1. \(2 \text{ points} \) Saved Work | Show Details
A conductor consists of a circular loop of radius \( R \) and two straight, long sections as shown in Figure P30.6. The wire lies in the plane of the paper and carries a current \( I \). Find an expression for the vector magnetic field at the center of the loop. (Use \( \mu_0 \) for \( \mu_\circ \), \( \pi \) for \( \pi \), \( I \), and \( R \) as necessary.)

The resultant magnetic field is directed \( \ldots \) into the page.

![Diagram of a circular loop and straight section with a current](image)

**Figure P30.6**

- **Circular loop:** Page 842: \( \vec{B} = \frac{\mu_0 I}{2R} \) (\( x = 0 \), in Eq. 30.7)

- For long straight: \( \frac{\mu_0 I}{2\pi R} \)

\[ \vec{B}_{\text{total}} = (1 + \frac{1}{\pi}) \frac{\mu_0 I}{2R} \] into the page.

**Note.**

When typing into WebAssign,

\[ \vec{B} = (1 + 1/\pi)(\mu_0 0 I)/(2R) \]
2. --/2 points Saved Work | Show Details Notes
A very long straight wire carries current I. In the middle of the wire a right-angle bend is made. The bend forms an arc of a circle of radius r, as shown in Figure P30.10. Determine the magnetic field at the center of the arc. (Use \( \mu_0 \) for \( \mu_0 \), \( \pi \) for \( \pi \), \( r \), and \( I \) as necessary.)

\[
B = \frac{\mu_0 I}{2r} (\frac{1}{\pi} + \frac{1}{4})
\]

What is the direction of the magnetic field?

- down
- left
- right
- into the plane of the computer screen
- out of the plane of the computer screen

Eq. (30.6) \( B = \frac{\mu_0 I}{2\pi r} \theta = \frac{\mu_0 I}{2\pi} \frac{z}{z} = \frac{\mu_0 I}{2\pi} \)

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Eq. (30.13) \( \phi \) is \( \phi \), \( d\phi \) = \( \mu_0 I \)

\( B = \frac{\mu_0 I}{2\pi r} \) for very long straight wire

Long \( \phi \)

\[ \mathbf{B} = \left[ \frac{1}{2} \frac{\mu_0 I}{2\pi r} + \frac{1}{4} \frac{\mu_0 I}{2\pi r} + \frac{1}{2} \frac{\mu_0 I}{2\pi r} \right] \text{ into the page} \]

\[ \mathbf{B} = \frac{\mu_0 I}{2\pi r} \left( \frac{1}{\pi} + \frac{1}{4} \right) \]
A current path shaped as shown in the figure produces a magnetic field at P, the center of the arc. If the arc subtends an angle of $\theta = 40.0^\circ$ and the radius of the arc is 0.500 m, what are the magnitude and direction of the field produced at P if the current is 3.00 A?

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi} \frac{ds}{a^2}$$

$$S = r\theta = 0.5 \times 40^\circ \left(\frac{2\pi}{360^\circ}\right) = 0.35 \text{ m}$$

$$B = \frac{\mu_0 I}{4\pi r^2} \int ds = \frac{\mu_0 I S}{4\pi r^2}$$

*Given*

- $\theta = 40^\circ$
- $r = 0.5 \text{ m}$
- $I = 3.0 \text{ A}$
- $\mu_0 = 4\pi \times 10^{-7}$

$$B = 420 \text{ nT}$$
Two long, parallel conductors carry currents $I_1 = 2.50 \, \text{A}$ and $I_2 = 1.50 \, \text{A}$, both directed into the plane of the computer screen as shown in the figure. Determine the magnitude and direction of the resultant magnetic field at $P$.

Take the $x$-direction to the right & the $y$-direction up in the plane of the paper.

* **Current 1 creates at $P$ a field**

$$B_1 = \frac{\mu_0 I}{2\pi a} = \frac{(2 \times 10^{-7} \, \text{T} \cdot \text{m}) (2.50 \, \text{A})}{\pi (0.05 \, \text{m})} = 10 \, \mu \text{T}$$

**$B_1 = 10 \, \mu \text{T}$** downward & leftward at angle $67.4^\circ$ below the $-x$-axis

* **Current 2**

$$B_2 = \frac{(2 \times 10^{-7} \, \text{T} \cdot \text{m})(1.50 \, \text{A})}{(0.12 \, \text{m})} = 2.5 \, \mu \text{T}$$

