**Significant Figures**
George A. Williams

Any physical measurement has some uncertainty in it. For example, it would be silly to measure a length with an ordinary lab meterstick and report that length as 0.94562 m. Clearly you could not measure either the 2 or 6 using a meterstick as a tool. This is taken into account by reporting only the number of figures that make sense. These are called significant figures. In the case above it would be three significant figures, 0.946 m. (Or possibly you might try to estimate tenths of a millimeter to get 0.9456 m.)

Clearly, if the input data of a calculation has a limited accuracy, then the output should not be quoted to any greater degree of accuracy. Therefore, we make the general rule that we do not quote an answer to more significant figures than the least accurate of the input data. Factors like 1/2 or 2 in formulas are, of course, exact and do not affect the number of significant figures appropriate for an answer. [Note: What we are discussing here is the tip of the iceberg of a subject called error analysis. This is covered in Elementary Physics Lab 2219 and can be treated at still higher levels of sophistication.]

The most unambiguous way to determine the number of significant figures is to use scientific, or powers-of-ten notation. This removes the ambiguity as to whether 4530 has three or four significant figures.

\[
4530 = 4.53 \times 10^3 \quad \text{(three significant figures)}
\]

\[
4530 = 4.530 \times 10^3 \quad \text{(four significant figures)}
\]

\[
0.567 = 5.67 \times 10^{-1} \quad \text{(three significant figures)}
\]

\[
0.005670 = 5.670 \times 10^{-3} \quad \text{(four significant figures)}
\]

The following (arbitrary) rule works for rounding off answers to the proper number of significant figures. If the extra digit is 6, 7, 8 or 9, round up. If it is 1, 2, 3 or 4, round down. If it is 5 round up if the digit to its left is odd; drop it if the digit to its left is even. Some pocket calculators have round-off procedures built in. They do not work as above, since they keep all 10 digits in their memory, even though only three or four are displayed. The calculator algorithm rounds 4.5451 to 4.55. The rule above assumes that the second extra digit has no meaning at all.

In order to avoid cumulative errors from round off, keep at least one extra digit for all intermediate steps of a calculation. Then round off at the end. If the answer to part (a) of a problem is an input number for part (b), quote the answer to (a) with the correct number of significant figures, but as input to (b) use one more digit. Most calculators do this automatically. Beware of cases where you subtract two numbers and get a small difference. In many, but not all cases, you must keep many more than the desired number of significant figures in order that the difference term has even one or two.

The goal in all of this is to quote answers to percentage accuracy which is not greater than or less than that allowed by the input data. Note the following: If your input data is 9.80 m/s (three significant figures), an error of ± 1 in the last digit is slightly poorer than 0.1% accuracy. If the answer to the problem comes out 1.016 m you would be tempted to round this to 1.02 m. But ± 1 in the third digit here is 1% error. So it is appropriate to keep one extra digit for answers where the first digit is 1, but not when it is larger. This is done almost automatically on a slide rule. For input data consider that 1.015 has 4 significant figures.
Grading. As far as exams in this course are concerned, we will take off points if too many or too few significant figures are reported. We all also take off points if the answer differs by more than ±1 in the last significant figure from the correct answer. This means that if 9.801 is the correct answer, 9.800 and 9.802 will also be marked correct, but not 9.799, 9.803 or 9.80. If the first digit of the answer is 1, use the procedure described above. The physical constants given to you will all be given to three significant figures. The input data in problems may have more or less than three.

For grading purposes we will use the convention that a number like 4000 in the input data for a problem will be treated as having 4 significant figures. (Which it may well not have in the real world.) In reporting your answers, use scientific notation. We will regard 45,000 as having 5 significant figures when reported in an answer. Use the number of significant figures given in the constants on the front page of the test, not values stored in your head or your calculator. An angle of 37° will be regarded as having 2 significant figures, 37.1°, three. (This is approximately right for all but angles near 90° and 0°.)