1. PSE6 30.P.003. [317970]

(a) A conductor in the shape of a square loop of edge length \( l = 0.440 \) m carries a current \( I = 11.0 \) A as in Fig. P30.3. Calculate the magnitude and direction of the magnetic field at the center of the square.

From symmetry, we can see that

\[ B_r = 4 B_i \]

where \( B_i \) is the magnetic field from 1 side.

Then using Biot-Savart law

\[ \frac{d \vec{B}}{dr} = \frac{\mu_0 I}{4\pi} \frac{d \vec{r} \times \hat{r}}{r^2} \]

\[ \vec{r} = -x \hat{i} + \frac{l}{2} \hat{j} \]

\[ r^2 = x^2 + \left(\frac{l}{2}\right)^2 \]

\[ \hat{r} = \frac{-x \hat{i} + \frac{l}{2} \hat{j}}{\left(x^2 + \left(\frac{l}{2}\right)^2\right)^{1/2}} = r_x \hat{i} + r_y \hat{j} \]

\[ d \vec{r} \times \hat{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & 0 & 0 \\ r_x & r_y & 0 \end{vmatrix} = r_y \ dx \hat{k} \]

Then

\[ \vec{B}_r = \int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{\mu_0 I}{4\pi} \frac{\frac{l}{2}}{\left(x^2 + \left(\frac{l}{2}\right)^2\right)^{3/2}} \hat{k} \ dx = \frac{\mu_0 I}{4\pi} \frac{l}{2} \left(\frac{2}{l}\right)^2 \frac{x}{\left(x^2 + \left(\frac{l}{2}\right)^2\right)^{1/2}} \bigg|_{-\frac{l}{2}}^{\frac{l}{2}} \]

1) a) \[ B_i = \frac{\mu_0 I}{4\pi r^2} \left( \frac{\ell}{(\frac{\ell}{2})^2 + (\frac{\ell}{2})^2)^{\frac{3}{2}}} - \frac{\ell}{(\frac{\ell}{2})^2 + (\frac{\ell}{2})^2} \right) \]
\[ = \frac{\mu_0 I}{2\pi \ell} \left( \frac{\ell}{\ell^2 + \ell^2} \right) \]
\[ = \frac{\mu_0 I}{\pi \ell \sqrt{2}} \]
\[ = \frac{(4\pi \times 10^{-7} N/A^2)(11.0A)}{(0.440m)\sqrt{2}} \approx 7.07 \times 10^{-6} T \]

\[ \vec{B} = 4\pi \vec{B}_i = 28.3 \times 10^{-6} \text{T} \approx 28.3 \mu T \]

direction - into the page

b) If this conductor is formed into a single circular turn and carries the same current, what is the value of the magnetic field at the center?

again - using Biot-Savart law

\[ d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2} \]

\[ \vec{B} = \int \frac{\mu_0 I}{4\pi} \frac{r d\theta \hat{k}}{r} \]

\[ = \frac{\mu_0 I}{2\pi} \hat{k} \]

so find \( r \)?

\[ 2\pi r = \frac{2\ell}{\pi} \]

\[ r = \frac{2\ell}{\pi} \]

\[ B = \frac{\mu_0 I}{2(2\ell)} = \frac{(4\pi \times 10^{-7} N/A^2)(11.0A)}{\pi (0.440m)} = 24.7 \times 10^{-6} \text{T} \]

= 24.7 \mu T
2. PSE6 30.P.007. [317993] The segment of wire in Figure P30.7 carries a current of \( I = 5.30 \, \text{A} \), where the radius of the circular arc is \( R = 3.50 \, \text{cm} \). Determine the magnitude and direction of the magnetic field at the origin.

From the right hand rule: the direction will be into the page.

Using Biot-Savart's Law, 3 segments:

1) \[
\begin{align*}
\vec{I} & = \int \vec{e} \, dl = \int dy \hat{j} \\
\vec{r} & = \hat{j} \\
\vec{dl} \times \hat{r} & = 0 \implies \text{no contribution}
\end{align*}
\]

Similarly:

2) \[
\begin{align*}
\int \vec{dl} = R \, d\theta \hat{R} \\
\vec{dl} \times \hat{R} & = R \, d\theta \hat{k}
\end{align*}
\]

Then:

\[
\begin{align*}
\frac{d\vec{B}}{4\pi} & = \frac{\mu_0 I}{r^2} \frac{\vec{dl} \times \hat{r}}{2 \pi R} = \frac{\mu_0 I \, Rd\theta \hat{k}}{4\pi R} = \frac{\mu_0 I \, d\theta \hat{k}}{4\pi R}
\end{align*}
\]

\[
\vec{B} = \frac{\mu_0 I}{4\pi R} \int_0^{\pi/2} d\theta \hat{k} = \frac{\mu_0 I}{4\pi R} \frac{\pi}{2} \hat{k} = \frac{\mu_0 I}{8R} \hat{k}
\]

\[
\vec{B} = \frac{4\pi \times 10^{-7} \, \text{N} \, \text{A}^{-1}}{2\pi R} \left( 3.50 \times 10^{-2} \, \text{m} \right) = 23.8 \times 10^{-6} \, \text{T} = 23.8 \, \text{mT}
\]
Two long, parallel conductors separated by 14.0 cm carry currents in the same direction. The first wire carries current $I_1 = 4.00 \, \text{A}$, and the second carries $I_2 = 8.00 \, \text{A}$. (Assume the conductors lie in the plane of the page.)

(a) What is the magnitude of the magnetic field created by $I_1$ and acting on $I_2$?

