 3: $R_3$ is shorted, that is no current goes through it.

3. so, $R_1 + R_2 = \frac{6.00V}{1.803-3A} \approx 3.3333 \times 3 \Omega$

Each time total $R$ has decreased and current increased.

3) into 1) $\Rightarrow R_3 = 6.0033 \Omega - 3.3333 \times 3 \Omega = 2.67 \Omega$

1) minus 2) $\Rightarrow R_2 = 2 \times 1.0033 \Omega = 2.0066 \Omega$

3) $\Rightarrow R_1 = 1.3333 \times 3 \Omega$

three unknowns, need three eqns

used $V_{tot} = I_{tot} \cdot R_{eq}$ for three cases.
The ammeter shown in Figure P28.20 reads 2.80 A. Find $I_1$, $I_2$, and $\mathcal{E}$.

$I_1 = 0.143$ A

$I_2 = 2.66$ A

$\mathcal{E} = 19.3$ V

$\rightarrow 2.80 A = I_1 + I_2$ (junction rule)

find $I_1$ by counter clockwise loop rule, with the direction for $I_1$ as given.

$\Rightarrow 15.0V - 7.00\Omega I_1 - 5.00\Omega \times 2.80A = 0$

(ideal Ammeter has zero resistance)

$\Rightarrow I_1 = 1.429 \times -1 A$ (plug in 0.143 A)

$I_2$ is now easily found from junction rule

$I_2 = 2.80 A - 1.429 A = 2.657 A$ (plug in 2.66)

$\mathcal{E} = I_2 \times 2.00\Omega + (2.80A)(5.00\Omega)$ (from another loop rule).

$= 19.3$ V
Taking \( R = 1.50 \, \text{k}\Omega \) and \( E = 180 \, \text{V} \) in Figure P28.25, determine the direction and magnitude of the current in the horizontal wire between \( a \) and \( e \).

There are five unknown currents, so we will need five equations.

We have three independent loops and two independent junctions for a total of five equations. (Trying to add more junctions or loops is redundant.)

To save on a little algebra, we can also notice that \( a \) and \( e \) must be at the same potential, so \( 4R \) and \( 3R \) are in parallel if we care to find their Req at some point. (There's more than one way to solve, and finding Req could save some work, though it's not necessary. It would take the place of the middle loop.)
Loops:
1. $2\epsilon - I_22R - I_33R = 0$
2. $\epsilon - I_1R - I_44R = 0$
3. $-I_44R + I_33R = 0 \Rightarrow I_3 = \frac{4}{3}I_4$
4. $I_4 = I_1 + I_5$ (sub)

Junctions:
3. $I_1 + I_2 = I_3 + I_4 \Rightarrow I_4 = \frac{3}{7}(I_1 + I_2)$

1 - 2 \Rightarrow $\epsilon - I_22R + I_1R - (\frac{4}{3}I_4)3R + I_44R = 0$

1 + 2 \Rightarrow $3\epsilon - I_22R - I_1R - 8I_44R = 0$

$\Rightarrow 3\epsilon - \frac{36}{7}I_1R - \frac{36}{7}I_2R = 0$

Use 0 - 2 \Rightarrow $I_1 = \frac{-\epsilon}{R} + 2I_2$

$\Rightarrow 3\epsilon - \frac{36}{7}(\frac{\epsilon}{R} + 2I_2R) - \frac{36}{7}I_2R = 0$

$\Rightarrow I_2 = \frac{82}{100} \frac{\epsilon}{R} = 62.4 \text{ mA}$

0 - 2 \Rightarrow $I_1 = 4.80 \text{ mA}$

3. $\Rightarrow I_4 = 28.8 \text{ mA}. (\frac{23}{7}(I_1 + I_2))$

4. $\Rightarrow I_5 = 2.40 \text{ mA} \text{ to the right, consistent with initial guess.}$
If you reduce $I_3$ & $I_4$ two loops and one
junction for three unknowns, a/c

Loops:

1. $E - I_1R - (I_1 + I_2)Req = 0$
2. $2E - 2I_2R - (I_1 + I_2)Req = 0$

$Req = \frac{12}{7}R$

These are just 1 & 2 from before.
$Req = \frac{12}{7}R$ has replaced the third loop.

