1. Consider the situation shown in the figure below. An electric field of 300 V/m is confined to a circular area \( d = 9.9 \) cm in diameter and directed outward perpendicular to the plane of the figure. If the field is increasing at a rate of 23.8 V/m·s, what are the direction and magnitude of the magnetic field at the point \( P \), 14.7 cm from the center of the circle?

- direction: upwards
- magnitude: \( 2.2 \times 10^{-18} \) T

Since \( \frac{dE}{dt} \) is out of the page, using the right-hand rule with \( \frac{dE}{dt} \) as thumb, \( \vec{B} \) is counterclockwise.

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 I + \frac{1}{c^2} \frac{d\Phi_E}{dt} \]

\[ 2\pi r B = \frac{1}{c^2} \frac{d\Phi_E}{dt} \]

\[ B = \frac{1}{2\pi r c^2} \frac{d\Phi_E}{dt} \]

\[ \Phi_E = \int_A \vec{E} \cdot d\vec{A} = \int_0^{2\pi} \int_0^{15.0} \vec{E} \cdot r dr d\theta = E \cdot \pi \cdot \left( \frac{d}{2} \right)^2 \]

\[ \Rightarrow \frac{d\Phi_E}{dt} = \frac{d^2 E}{dt} \]

\[ \Rightarrow B(0) = \frac{1}{2\pi r c^2} \left( \frac{d^2 E}{dt} \right) \]

\[ B(447\text{m}) = \frac{(0.99)^2 (23.8 \frac{V}{\text{m} \cdot \text{s}})}{8(147\text{m})(3 \times 10^8 \frac{\text{m}}{\text{s}})^2} = 2.2 \times 10^{-18} \text{ T} \]

\[ = 2.2 \times 10^{-18} \text{ T} \]
An electron moves through a uniform electric field $\vec{E} = (2.30 \hat{i} + 4.30 \hat{j})$ V/m and a uniform magnetic field $\vec{B} = 0.40$ kT. Determine the acceleration of the electron when it has a velocity $\vec{v} = 10.0 \hat{i}$ m/s.

$$\vec{a} = (-4.045 \times 10^1 \hat{i} + 5.276 \times 10^1 \hat{j} + 0 \hat{k}) \text{ m/s}^2$$

$$\vec{a} = -e \left( \vec{v} \times \vec{B} + \vec{E} \right) = m_e \vec{a}$$

$$\vec{a} = -e \left( \left( v_x \hat{i} \times B_z \hat{k} + E_x \hat{i} + E_y \hat{j} \right) \right)$$

$$\vec{a} = -e \left( -v_x B_z \hat{j} + E_y \hat{j} + E_x \hat{i} \right)$$

$$\vec{a} = -e \left( E_x \hat{i} + (E_y - v_x B_z) \hat{j} \right)$$

$$\vec{a} = \frac{(1.602 \times 10^{-19} \text{ C})}{9.109 \times 10^{-31} \text{ kg}} \left( 2.30 \frac{\text{V}}{\text{m}} \hat{i} + \left( 4.30 \frac{\text{V}}{\text{m}} - (10 \frac{\text{m}}{s}) (0.47) \right) \hat{j} \right)$$

$$\vec{a} = \left( -4.045 \times 10^1 \frac{\text{m}}{s^2} \right) \hat{i} - \left( 5.276 \times 10^1 \frac{\text{m}}{s^2} \right) \hat{j}$$
3. The speed of an electromagnetic wave traveling in a transparent nonmagnetic substance is given by the following equation, in which $\kappa$ is the dielectric constant of the substance.

$$\nu = \frac{1}{\sqrt{\kappa \mu_0 \varepsilon_0}}$$

Determine the speed of light in cubic zirconia, which has a dielectric constant at optical frequencies of 4.84.

$$\nu = \frac{1}{\sqrt{\kappa \mu_0 \varepsilon_0}} = \frac{C}{\sqrt{\kappa}}$$

$$\nu = \frac{3 \times 10^8 \text{ m/s}}{\sqrt{4.84}} = 1.36 \times 10^8 \text{ m/s}$$
An electromagnetic wave in vacuum has an electric field amplitude of 275 V/m. Calculate the amplitude of the corresponding magnetic field.

