Problem 1.

a. Calculate the approximate self-inductance $L$ of a toroid with $N$ windings, a radius of curvature $R$, and cross-sectional area $A^2$ where $l < R$. Since $l < R$, treat as a solenoid of length $2\pi r$.

$$B = \frac{\mu_0 I N}{2\pi R}$$

$$\phi_B = NBL^2$$

$$L = \frac{\phi_B}{I} = \frac{\mu_0 N^2 L^2}{2\pi R}$$

b. What is the magnetic energy in a cylindrical annulus of inner radius $a$ and outer radius $b$ and height $h$ centered on a long straight wire carrying a current $I$?

$$U = \int udV = \int h2\pi r dr$$

$$U = \frac{1}{2\mu_0} \int B^2$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B^2 = \frac{\mu_0 I^2}{4\pi^2 r^2}$$

$$U = \int_a^b \frac{\mu_0 I^2 h}{4\pi r} dr = \frac{\mu_0 I^2 h}{4\pi} \ln \left( \frac{b}{a} \right)$$

c. An inductor and a pair of resistors are hooked up to a battery, as shown. The circuit operates for some long time with the switch closed. Then the switch is opened. Find the total energy $U$ dissipated through the resistor after the switch is opened.

Before the switch is opened - the energy stored in inductor is

$6V + 3\Omega$ $\rightarrow 5H$

$U_L = \frac{1}{2}L I^2$ \quad \Rightarrow \quad I = \frac{V}{R} = \frac{6V}{2\Omega} = 3A$

All this energy will be dissipated through the $3\Omega$ resistor after the switch is opened.

$$U_{KT} = \frac{1}{2}L I^2 = \frac{1}{2}(5H)(3A)^2 = 22.5J$$

d. A wire loop (#1) is in series with a resistor and a battery. A second wire loop (#2) is coaxial with the first and lies in the same plane, as shown. Just after the switch is closed, a current is induced in the second loop. Is the induced current in the clockwise or counterclockwise direction? Justify your answer.

Current in loop #1 is counter clockwise.

Prior to closing switch no magnetic field existed in loop #2, after switch is closed the current in loop #1 creates a magnetic field pointing out of page. To resist this change, loop #2 induces a current that will create a magnetic field pointing into the page.

$\Rightarrow$ the current is clockwise as shown. (short answer: Lenz law)
Problem 2.

a. An LC circuit at \( t = 0 \) s has its capacitor uncharged and a current \( I = 0.25 \) A running through it. Find the maximum potential \( V \) that will be generated across the capacitor.

\[
U_C(\text{max}) = \frac{1}{2} L I_{\text{max}}^2 = \frac{1}{2} C V_{\text{max}}^2
\]

\[
\Rightarrow V_{\text{max}} = \sqrt{\frac{L}{C}} I_{\text{max}} = \left(\frac{2.5 \times 10^{-3} \text{ H}}{1.2 \times 10^{-3} \text{ F}}\right)^{\frac{1}{2}}(0.25 \text{ A})
\]

\[
I_{\text{max}} = 0.36 \text{ V}
\]

b. For the LC circuit in part 2a, at what time \( t \) will the current \( I \) first reach zero?

\[
t = \frac{1}{4} T \quad \Rightarrow T = \frac{1}{f} = \frac{2\pi}{\omega} \quad \Rightarrow \omega = \frac{1}{\sqrt{LC}}
\]

\[
t = \frac{1}{4} \left(2\pi \sqrt{\frac{1}{LC}}\right) = \frac{\pi}{2L} \left(2.5 \times 10^{-3} \text{ H} \cdot 1.2 \times 10^{-3} \text{ F}\right)^{\frac{1}{2}} = 2.7 \times 10^{-3} \text{ s}
\]

c. A driven RLC circuit is set up as shown. If the AC power supply is operating at 60 Hz, how much power is drawn by this circuit?

\[
\omega = 2\pi(60 \text{ Hz}) = 377 \quad X_L = \omega L = 2.64 \Omega
\]

\[
P = I_{\text{rms}} V_{\text{rms}} \cos \phi
\]

\[
\phi = \tan^{-1} \left(\frac{X_L - X_C}{R}\right) = 14.7^\circ
\]

\[
I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}} = 23.2 \text{ A}
\]

\[
P = (23.2 \text{ A} \times 120 \text{ V}) \cos (14.7^\circ) = 2700 \text{ W}
\]
Problem 3.

a. A parallel-plate capacitor, consisting of two circular plates of radius $R$, generates a current flow, associated with a time-varying charge on its plates. This charge is produces an electric field of strength $E = E_0 \exp(-t/\tau)$ between the plates. Find an expression for the magnetic field $B$ between the plates at some distance $r < R$ from the axis of the capacitor.

\[
\mathbf{B} = -\frac{\mu_0 E_0}{2r} E_0 e^{-t/\tau}
\]

b. The average intensity $\bar{S}$ of an electromagnetic plane wave in a vacuum is 0.05 W/m$^2$. What is the peak amplitude of the electric field, $E_p$?

\[
\bar{S} = \frac{E_p^2}{2\mu_0 c} \Rightarrow E_p = \sqrt{\frac{2\mu_0 c \bar{S}}{}} = \sqrt{2 \cdot (4\pi \times 10^{-7} \text{N/A}^2 \cdot 3.0 \times 10^8 \text{m/s} \cdot 0.05 \text{W/m}^2)} = 6.14 \text{ N/C}
\]

c. A radio station transmitter radiates 100,000 W of power uniformly in all directions. What is the average force on a 0.5 m$^2$ umbrella (flat, and oriented as shown) at a distance of 1 km from the transmitter?

\[
F = \frac{P A_n}{4\pi d^2} = \frac{1 \times 10^5 \text{ W} \cdot 0.5 \text{ m}^2}{4\pi (1 \cdot 10^3 \text{ m})^2 (2 \cdot 10^{-8} \text{ m/s})} = 1.33 \times 10^{-11} \text{ N}
\]