1. PSE6 23.P.021. Four point charges are at the corners of a square of side a, as shown in Figure P23.21. (A = 4, B = 5, C = 2.)

(a) Determine the magnitude and direction of the electric field at the location of charge q generated by the other three charges. (Use k for the Coulomb constant and q and a as necessary.)

\[ \vec{E}_A = \frac{kq}{a^2} \hat{i} \quad \vec{E}_C = \frac{kq}{a^2} \hat{j} \]

\[ \vec{E}_B = \frac{kq}{a^2} \left( \frac{\sqrt{2}}{2} \hat{i} + \frac{\sqrt{2}}{2} \hat{j} \right) \]

\[ \vec{E}_T = \frac{kq}{a^2} \left( \left( 4 + \frac{5\sqrt{2}}{4} \right) \hat{i} + \left( 2 + \frac{5\sqrt{2}}{4} \right) \hat{j} \right) = \frac{kq}{a^2} \left( 5.77 \hat{i} + 3.77 \hat{j} \right) \]

\[ E_T = |\vec{E}_T| = \sqrt{(E_x)^2 + (E_y)^2} = \sqrt{(5.77)^2 + (3.77)^2} \left( \frac{kq}{a^2} \right) \]

\[ = 6.89 \frac{kq}{a^2} \]

\[ \theta = \arctan \left( \frac{E_y}{E_x} \right) = \arctan \left( \frac{3.77}{5.77} \right) = 33.2^\circ \]

(b) What is the resultant force on q?

\[ \vec{F} = \vec{E}_T \cdot q = 6.89 \frac{kq^2}{a^2} \]

\[ \theta = 33.2^\circ \]
2. A continuous line of charge lies along the x axis, extending from \( x = x_0 \) to positive infinity. The line carries charge with a uniform linear charge density \( \lambda_0 \). What are the magnitude and direction of the electric field at the origin? (Type your answer using \( \lambda_0 \), \( x_0 \), and \( k \).

\[
\vec{E} = \int \frac{k dq \hat{r}}{r^2} \quad dq = \lambda_0 dx \quad r = x \\
\hat{r} = -\hat{i} \quad \text{integrate from } x_0 \text{ to } \infty \\
= \int_{x_0}^{\infty} \frac{k \lambda_0 dx \hat{i}}{x^2} = -\frac{k \lambda_0}{x} \bigg|_{x_0}^{\infty} = 0 - \left( -\frac{k \lambda_0}{x_0} \right) \hat{i}
\]

\[
\vec{E} = \left| \frac{k \lambda_0}{x_0} \right| \hat{i}
\]

3. A uniformly charged ring of radius 10.0 cm has a total charge of 83.0 \( \mu \text{C} \). Find the electric field on the axis of the ring at the following distances from the center of the ring.

Use Coulomb\'s Law for electric field & integrate around ring.

By symmetry, we can see that the y & z components of \( \vec{E} \) cancel leaving only the x component.
3) cont.

\[ \vec{E} = \int \frac{k \, dq \, \hat{r}}{r^2} \]

\[ dq = \frac{Q \, d\theta}{2\pi \alpha} \quad r = \sqrt{x^2 + a^2} \]

\[ \hat{r} = \frac{\vec{r}}{r} = \frac{x \, \hat{i} + y \, \hat{j} + z \, \hat{k}}{\sqrt{x^2 + a^2}} \]

\[ \vec{E} = \int_{0}^{2\pi} \frac{k \, Q \, x \, \frac{d\theta}{(x^2 + a^2)^{3/2}}}{2\pi \, (x^2 + a^2)^{3/2}} = 2\pi \, k \, Q \, x \int_{0}^{2\pi} \frac{d\theta}{2\pi \, (x^2 + a^2)^{3/2}} \]

\[ \vec{E} = 7.35 \times 10^6 \text{ N/C} \hat{i} \]

a) (a) 1.00 cm

\[ \vec{E} = \frac{k \, Q \, x \, \hat{i}}{(x^2 + a^2)^{3/2}} \quad x = 0.0100 \text{ m} \quad Q = 8.30 \times 10^{-6} \text{ C} \]

\[ k = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2 \]

\[ a = 0.100 \text{ m} \]

\[ \vec{E} = \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(8.30 \times 10^{-6} \text{ C})(0.0100 \text{ m})}{((0.0100 \text{ m})^2 + (0.100 \text{ m})^2)^{3/2}} = 7.35 \times 10^6 \text{ N/C} \hat{i} \]

\[ = 7.35 \text{ MN/C} \hat{i} \]

b) (b) 5.00 cm

\[ x = 0.0500 \text{ m} \]

\[ \vec{E} = \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(8.30 \times 10^{-6} \text{ C})(0.0500 \text{ m})}{((0.0500 \text{ m})^2 + (0.100 \text{ m})^2)^{3/2}} = 26.7 \text{ MN/C} \hat{i} \]

c) (c) 30.0 cm

\[ x = 0.300 \text{ m} \]

\[ \vec{E} = \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(8.30 \times 10^{-6} \text{ C})(0.300 \text{ m})}{((0.300 \text{ m})^2 + (0.100 \text{ m})^2)^{3/2}} = 7.08 \text{ MN/C} \hat{i} \]

d) (d) 100 cm

\[ x = 1.00 \text{ m} \]

\[ \vec{E} = \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(8.30 \times 10^{-6} \text{ C})(1.00 \text{ m})}{((1.00 \text{ m})^2 + (0.100 \text{ m})^2)^{3/2}} = 0.735 \text{ MN/C} \hat{i} \]
A proton accelerates from rest in a uniform electric field of 670 N/C. At some later time, its speed is $1.50 \times 10^6$ m/s (nonrelativistic, because $v$ is much less than the speed of light).

