This problem must be handed in on paper according to the rules for group homework problems.

[Problem 8-69] A ball having mass \( m \) is connected by a strong string of length \( L \) to a pivot point and held in place in a vertical position. A wind exerting constant force of magnitude \( F \) is blowing from left to right as in the figure. (a) If the ball is released from rest, show that the maximum height \( H \) reached by the ball, as measured from its initial height, is

\[
H = \frac{2L}{1 + \left(\frac{mg}{F}\right)^2}
\]

Check that the above result is valid both for cases when \( 0 < H < L \) and for \( L < H < 2L \). (b) Compute the value of \( H \) using the values \( m = 2.00 \text{ kg} \), \( L = 2.00 \text{ m} \), and \( F = 14.7 \text{ N} \). (c) Using these same values, determine the equilibrium height of the ball. (d) Could the equilibrium height ever be larger than \( L \)? Explain.

(a) We can use conservation of energy to solve this problem.

\[
\begin{align*}
\mathcal{E}_i^0 + \mathcal{E}_f^0 + W &= \mathcal{E}_f^0 + mgH \\
(\mathcal{E}_i^0 &= 0) \quad (H = 0) \quad (\mathcal{E}_f^0 = 0)
\end{align*}
\]

\[
W = F \cdot x = mgH
\]

By the Pythagorean Theorem:

\[
x = \sqrt{L^2 - (L-H)^2} = \sqrt{2LH - H^2}
\]

Since \( F \) is parallel to \( x \),

\[
W = F \cdot x = F \cdot (\cos 0^\circ) = F \cdot x = F \sqrt{2LH - H^2} = mgH
\]

Square both sides:

\[
F^2 (2LH - H^2) = (mg)^2 H^2 \Rightarrow F^2 (2L - H) = (mg)^2 H
\]

Solving for \( H \):

\[
H = \frac{2LF^2}{F^2 + (mg)^2} \left( \frac{1/F^2}{1/F^2} \right) \text{ multiply by 1}
\]

\[
H = \frac{2L}{1 + \left(\frac{mg}{F}\right)^2} = H
\]
Checking the cases:

1) For \( H \to 0 \), it is easier to use the formula for \( F^2 \):

\[ F^2 = \frac{(mgH)^2}{2LH-H^2} = \frac{(mg)^2 H}{2L - H} \]

So

\[ \lim_{H \to 0} F^2 = \frac{0}{2L} = 0 \Rightarrow \text{no } F, \text{ thus there is no height } H \text{ if there is no } F. \]

2) If \( H = L \):

\[ F \sqrt{2L(L) - (L)^2} = mgL \]
\[ F \sqrt{2L^2 - L^2} = mgL \]
\[ FL = mgL \Rightarrow F = mg \checkmark \]

3) For \( H \to 2L \), it is easier to see how \( H \) behaves as \( F \to \infty \):

\[ \lim_{F \to \infty} H = \frac{2L}{1 + \left( \frac{mg}{F} \right)^2} = 2L. \]

This limit exists, but it would be hard to approach experimentally.

(b) \[ H = \frac{2(2 \text{ m})}{1 + \left[ \frac{(2 \text{ kg})(9.8 \text{ m/s}^2)}{14.7 \text{ N}} \right]^2} = 1.44 \text{ m} \]
(c) Let $\theta = \text{equilibrium angle with the vertical}:

\[ \Sigma F_x = -T \sin \theta + F = m_x \]
\[ \Sigma F_y = mg - T \cos \theta = m_y \]

So, 
\[ T \sin \theta = F \]
\[ T \cos \theta = mg \]

Dividing:
\[ \tan \theta = \frac{F}{mg} = \frac{14.7 \text{N}}{19.6 \text{N}} = 0.75 \]
\[ \therefore \theta = 36.9^\circ \]

Thus, 
\[ H_{eq} = L - L \cos \theta = L (1 - \cos \theta) = (2m)(1 - \cos 36.9^\circ) = 0.400 \text{m} \]

(d) As $F \to \infty$, $\tan \theta \to \infty$, $\theta \to 90^\circ$, so $H_{eq} \to L$.

A very strong wind pulls the string out horizontal, parallel to the ground.

\[ \therefore (H_{eq})_{\text{max}} = L \]