\[
\frac{E_{\text{max}}}{B_{\text{max}}} = \frac{c}{\mu_0}
\]

\[
\Rightarrow B_{\text{max}} = \frac{E_{\text{max}}}{c}
\]

\[
B_{\text{max}} = \frac{275 \text{ V/m}}{3 \times 10^8 \text{ m/s}} = 9.17 \times 10^{-7} \text{ V.s/m}^2
\]

\[
B_{\text{max}} = 917 \text{ nT}
\]
A microwave oven is powered by an electron tube called a magnetron, which generates electromagnetic waves of frequency 2.01 GHz. The microwaves enter the oven and are reflected by the walls. The standing wave pattern produced in the oven can cook food unevenly, with hot spots in the food at antinodes and cool spots at nodes, so a turntable is often used to rotate the food and distribute the energy. If a microwave oven intended for use with a turntable is instead used with a cooking dish in a fixed position, the antinodes can appear as burn marks on foods such as carrot strips or cheese. The separation distance between the burns is measured to be 7 cm ±5%. From these data, calculate the speed of the microwaves.

\[ \sqrt[2]{2.814\times10^8} \text{ m/s ±5%} \]

\[ d : \text{distance between burns} \]

\[ d = \frac{\lambda}{2} \Rightarrow \lambda = 2d \]

\[ v = \lambda f = 2df \]

\[ v = 2 \left(0.07 \text{ m}\right) \left(2.01 \times 10^7 \text{ Hz}\right) = 2.81 \times 10^8 \text{ m/s} \]
An AM radio station broadcasts isotropically (equally in all directions) with an average power of 4.00 kW. A dipole receiving antenna 65.0 cm long is at a location 4.00 miles from the transmitter. Compute the amplitude of the emf that is induced by this signal between the ends of the receiving antenna.

\[ \varepsilon_{\text{emf}} = E_{\text{max}} \cdot l \]

\[ \text{Power} = P \text{/ Area} = \frac{E_{\text{max}}^2}{2 \pi \varepsilon_0 C} \cdot \text{Area} \]

\[ \text{Area} = \text{Area of sphere with radius 4.00 miles} \]

\[ \text{(1 mile = 1609 m)} \]

\[ E_{\text{max}} = \sqrt{\frac{2 \mu_0 C P}{A}} \]

\[ \varepsilon_{\text{emf}} = \sqrt{\frac{2 \mu_0 C P}{A}} \cdot l \]

\[ \varepsilon_{\text{emf}} = \sqrt{\frac{2 \cdot (4 \pi \times 10^{-7} \text{ F/m}) \cdot (3 \times 10^6 \text{ W/m}) \cdot (4 \times 10^3 \text{ W})}{4 \pi \cdot (4 \pi \times 1609 \text{ m})^2}} \cdot 0.65 \text{ m} \]

\[ \varepsilon_{\text{emf}} = (0.076 \text{ V/m}) \cdot (0.65 \text{ m}) = 49.4 \text{ mV} \]
In a region of free space, the electric field at an instant of time is \( \mathbf{E} = [(90.0) \hat{i} + (36.0) \hat{j} + (-72.0) \hat{k}] \text{ N/C} \), and the magnetic field is \( \mathbf{B} = [(0.200) \hat{i} + (0.080) \hat{j} + (0.290) \hat{k}] \text{ \mu T} \).

(a) Show that the two fields are perpendicular to each other by calculating the following quantities.

\[
\begin{align*}
E_x B_x &= \frac{18}{\mu_0} \text{ N} \cdot \text{m} / \text{T} \\
E_y B_y &= \frac{2.88}{\mu_0} \text{ N} \cdot \text{m} / \text{T} \\
E_z B_z &= \frac{-20.88}{\mu_0} \text{ N} \cdot \text{m} / \text{T} \\
E_x B_x + E_y B_y + E_z B_z &= 0 \text{ N} \cdot \text{m} / \text{T}
\end{align*}
\]

(b) Determine the component representation of the Poynting vector for these fields. Express the answer using \( i, j, \) and \( k \) to represent \( \hat{i}, \hat{j}, \) and \( \hat{k}, \) respectively. Use three decimal places.

\[
\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} = \frac{1}{\mu_0} \begin{vmatrix} i & \hat{j} & \hat{k} \\ E_x & E_y & E_z \\ B_x & B_y & B_z \end{vmatrix}
\]

\[
\mathbf{E} = \begin{bmatrix} 30 \hat{i} + 16.0 \hat{j} + 75 \hat{k} \end{bmatrix} \text{ \mu T} \\
\mathbf{B} = \begin{bmatrix} 0.16 \hat{i} + 0.58 \hat{j} + 0.4 \hat{k} \end{bmatrix} \text{ \mu T}
\]

\[
\frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} = \frac{1}{\mu_0} \begin{bmatrix} -160 \hat{i} \\
12 \hat{j} - 12 \hat{k} \\
-24 + 43.5 \hat{j} \end{bmatrix} \text{ \mu W/m}^2
\]

\[
\mathbf{S} = \begin{bmatrix} 12.9 \hat{i} - 32.2 \hat{j} + 19.5 \hat{k} \end{bmatrix} \text{ \mu W/m}^2
\]

Clegg, Spencer
A possible means of space flight is to place a perfectly reflecting aluminized sheet into orbit around the Earth and then use the light from the Sun to push this "solar sail."