But we can use $Req$: $V_c - V_a = (I_1 + I_2)Req$

Once we find $I_1$ & $I_2$

Then $I_4 = \frac{V_c - V_a}{4R}$

So $I_4 \rightarrow I_5 = I_4 - I_1$

Where $V_c - V_a$ is the drop across 3 parallel branches.

These last two steps are the same as previously, seen by substituting in $V_c - V_a = (I_1 + I_2)\frac{12}{7}R$. 
A dead battery is charged by connecting it to the live battery of another car with jumper cables. Assume that $V = 5.5$ V and $R = 0.60 \, \Omega$. (Take downward current flow as positive.)

**Determine the current in the starter.**

Three unknowns $I_{live}$, $I_{dead}$, $I_{starter}$.

Need three Eqs., two loops & one junction.

Loops:

1. $12V - 0.12I_{live} - 0.6I_{starter} = 0$
2. $12V - 0.12I_{live} - I_{dead} R = 0$
3. $V = 0$

Used this third loop at very end. (Junction 1, 2, 3)

Just to plug in numbers.

$\Rightarrow I_{live} = \frac{6.50 - 0.60}{0.12}$

Plug in (3) $I_{live} = \frac{650 - 60}{12}$ (Eqn. - Eqn.)

$\Rightarrow I_{live} = \frac{650 + 60}{12}$ $I_{starter}$

1. $\Rightarrow I_{starter} = \frac{(12V - 6.50)}{0.60 + 0.60}$ = 170 A.

$I_{dead} = \frac{(I_{starter} - 0.6)}{R} = \frac{178.3}{0.60}$ A using a third loop.
A 11.0 \mu F capacitor is charged by a 9.0 V battery through a resistance \( R \). The capacitor reaches a potential difference of 4.00 V at a time 3.00 s after charging begins. Find \( R \).

We know \( V = \frac{Q}{C} \) for the capacitor, and \( Q(t) \) is increasing with time by eqn 28.14

\[ Q(t) = Q(1 - e^{-\frac{t}{RC}}) \]  
(as charge accumulates on the capacitor, current decreases by eqn 28.15

\[ I(t) = \frac{Q}{R} e^{-\frac{t}{RC}} \]  
\( I(t)R = V \) is the voltage across the resistor, NOT the capacitor

So \( \frac{Q(3\text{ s})}{C} = \frac{4.00\text{ V}}{\frac{Q}{C}(1 - e^{-\frac{t}{RC}})} = 4(1 - e^{-\frac{3}{RC}}) \)

\[ R = \frac{-t}{C \ln\left(\frac{4.00\text{ V}}{E} + 1\right)} = 464 \text{ k}\Omega \]
In the circuit, the switch S has been open for a long time and $R = 115 \text{ k}\Omega$. It is then suddenly closed.

(a) Determine the time constant before the switch is closed.

With the switch open, $R_{eq} = R_1 + R_2 = 165 \times 3 \Omega$

1.65 s

$\tau = RC$ by def'n $\tau = 1.65 \text{ s}$

(b) Determine the time constant after the switch is closed.

There is a short, so the battery and capacitor do not see each other, as if on independent circuits.

1.15 s

$\tau = 115 \times 3 \Omega \times 10.05 \times 6 \text{ F} = 1.15 \text{ s}$

(c) Let the switch be closed at $t = 0$. Find the current in the switch as a function of time. (Use $t$ for time. Express your answer in milliamps and seconds.)

$I(t) = \frac{0.2 + \exp(-t/1.15)}{0.087} \text{ mA}$

The switch shorts current from the battery and the capacitor.