(a) Find the acceleration of the proton.

Use Newton's 2nd Law

$$\vec{F} = q\vec{E} \quad \text{and} \quad \vec{F} = ma \quad \Rightarrow \quad q\vec{E} = ma$$

$$\vec{a} = \frac{q}{m} \vec{E} \quad \text{(obviously, same direction, so treat as scalar)}$$

$$a = \frac{q}{m} E = \frac{(1.602 \times 10^{-19} \text{C})(670 \text{N/C})}{(1.67 \times 10^{-27} \text{kg})} = 6.43 \times 10^{10} \text{m/s}^2$$

(b) How long does it take the proton to reach this speed?

Use kinematics

$$V = V_0 + at \quad \Rightarrow \quad t = \frac{V}{a} = \frac{1.50 \times 10^6 \text{m/s}}{6.43 \times 10^{10} \text{m/s}^2} = 2.33 \times 10^{-5} \text{s}$$

(c) How far has it moved in this time?

$$X = x_0 + v_0 t + \frac{1}{2} at^2 = \frac{1}{2} \left(6.43 \times 10^{10} \text{m/s}^2\right)(2.33 \times 10^{-5} \text{s})^2 = 17.5 \text{ m}$$

(d) What is its kinetic energy at this time?

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \left(1.67 \times 10^{-27} \text{kg}\right)(1.50 \times 10^6 \text{m/s})^2 = 1.88 \times 10^{-15} \text{J}$$
5. A small, \( m = 7.00 \text{ g} \) plastic ball is suspended by a 20.0 cm long string in a uniform electric field as shown in Figure P23.54. If the ball is in equilibrium when the string makes a 15.0° angle with the vertical, what is the net charge on the ball?

Use Newton's 2nd Law

\[ \sum F = 0. \]

\[ F_x = qE - T\sin 15^\circ = 0 \]

\[ F_y = T\cos 15^\circ - mg = 0 \]

Solve for \( T \) and \( q \)

\[ T = \frac{mg}{\cos 15^\circ} \]

\[ q = \frac{T\sin 15^\circ}{E} = \frac{mg\tan 15^\circ}{E} \]

\[ q = \frac{(0.00700 \text{ kg})(9.81 \text{ m/s}^2)(\tan 15^\circ)}{\left(1.00 \times 10^3 \text{ N/C}\right)} = 1.84 \times 10^{-5} \text{ C} \]

\[ = 18.4 \text{ nC} \]

6. A line of positive charge is formed into a semicircle of radius \( R = 45.0 \text{ cm} \), as shown in Figure P23.63. The charge per unit length along the semicircle is described by the expression \( \lambda = \lambda_0 \cos \theta \). The total charge on the semicircle is 15.0 \( \mu \text{C} \).

Calculate the total force on a charge of 2.00 \( \mu \text{C} \) placed at the center of curvature.

Integrate again

\[ \vec{F} = q \vec{E} \]

\[ \vec{E} = \sum \frac{k dq}{r^2} \hat{r} \]

Figure P23.63
\[
6) \text{cont.} \quad dq = \lambda R \, d\Omega = \lambda_0 R \cos \theta \, d\theta
\]

\[
r = R \quad \hat{r} = -\sin \theta \hat{i} - \cos \theta \hat{j}
\]

\[
\vec{E} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} k \frac{\lambda_0 R \cos \theta}{R^2} (-\sin \theta \hat{i} - \cos \theta \hat{j}) \, d\theta
\]

\[
= k \frac{\lambda_0}{R} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -\sin \theta \cos \theta \hat{i} - \cos^2 \theta \hat{j} \, d\theta
\]

\[
= k \frac{\lambda_0}{R} \left( 0 \hat{i} - \frac{\pi}{2} \hat{j} \right) = -k \frac{\lambda_0 \pi}{2R} \hat{j}
\]

find \( \lambda_0 \)

\[
\vec{Q} = \int d\vec{q} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \lambda_0 R \cos \theta \, d\theta = 2 \lambda_0 R
\]

\[
\Rightarrow \lambda_0 = \frac{Q}{2R} \Rightarrow E = k\pi \frac{Q}{4R^2} = \frac{(8.99 \times 10^9 \text{Nm}^2/\text{C}^2)(15.0 \times 10^{-6} \text{C})\pi}{4(0.450 \text{m})^2}
\]

\[
E = 5.23 \times 10^5 \text{N/C} \quad \text{down}
\]

\[
F = qE = (2.00 \times 10^{-6} \text{C})(5.23 \times 10^5 \text{N/C}) = 1.05 \text{N} \quad \text{down}
\]