**$B_2 = 2.5 \, \mu \text{T}$** to the right & down, at an angle of $22.6^\circ$. 
\[ \vec{B} = \vec{B}_1 + \vec{B}_2 \]

\[ \vec{B} = (10 \mu T)(\hat{i} \cos 67.4^\circ - \hat{j} \sin 67.4^\circ) \]
\[ + (2.5 \mu T)(\hat{i} \cos 22.6^\circ - \hat{j} \sin 22.6^\circ) \]

\[ = (-3.843 \mu T + 2.308) \hat{i} + (-9.23 \mu T + 9.60) \hat{j} \]

\[ \vec{B} = (-1.535 \mu T)(\hat{i}) + (-10.19 \mu T)(\hat{j}) \]

\[ |\vec{B}| = \sqrt{(-1.535 \mu T)^2 + (-10.19 \mu T)^2} \quad T = 1.03 \times 10^{-5} T \]

\[ \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{y}{x} \]

\[ \theta = 81.43^\circ \]

\[ 180^\circ - \theta = 180^\circ - 81.43^\circ \]

\[ \chi = 98.6^\circ \]
The unit of magnetic flux is named for Wilhelm Weber. The practical-size unit of magnetic field is named for Johann Karl Friedrich Gauss. Both were scientists at Göttingen, Germany. Along with their individual accomplishments, together they built a telegraph in 1833. It consisted of a battery and switch, at one end of a transmission line 3 km long, operating an electromagnet at the other end. (André Ampère suggested electrical signaling in 1821; Samuel Morse built a telegraph line between Baltimore and Washington, D.C. in 1844.) Suppose Weber and Gauss's transmission line was as diagrammed in the figure below. Two long, parallel wires, each having a mass per unit length of 30.0 g/m, are supported in a horizontal plane by strings 6.00 cm long. When both wires carry the same current I, the wires repel each other so that the angle \( \theta \) between the supporting strings is 16.0°.

(a) Are the currents in the same direction or in opposite directions? \* opposite b/c they repel each other.

(b) Find the magnitude of the current.

\[
I = 58.7 \text{ A}
\]

\[
\frac{F_B}{F_g} = \frac{\mu_0 I^2 l}{2 \pi \alpha m g} = \tan 8^\circ
\]

**Given**

- \( \mu_0 = 4\pi \times 10^{-7} \text{ T m/A} \)
- \( l = 6.0 \times 10^{-2} \text{ m} \) (strings)
- \( m = 30.0 \text{ g/m (wires)} \)

**Solution**

\[
I^2 = \frac{m g 2\pi \alpha}{l \mu_0} \tan 8^\circ
\]

\[
I^2 = (30 \times 10^3 \text{ kg/m})(9.8 \text{ m/s}^2)(2\pi)(0.0167 \text{ m}) \tan 8^\circ
\]

\[
I^2 = 3.45 \times 10^3 \Rightarrow I = 58.7 \text{ A}
\]
The figure below is a cross-sectional view of a coaxial cable. The center conductor is surrounded by a rubber layer, which is surrounded by an outer conductor, which is surrounded by another rubber layer. In a particular application, the current in the inner conductor is \( I_1 = 1.18 \text{ A} \) out of the monitor, and the current in the outer conductor is \( I_2 = 3.16 \text{ A} \) into the monitor. Determine the magnitude and direction of the magnetic field at point \( a \).

Determine the magnitude and direction of the magnetic field at point \( b \).

From Ampere's law,

\[
B_a = \frac{\mu_0 I_a}{2\pi r_a}
\]

\( I_a = \text{net current through the area of circle of radius } r_a \)

\[
B_a = \left( \frac{4\pi \times 10^{-7} \text{T.m/A}}{2\pi (1 \times 10^{-3} \text{m})} \right) (1.18 \text{ A}) = 2.36 \times 10^{-4} \text{T}
\]

\[
B_a = 236 \mu\text{T}
\]

\[
B_b = \frac{\mu_0 I_b}{2\pi r_b}
\]

\( I_b = 3.16 - 1.18 = 1.98 \text{ A} \)

\[
B_b = \left( \frac{4\pi \times 10^{-7} \text{T.m/A}}{2\pi (3 \times 10^{-3} \text{m})} \right) (1.98 \text{ A}) = 1.32 \times 10^{-4} \text{T}
\]

\[
B_b = 132 \mu\text{T}
\]
The magnetic coils of a tokamak fusion reactor are in the shape of a toroid having an inner radius of 0.700 m and an outer radius of 1.30 m. The toroid has 920 turns of large-diameter wire, each of which carries a current of 15.0 kA.

(a) Find the magnitude of the magnetic field inside the toroid along the inner radius.

(b) Find the magnitude of the magnetic field inside the toroid along the outer radius.