$I_{\text{batt}} = \frac{10.0 \text{ V}}{50 \times 3 \Omega} = 200 \times 10^{-3} \text{ A} = 0.2 \text{ mA}$

$I_{\text{cap}} = I_0 e^{-t/\tau}$ for discharging

since the switch was open for a long time, the capacitor fully charged to a voltage $= 10.0 \text{ V}$

so $I_0 = \frac{\tau \cdot \epsilon}{RC} = \frac{10.0 \text{ V}}{115 \times 3 \Omega} \text{ discharges through only}$

the one resistor

so $I = I_{\text{batt}} + I_{\text{cap}} = 0.2 + 0.087 \exp(-t/1.15)$
A particular galvanometer serves as a 2.00 V full-scale voltmeter when a 2900 Ω resistor is connected in series with it. It serves as a 0.500 A full-scale ammeter when a 0.240 Ω resistor is connected in parallel with it. Determine the internal resistance of the galvanometer and the current required to produce full-scale deflection.

\[
\begin{align*}
I_g &= 0.648 \text{ mA} \\
\text{Series resistor: } & \quad 2.00 \text{V} = I_g (r + 2900 \Omega) \\
\text{Parallel: } & \quad I_g r = (0.500 \text{A} - I_g) \cdot 240 \Omega \\
\text{Parallel voltage equals } & \quad V \\
I = 0.500 \text{A} = I_g + I \cdot 240 \Omega \\
\text{at junction.} \\
\text{Most current goes through } & \quad 240 \Omega \text{ shunt resistor since } 240 \ll r \\
\text{Plug parallel into series: } & \quad 2.00 \text{V} = 0.500 \text{A} \cdot \left(240 \Omega + (1240 \Omega + 2900 \Omega)\right) \\
\implies I_g &= 0.648 \times 10^{-3} \text{A} \\
\text{Solve either equation: } & \quad r = 185.52 \Omega
\end{align*}
\]
A battery has an emf of 10.00 V and an internal resistance of 1.20 Ω.

(a) What resistance across the battery will extract from it a power of 12.8 W? (If there is no resistance to make this possible, enter NA.)

\[ R_{\text{lower}} = \frac{V}{I} \]

0.281 Ω

\[ R_{\text{higher}} = \frac{V}{I} \]

5.13 Ω

We can plug these in or use \( P = I^2 R \), where

(b) What resistance across the battery will extract from it a power of 21.2 W? (If there is no resistance to make this possible, enter NA.)

\[ R = \frac{V^2}{P} \]

NA Ω

\[ P = \frac{\epsilon^2 R}{(R + r)^2} \]

\[ \Rightarrow P R^2 + 2(\epsilon P - \epsilon^2) r + \epsilon P^2 = 0 \]

Quadratic formula

\[ R = \frac{\epsilon^2 - 2\epsilon P \pm \sqrt{(\epsilon^2 - 2\epsilon P)^2 - 4\epsilon^2 P^2}}{2P} \]

\[ = 5.13 \text{ Ω or } 1.2806 \text{ Ω} \]

b) Something, but check the discriminant (under the square root).

\[ (\epsilon^2 - 2\epsilon P)^2 - 4\epsilon^2 P^2 = -176 \text{ leads to an imaginary number} \]

\[ (\epsilon^2 - 2\epsilon P)^2 - 4\epsilon^2 P^2 \geq 0 \Rightarrow \epsilon^2 - 4\epsilon P \epsilon^2 \geq 0 \Rightarrow P \geq \frac{\epsilon^2}{4\epsilon} = 2.08 \text{ W} \]

So \( P = 20.83 \text{ W is the largest allowed power at } \frac{dP}{dR} = 0 \Rightarrow R = R \)
A power supply has an open-circuit voltage of 34.0 V and an internal resistance of 2.00 Ω. It is used to charge two storage batteries connected in series, each having an emf of 6.00 V and internal resistance of 0.300 Ω.

