1. PSE6 27.P.024. [317703] The rod in figure P27.24 is made of two materials. The figure is not drawn to scale. Each conductor has a square cross section 5.00 mm on a side. The first material has a resistivity of $2.50 \times 10^{-3} \, \Omega \cdot m$ and is 25 cm long, while the second material has a resistivity of $6.00 \times 10^{-3} \, \Omega \cdot m$ and is 40 cm long. What is the resistance between the ends of the rod?

$$R = \frac{\rho l}{A}$$

for each material and resistors in series add like:

$$R_T = R_1 + R_2.$$

Then, for each resistor,

$$R_1 = \frac{\rho_1 l_1}{A_1},$$

$$R_2 = \frac{\rho_2 l_2}{A_2},$$

but $A_1 = A_2 = A = (5.00 \text{ mm})^2 = 2.5 \times 10^{-5} \text{ m}^2$

so

$$R_T = \frac{\rho_1 l_1}{A} + \frac{\rho_2 l_2}{A}$$

$$R_T = \frac{\rho_1 l_1 + \rho_2 l_2}{A}$$

$$= \frac{(2.50 \times 10^{-3} \, \Omega \cdot \text{m})(0.250 \text{ m}) + (6.00 \times 10^{-3} \, \Omega \cdot \text{m})(0.400 \text{ m})}{2.5 \times 10^{-5} \text{ m}^2}$$

$$R_T = 121 \, \Omega$$
2. PSE6 27.P.068. [317765] An oceanographer is studying how the ion concentration in sea water depends on depth. She does this by lowering into the water a pair of concentric metallic cylinders (Fig. P27.68) at the end of a cable and taking data to determine the resistance between these electrodes as a function of depth. The water between the two cylinders forms a cylindrical shell of inner radius \( r_a \), outer radius \( r_b \), and length \( L \) much larger than \( r_b \). The scientist applies a potential difference \( \Delta V \) between the inner and outer surfaces, producing an outward radial current \( I \). Let \( \rho \) represent the resistivity of the water.

![Figure P27.68](image)

(a) Find the resistance of the water between the cylinders in terms of \( L, \rho, r_a, \) and \( r_b \). (Use \( \pi \) for \( \pi \), \( V \) for \( \Delta V \), \( \rho \) for \( \rho \), \( r_a \) for \( r_a \), \( r_b \) for \( r_b \), \( I \) and \( L \) as necessary.)

\[
R = \frac{\rho L}{r_b - r_a}
\]

(b) Express the resistivity of the water in terms of the measured quantities \( L, r_a, r_b, V, \) and \( I \). (Use \( \pi \) for \( \pi \), \( V \) for \( \Delta V \), \( \rho \) for \( \rho \), \( r_a \) for \( r_a \), \( r_b \) for \( r_b \), \( I \) and \( L \) as necessary.)

\[
\rho = \frac{\pi r_b^2 I}{\Delta V}
\]
2. a.) \( R = \frac{V}{I} \)

\[
V = \int_{r_a}^{r_b} E \, dr \quad \text{and} \quad E = \frac{\lambda}{2\pi \varepsilon_0 r} \quad \text{(from Gauss's Law)}
\]

\[
V = \int_{r_a}^{r_b} \frac{\lambda}{2\pi \varepsilon_0 r} \, dr = \frac{\lambda}{2\pi \varepsilon_0} \ln \left( \frac{r_b}{r_a} \right)
\]

For \( I \), use \( J = \frac{I}{A} \), so \( I = JA = J 2\pi r_b L \)

Also, \( J = \sigma E = \frac{E}{\rho} \) (because \( \sigma = \frac{1}{\rho} \))

Then \( I = \frac{E}{\rho} 2\pi r_b L = \left( \frac{\lambda}{2\pi \varepsilon_0 r_b} \right) \frac{2\pi r_b L}{\rho} = \frac{\lambda L}{\rho \varepsilon_0} + I \)

Plug \( V \) and \( I \) into \( R = \frac{V}{I} \)

\[
R = \left[ \frac{\lambda}{2\pi \varepsilon_0} \ln \left( \frac{r_b}{r_a} \right) \right] \frac{\lambda L}{\rho \varepsilon_0} = \frac{\rho}{2\pi L} \ln \left( \frac{r_b}{r_a} \right)
\]

b.) \( \frac{V}{I} = \frac{\rho}{2\pi L} \ln \left( \frac{r_b}{r_a} \right) \)

Solve for \( \rho \):

\[
\rho = \frac{2\pi L V}{V \left( \ln \left( \frac{r_b}{r_a} \right) \right)}
\]
3. PSE 5 27.P.060. [317714] An electric utility company supplies a customer's house from the main power lines 120 V with two copper wires, each of which is 50.0 m long and has a resistance of 0.148 Ω per 300 m.

(a) Find the voltage at the customer's house for a load current of 110 A.

V

(b) For this load current, find the power the customer is receiving.

kW

(c) Find the electric power lost in the copper wires.

W

\[ R_1 = R_2 = \text{Resistance of one copper wire} = \left( \frac{0.148 \Omega}{300 \text{m}} \right) \times 50 \text{m} \]
\[ R_L = \text{Resistance of the load.} \]
\[ R_3 = \text{combined or equivalent resistance of } R_1 \text{ & } R_2 \]
\[ R_3 = R_1 + R_2 = 2R_1 = 2 \left( \frac{0.148 \Omega}{300 \text{m}} \right) \times 50 \text{m} = 4.93 \times 10^{-2} \Omega \]

Now, the sum of the voltage drops equals the total voltage:
\[ 120V = V_1 + V_2 \]
\[ V_1 = I (2R_1) \]
So \[ 120V = I (2R_1) + V_2 \]
\[ V_2 = 120V - I (2R_1) = 120V - (110A)(4.93 \times 10^{-2} \Omega) \]
\[ V_2 = 114.6V \]

\[ P = I V = I V_2 = (110A)(14.6V) = 12.6 \times 10^3 \text{ W} = 12.6 \text{ kW} \]

\[ P = I^2 R = I^2 (2R_1) = (110A)^2 (4.93 \times 10^{-2} \Omega) \]
\[ P = 597 \text{ W} \]
4. Equivalent resistance [384840] Three light bulbs each draw 60 W of power at 120 V. Find the total power drawn when (a) the light bulbs are hooked together in series, and (b) when they are connected in parallel.

a) Series: \[ \text{W} \]

b) Parallel: \[ \text{W} \]

\[ R_L = \text{Resistance of one light bulb.} \]
\[ P = IV = \frac{V^2}{R} \]
\[ \text{So } R_L = \frac{V^2}{P} = \frac{(120V)^2}{60W} = 240 \Omega \]

a) For in series, \( R_T = R_1 + R_2 + R_3 \), with \( R_1 = R_2 = R_3 = R_L \) then \( R_T = 3R_L = 720 \Omega \)
\[ P = \frac{V^2}{R_T} = \frac{V^2}{3R_L} = \frac{(120V)^2}{720 \Omega} = 120 \text{W} = P \]

b) For in parallel, \[ \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]
\[ \frac{1}{R_T} = \frac{1}{R_L} + \frac{1}{R_L} + \frac{1}{R_L} = \frac{3}{R_L} \]
\[ R_T = \frac{R_L}{3} = 80 \Omega \]
\[ P = \frac{V^2}{R_T} = \frac{(120V)^2}{80 \Omega} = 180 \text{W} = P \]