Suppose a sail of area $7.10 \times 10^5 \text{ m}^2$ and mass $7000 \text{ kg}$ is placed in orbit facing the Sun.

(a) What force is exerted on the sail?

\[ F = 6.48 \text{ N} \]

(b) What is the sail's acceleration?

\[ a = 9.26 \times 10^{-4} \text{ m/s}^2 \]

(c) What time interval is required for the sail to reach the Moon, $3.84 \times 10^8 \text{ m}$ away?

Ignore all gravitational effects, assume the acceleration calculated in part (b) remains constant, and assume a solar intensity of $1370 \text{ W/m}^2$.

\[ t = 10.5 \text{ days} \]

\[ \text{Perfectly reflecting} \implies P \text{ressure} = \frac{2S}{c} \]

\[ F = PA = \frac{2S}{c} \cdot A \]

\[ F = 2 \left( \frac{1370 \frac{W}{m^2}}{3 \times 10^8 \frac{W}{J}} \right) (7.10 \times 10^5 \text{ m}^2) = 6.48 \text{ N} \]

\[ a = \frac{F}{m} = \frac{6.48 \text{ N}}{7000 \text{ kg}} = 9.26 \times 10^{-4} \text{ m/s}^2 \]

\[ x = \frac{1}{2} a t^2 \implies t = \sqrt{\frac{2x}{a}} \]

\[ t = \sqrt{\frac{2 \left( 3.84 \times 10^8 \text{ m} \right)}{(9.26 \times 10^{-4} \text{ m/s})}} \]

\[ t = 9.107 \times 10^5 \text{ sec} \times \frac{1 \text{ day}}{24 \times 3600 \text{ s}} = 10.5 \text{ days} \]
Two hand-held radio transceivers with dipole antennas are separated by a great fixed distance. Assuming the transmitting antenna is vertical, what percentage of the maximum received power will occur in the receiving antenna when it is inclined from the vertical by each of the following angles?

(a) $15.0^\circ$  
$93\%$

(b) $40.0^\circ$  
$59\%$

(c) $80.0^\circ$  
$3\%$

Component of $\mathbf{E}$ along receiving antenna if $\mathbf{E}_\text{cos}\theta$

$\mathbf{I} = \int \alpha |\mathbf{E}_\text{cos}\theta|^2$

Intensity $\propto \cos^2 \theta$

$I_{15^\circ} = \cos^2(15^\circ) = 0.93 = 93\%$

$I_{40^\circ} = \cos^2(40^\circ) = 0.59 = 59\%$

$I_{80^\circ} = \cos^2(80^\circ) = 0.03 = 3\%$
A very large flat sheet carries a uniformly distributed electric current with current per unit width \( J_s \). The current creates a magnetic field on both sides of the sheet, parallel to the sheet and perpendicular to the current, with magnitude
\[
B = \frac{1}{2} \mu_0 J_s \cdot J.
\]
The current oscillates in time according to
\[
J_s = J_{\text{max}}(\cos \omega t) \hat{J} = J_{\text{max}}[\cos (-\omega t)] \hat{J},
\]
and the sheet radiates an electromagnetic wave as shown in the figure below. The magnetic field of the wave is described by the wave function
\[
B = \frac{1}{2} \mu_0 J_{\text{max}} \cos k \cdot x \cdot \sin(k \cdot z - \omega t) \hat{k}.
\]

(a) Find the wave function for the electric field in the wave. (Use \( \mu_0 \) for \( \mu_0 \), \( J_\text{m} \) for \( J_{\text{max}} \), omega for \( \omega \), \( c \), \( k \), \( x \), \( t \), as necessary.)
\[
E = \frac{1}{2} \frac{J_\text{m} c \mu_0}{2} \cos(kx-\omega t) \hat{k}
\]

(b) Find the Poynting vector as a function of \( x \) and \( t \).
\[
S = \frac{1}{4\mu_0 \varepsilon_0} \mu_0 c^2 (\cos(kx-\omega t))^2
\]

(c) Find the intensity of the wave.
\[
\frac{1}{8} \frac{\mu_0 \varepsilon_0}{\mu_0 \varepsilon_0} c^2
\]

(d) What If? If the sheet is to emit radiation in each direction (normal to the plane of the sheet) with an intensity of 535 W/m², what maximum value of sinusoidal current density \( (J_{\text{max}}) \) is required?
\[
3.37 \ A/m^2
\]