(a) If the charging current is to be 3.20 A, what additional resistance should be added in series?

\[
\begin{align*}
4.28 & \quad \text{Ω} \\
\text{34.0 V is the emf of the battery, without considering internal resistance.}
\end{align*}
\]

(b) At what rate does the internal energy increase in the supply, in the batteries, and in the added series resistance?

\[
\begin{align*}
\text{inside the supply} & \quad 20.5 \quad \text{W} \\
\text{in the batteries} & \quad 6.14 \quad \text{W} \\
\text{in the added resistor} & \quad 43.8 \quad \text{W}
\end{align*}
\]

What R limits current to 3.20A?

\[
\text{So, loop can } R = 4.275 \text{Ω}
\]

\[\text{b) Current is the same for each element since nothing is in parallel (no junctions).}
\]

(c) At what rate does the chemical energy increase in the batteries?

\[
\begin{align*}
38.4 & \quad \text{W}
\end{align*}
\]

\[\text{The loop rule, which included 2x6.00W=12.0V delivered to the batteries, determined the current.}\]
So we know the current and the voltage across the batteries. \[ P = IV = 3.20 \text{A} \times 12.0 \text{V} = 38.4 \text{ W} \]
Switch $S$ has been closed for a long time, and the electric circuit shown in the figure below carries a constant current. Take $C_1 = 3.00 \ \mu F$, $C_2 = 6.00 \ \mu F$, $R_1 = 4.00 \ \text{k}\Omega$, and $R_2 = 7.00 \ \text{k}\Omega$. The power delivered to $R_2$ is 2.20 W.

(a) Find the charge on $C_1$.

\[ Q = \int \text{d}Q = \int \frac{\text{d}Q}{C} = \frac{Q}{C_1} = 213 \ \mu C \]

(b) Now the switch is opened. After many milliseconds, by how much has the charge on $C_2$ changed?

\[ Q = \int \text{d}Q = \int \frac{\text{d}Q}{C_2} = \frac{Q}{C_2} = 425 \ \mu C \]

5) With the switch open, there is no current.

All of the voltage from the battery appears across $C_1$ and $C_2$, and none is dissipated in the resistors.

But what was the voltage from the battery?

\[ V = \frac{Q}{C} + \frac{Q}{C_2} = 195.03 \text{V} \]

After enough time to charge $C_2$, \[ Q = C_2 V = 1.617 \times 10^{-3} \text{C} \]

The old charge was \[ Q = C_2 I R_2 = 7.447 \times 10^{-4} \text{C} \]

\[ \Rightarrow \Delta Q = 4.25 \times 10^{-4} \text{C} \]
The switch in Figure P28.66a closes when $\Delta V_c > 2\Delta V / 3$ and opens when $\Delta V_c < \Delta V / 3$. The voltmeter reads a voltage as plotted in Figure P28.66b. What is the period $T$ of the waveform in terms of $R_a$, $R_b$, and $C$? (Use $R_a$ for $R_a$, $R_b$ for $R_b$, and $C$ as necessary.)

$$T = \frac{3}{(R_a + 2R_b)C \ln(2)}$$

When the switch closes $C$ discharges with $q = C\int_0^t \frac{1}{\tau} dt = \frac{Q}{e^{t/\tau}}$ to solve

$$\frac{Q}{C} = \frac{V}{3} = \frac{2V}{3}e^{-t_1/\tau}$$

for $t_1 = 2\ln 2 = R_bC \ln 2$.

When it opens, it charges with $\frac{\tau}{\tau} = (R_4 + R_b)C$ and stops at $\frac{2V}{3}$. Use $q = Q(1 - e^{-t/\tau})$

but initial voltage should be $V_3$, so

$$\frac{Q}{C} = \left(\frac{Q}{C} + \frac{2V}{3}e^{t_2/\tau}\right) = \frac{V}{3} - \frac{2V}{3}e^{-t_2/\tau}$$

with $q = \frac{2V}{3}$ when $t = t_2$.

$$t_2 = (R_4 + R_b)C \ln 2$$

Then $t_1 + t_2 = T = (R_a + 2R_b)C \ln 2$. 

$\Rightarrow$ $Q = \frac{2V}{3}C$