**Magnitudes of the Fields Inside a Toroid**

\[ B = \frac{\mu_0 NI}{2\pi r} \]

### Given

- \( r_{\text{inner}} = 0.7 \text{ m} \)
- \( r_{\text{outer}} = 1.3 \text{ m} \)
- \( N = 920 \text{ turns} \)
- \( I = 15 \text{ kA} \)

#### a) \( B_{\text{inner}} \)

\[ B_{\text{inner}} = \frac{\mu_0 NI}{2\pi r_{\text{inner}}} = \frac{(4\pi \times 10^{-7} \text{ T.m/A}) (920) (15 \times 10^3 \text{ A})}{2\pi (0.7 \text{ m})} \]

\[ B_{\text{inner}} = 3.943 \text{ T} \]

#### b) \( B_{\text{outer}} \)

\[ B_{\text{outer}} = \frac{\mu_0 NI}{2\pi r_{\text{outer}}} = \frac{(4\pi \times 10^{-7} \text{ T.m/A}) (920) (15 \times 10^3 \text{ A})}{2\pi (1.3 \text{ m})} \]

\[ B_{\text{outer}} = 2.123 \text{ T} \]
A single-turn square loop of wire, 2.00 cm on each edge, carries a clockwise current of 0.160 A. The loop is inside a solenoid, with the plane of the loop perpendicular to the magnetic field of the solenoid. The solenoid has 30 turns/cm and carries a clockwise current of 15.0 A. Find the force on each side of the loop and the torque acting on the loop.

\[ F = \mu N \text{ directed away from the center} \]

\[ \tau = D N \cdot m \]

Force exerted on side \( AB \):

\[ (\vec{F}_{AB}) = I \vec{L} \times \vec{B} = (0.160 A) \left[ (2.16 \times 10^{-2} \text{ m}) \hat{j} \times (5.65 \times 10^{-2} \text{ T}) \hat{i} \right] \]

\[ (\vec{F}_{AB})_{AB} = (1.808 \times 10^{-4} \text{ N}) \hat{k} = 181 \mu \text{N} \hat{k} \]

Similarly, each side of the square loop experiences a force, lying in the plane of the loop, of 226 \( \mu \text{N} \) directed away from the center.

\[ \vec{A} = I \vec{A} = (0.200 A) (2.00 \times 10^{-2} \text{ m})^2 (-\hat{i}) = -80.0 \mu \text{A} \cdot \text{m}^2 \hat{i} \]

\[ \vec{C} = \vec{A} \times \vec{B} = (-80.0 \mu \text{A} \cdot \text{m}^2 \hat{i}) \times (-5.65 \times 10^{-2} \text{ T} \hat{i}) = 0 \]

Net torque exerted on the square loop by the field of the solenoid should be zero.
A solenoid 2.50 cm in diameter and 34.0 cm long has 300 turns and carries 12.0 A.

(a) Calculate the flux through the surface of a disk of radius 5.00 cm that is positioned perpendicular to and centered on the axis of the solenoid, as in the figure (a) above.

\[ \Phi_B = \frac{6.53 \mu Wb}{0.50 cm} \]

(b) The figure (b) above shows an enlarged end view of the same solenoid. Calculate the flux through the blue area, which is defined by an annulus that has an inner radius of 0.400 cm and outer radius of 0.800 cm.

\[ d = 2.50 cm \Rightarrow r = 1.25 cm \]

\[ A = \pi \left( \frac{1.25^2}{2} - \frac{0.4^2}{2} \right) \]

\[ \Phi_B = 2.005 \mu Wb \]
A circular coil of 6 turns and a diameter of 30.0 cm is oriented in a vertical plane with its axis perpendicular to the horizontal component of the Earth's magnetic field. A horizontal compass placed at the center of the coil is made to deflect 45.0° from magnetic north by a current of 0.560 A in the coil.

(a) What is the horizontal component of the Earth's magnetic field?

\[
\vec{B}_n = B \sin 45^\circ = \frac{\mu_0 I}{2\pi R} 
\]

\[
\begin{align*}
N &= 6 \text{ turns} \\
2R &= d = 30.0 \text{ cm} = 0.300 \text{ m} \\
\theta &= 45^\circ \\
I &= 0.560 \text{ A} \\
B &= 14.07 \mu T
\end{align*}
\]

(b) The current in the coil is switched off. A "dip needle" is a magnetic compass mounted so that it can rotate in a vertical north-south plane. At this location, a dip needle makes an angle of 13.0° from the vertical. What is the total magnitude of the Earth's magnetic field at this location?

\[
B = \frac{4\pi \times 10^{-7} (6)(0.560 \text{ A})}{\sin(13.0^\circ)} = \frac{602.6 \mu T}{\sin(13^\circ)}
\]

\[
B_n = \frac{4\pi \times 10^{-7} (6)(0.560 \text{ A})}{2(0.15)} = 14.07 \mu T
\]

\[
B = \frac{14.07 \mu T}{\sin(13^\circ)} = 602.6 \mu T
\]
A thin copper bar of length $L = 12.0 \, \text{cm}$ is supported horizontally by two (nonmagnetic) contacts. The bar carries current $I_1 = 85 \, \text{A}$ in the $-x$ direction, as shown in the figure below. At a distance $h = 0.500 \, \text{cm}$ below one end of the bar, a long straight wire carries a current $I_2 = 200 \, \text{A}$ in the $z$ direction. Determine the magnitude and direction of the magnetic force exerted on the bar.