2) \[
\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} = -\left(\frac{\omega}{2} \mu_0 J_\text{m} \sin(kx-\omega t)\right)
\]

\Rightarrow \ E = \frac{1}{2} \frac{\omega}{k} \mu_0 J_\text{m} \cos(kx-\omega t)

\frac{\vec{E}}{\frac{\omega}{k} \mu_0 J_\text{m} \cos(kx-\omega t)} \hat{J} \quad \text{must be} \quad \hat{B} \quad \text{and direction of prop.}\ 

b) \[ \vec{S} = \frac{1}{\mu_0} \left[ \vec{E} \times \vec{B} \right] \]

\[ = \frac{1}{\mu_0} \left( \frac{1}{2} c \mu_0 J_m \cos(kx-\omega t) \right) \cdot \frac{1}{2} \mu_0 J_m \cos(kx-\omega t) \]

\[ \frac{3}{4} = \frac{1}{\mu_0 c} \int J_m^2 \cos^2(kx-\omega t) \frac{dx}{k^2} \]

\[ I = \left< \vec{S} \right> = \frac{1}{4} \mu_0 c \int J_m^2 \left< \cos^2(kx-\omega t) \right> \frac{dx}{k^2} \]

\[ I = \mu_0 c \int J_m^2 \frac{dx}{k^2} \]

\[ I = \frac{\mu_0 c \int J_m^2}{8} \]

d) \[ J_m = \sqrt{\frac{8I}{\mu_0 c}} \]

\[ J_m = \sqrt{\frac{8 \left( \frac{535 \mu_2}{m^2} \right)}{4 \pi \times 10^7 \times 3 \times 10^6 \mu_2}} \]

\[ = 3.37 \frac{A}{m^2} \]

\[ \text{solve for } J_m \]
This just in! An important news announcement is transmitted by radio waves to people sitting next to their radios 90 km from the station and by sound waves to people sitting across the newsroom 2.80 m from the newscaster. Take the speed of sound in air to be 343 m/s. Who is the last to receive the news?

radio listeners

studio listeners

What is the difference between the two signal arrival times?

\[ t_{\text{light}} = \frac{\text{distance from station}}{c} \]

\[ t_{\text{light}} = \frac{90 \text{ km}}{3 \times 10^5 \text{ km/s}} = 3 \times 10^{-4} \text{ s} \]

\[ d = \text{Rate} \times \text{Time} \]

\[ t_{\text{sound}} = \frac{\text{distance in room}}{v_{\text{sound}}} = \frac{2.80 \text{ m}}{343 \text{ m/s}} = 8.16 \times 10^{-3} \text{ s} \]

\[ t_s > t_l \]

\[ \Rightarrow \text{studio listeners hear news last.} \]
A 0.80 m diameter mirror focuses the Sun's rays onto an absorbing plate 2.40 cm in radius, which holds a can containing 1.00 L of water at 20.0°C.

(a) If the solar intensity is 1.00 kW/m², what is the intensity on the absorbing plate?

\[ P = \frac{1 \text{kW}}{\text{m}^2} \]

(b) What are the maximum magnitudes of the fields \( \vec{E} \) and \( \vec{B} \)?

\( 14.5 \text{kN/C} \) (electric field)

\( 48.2 \mu \text{T} \) (magnetic field)

(c) If 32.0% of the energy is absorbed, how long does it take to bring the water to its boiling point?

\( 34.7 \text{ min} \)

\[ \text{Power} = P = 1 \text{kW} = \frac{1 \text{ kJ}}{\text{sec}} \]

\[ \text{Energy} = \frac{\text{Mass} \times \text{Specific Heat} \times \Delta T}{1} \]

\[ t = \frac{\text{Energy}}{\text{Power}} = \frac{m \cdot c \cdot \Delta T}{P} \]

\[ t = \frac{(1 \text{ kg}) \cdot (4.18 \times 10^3 \text{ J/kg°C}) \cdot (100°C - 20°C)}{(0.32) \cdot (278 \times 10^3 \text{ J/kg°C}) \times 1 \text{ kW}} = 2082 \text{ sec} = 34.7 \text{ min} \]
\[ \mu_0 = 4\pi \times 10^{-7} \text{T m/A} \]

\[ C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.99 \times 10^8 \text{m/s} \]

\( \text{Electromagnetic constant} \)

\[ \frac{E_{\text{max}}}{B_{\text{max}}} = c = c \frac{E}{B} \]

\[ E = \frac{I^2}{\mu_0 C} = \frac{C P_i^2}{\mu_0} \]

\[ \lambda = \frac{C}{S} \]

\( \omega = \text{angular freq.} \)