Use Amperes Law

\[
\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{1en}
\]

\[
B_1 \cdot 2\pi d = \mu_0 I_1
\]

\[
B_1 = \frac{\mu_0 I_1}{2\pi d} = \frac{(4\pi \times 10^{-7} \, \text{N/A}^2)(4.00 \, \text{A})}{2\pi (14.0 \times 10^{-2} \, \text{m})} = 5.71 \times 10^{-6} \, \text{T}
\]

The right-hand rule tells us that $B_1$ is directed out of the page.

(b) What is the force per unit length exerted on $I_2$ by $I_1$?

\[
\mathbf{F} = I_1 \mathbf{l} \times \mathbf{B} = I_2 (\mathbf{\hat{i}} \times \mathbf{B}_1 \mathbf{\hat{k}})
\]

\[
\mathbf{\hat{i}} \times \mathbf{\hat{k}} = -\mathbf{\hat{j}}
\]

\[
\frac{\mathbf{F}_2}{\ell} = -I_2 B_1 \mathbf{\hat{j}} = -(8.00 \, \text{A})(5.71 \times 10^{-6} \, \text{T}) \mathbf{\hat{j}}
\]

\[
= 4.57 \times 10^{-5} \, \text{N/m} \mathbf{\hat{j}} \quad \text{in -y direction}
\]
(c) What is the magnitude of the magnetic field created by \( I_2 \) at the location of \( I_1 \)?

Again, use Ampere's law

\[
\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}
\]

\[
B_2 \cdot 2\pi d = \mu_0 I_2
\]

\[
B_2 = \frac{\mu_0 I_2}{2\pi d} = \frac{(2\pi \times 10^{-7})(8.00A)}{2\pi (14.0 \times 10^{-2} \text{m})} = 1.14 \times 10^{-5} \text{ T}
\]

Use right hand rule: \( B_2 \) is directed into the page.

(d) What is the force per length exerted by \( I_2 \) on \( I_1 \)?

\[
\mathbf{F} = I_2 \mathbf{l} \times \mathbf{B} = I_1 (\mathbf{l} \hat{i} \times (-B_2 \hat{k})) = \hat{i} \times (-\hat{k}) = \hat{j}
\]

\[
\mathbf{F} = I_1 B_2 \mathbf{j} = (4.00 \text{A})(1.14 \times 10^{-5} \text{ T}) \hat{j}
\]

\[
\mathbf{F} = \frac{4.57 \times 10^{-5} \text{ N/m}}{\mathbf{j}} \text{ in the +y direction}
\]
4. In Figure P30.17, the current in the long, straight wire is \( I_1 = 7.00 \text{ A} \) and the wire lies in the plane of the rectangular loop, which carries the current \( I_2 = 10.0 \text{ A} \). The dimensions are \( c = 0.100 \text{ m} \), \( a = 0.150 \text{ m} \), and \( l = 0.500 \text{ m} \). Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire.

\[ \vec{F} = I \vec{l} \times \vec{B} \]

\[ \int B \cdot dl = \mu_0 I \text{enc} \]

\[ \vec{B}_1 = \frac{\mu_0 I_1}{2\pi r} \hat{k} \]

**Section 1**

\[ \vec{B}_1 = \frac{-\mu_0 I_1}{2\pi c} \hat{k} = \frac{(2\pi \times 10^{-7} \text{ N/Am}^2)(7.00 \text{ A})}{2\pi (0.100 \text{ m})} = 1.40 \times 10^{-5} \text{ T} \hat{-k} \]

\[ \vec{F}_1 = I_2 (\hat{l} \times \vec{B}_1 (-\hat{k})) = I_2 \vec{l} B_1 (\hat{j} \times (-\hat{k})) = I_2 \vec{l} B_1 (-\hat{i}) \]

\[ = (10.0 \text{ A})(0.500 \text{ m})(1.40 \times 10^{-5} \text{ T})(\hat{i}) = -7.00 \times 10^{-5} \text{ N} \hat{i} \]

Section 2 & Section 4 cancel - although \( B \) varies along both sections, it varies in the same way - the only significant difference is the direction of current, which is opposite, mathematically this is obvious,
\[ \vec{F}_2 + \vec{F}_y = \int d\vec{F}_2 + \int d\vec{F}_y = \int I_z \, d\vec{l}_x \times \vec{B}_i + \int I_z \, d\vec{l}_y \times \vec{B}_i \]
\[ d\vec{l}_x = dx \hat{i} \]
\[ d\vec{l}_y = -dy \hat{j} \]
\[ \vec{B}_i = -\frac{\mu_0 I_1}{2\pi x} \hat{k} \]
\[ \hat{\times}(-\hat{k}) = \hat{j} \]
\[ (\vec{F}_2 + \vec{F}_y) \bigg|_{c+a}^{c} = \int \frac{I_z I_1 \mu_0}{2\pi x} (\hat{j} \times \hat{k}) \, dx = 0 \]

Section 3
\[ \vec{B}_i = -\frac{\mu_0 I_1}{2\pi (c+a)} = \frac{(2\pi \times 10^{-7} \, N/A^2)(7.00A)}{2\pi (0.100 + 0.150)m} = 5.60 \times 10^{-6} \, T(-\hat{k}) \]
\[ \vec{l} = -\hat{l} \]
\[ \vec{F}_3 = I_z (\vec{l} \times \vec{B}, (-\hat{k})) = I_z \, l \, B_i \, (\hat{j} \times (-\hat{k})) = I_z \, l \, B_i \, \hat{k} \]
\[ = (10.0A)(0.500m)(5.6 \times 10^{-6} T) \, \hat{k} = 2.80 \times 10^{-5} \, N \, \hat{k} \]
\[ \vec{F}_T = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_y = (-7.00 \times 10^{-5} + 2.8 \times 10^{-5}) \, N \, \hat{i} \quad \text{to the left} \]
\[ = 4.20 \times 10^{-5} \, N \, (-\hat{i}) \]