The current $I_2$ creates a magnetic field $\vec{B} = \frac{\mu_0 I_2}{2\pi \sqrt{h^2 + x^2}}$, where $\mu_0$ is the permeability of free space.

The magnetic force on an element of the bar $d\vec{F}$ is given by $d\vec{F} = I_1 \vec{d} \times \vec{B} = I_1 \frac{\mu_0 I_2}{2\pi \sqrt{h^2 + x^2}} \sin \theta \vec{d} \times \vec{B}$.

Integrating over the length of the bar, we get

$$d\vec{F} = \frac{\mu_0 I_1 I_2}{2\pi \sqrt{h^2 + x^2}} \frac{x}{\sqrt{h^2 + x^2}} \vec{x} \quad \text{into the page}$$

Thus, the total force $\vec{F}$ is given by

$$\vec{F} = \frac{\mu_0 I_1 I_2}{2\pi} \left[ \ln \left( \frac{h^2 + L^2}{h^2} \right) - \ln \frac{h^2}{L^2} \right].$$
\[ \vec{F} = \frac{\mu_0 I_1 I_2 (-\hat{k})}{4\pi} \left[ \ln(h^2 + l^2) - \ln h^2 \right] \]

*Given:

\( I_1 = 85\,\text{A} \), \( I_2 = 200\,\text{A} \)

\( h = 0.5 \times 10^{-2}\,\text{m} \)

\( l = 0.12\,\text{m} \)

Then

\[ \vec{F} = (10^7)(85\,\text{A})(200\,\text{A})(-\hat{k}) \left[ \ln \left( (0.5 \times 10^{-2}\,\text{m})^2 + (0.12\,\text{m})^2 \right) \right. \]

\[ \left. - \ln \left( (0.5 \times 10^{-2}\,\text{m})^2 \right) \right] \]

\[ \vec{F} = 1.08 \times 10^2\,\text{N} (-\hat{k}) \]

*Answer wants magnitude

\[ |\vec{F}| = 1.08 \times 10^2\,\text{N} \]

into page
A wire carrying a current $I$ is bent into the shape of an exponential spiral $r = e^\theta$ from $\theta = 0$ to $\theta = 2\pi$, as in the figure below. To complete a loop, the ends of the spiral are connected by a straight wire along the $x$ axis. Find the magnitude and direction of the magnetic field $\mathbf{B}$ at the origin. (Use $\mu_0$ for $\mu_0$ and $\pi$ for $\pi$ as necessary.) 

Hints: Use the Biot-Savart law. The angle $\beta$ between a radial line and its tangent line at any point on the curve $r = f(\theta)$ is related to the function in the following way.

$$\tan \beta = r \left( \frac{dr}{d\theta} \right)^{-1}$$

Thus, in this case $r = e^\theta$, $\tan \beta = 1$, and $\beta = \pi/4$. Therefore, the angle between $ds$ and $\hat{r}$ is $\pi - \beta = 3\pi/4$. Also, use the following.

$$ds = dr \left( \sin \left( \frac{\pi}{4} \right) \right)^{-1} = \sqrt{2} dr$$

$$B = \frac{\mu_0 I (1 - \exp(-2\pi))}{4\pi}$$

The direction of the magnetic field is out of the plane.
Notes for:

* Chapter 30:

1. Magnetic flux \( \Phi_B = \int \vec{B} \cdot d\vec{A} \)

2. Biot-Savart Law:

\[
\frac{d\vec{B}}{dr} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}
\]

"The magnetic field \( d\vec{B} \) at a point \( P \) due to a length element \( d\vec{s} \) that carries a steady current \( I \)"

3. Ampère's Law:

\[ \int \vec{B} \cdot d\vec{s} = \mu_0 I \]

4. Magnetic force per unit length blw two parallel wires separated by a distance \( a \) & carrying currents \( I_1, I_2 \)

\[
\frac{F_B}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}
\]

5. Magn. of Magnetic field a distance \( r \) from a long straight wire carrying an \( I \):

\[
B = \frac{\mu_0 I}{2\pi r}
\]