Problem 1. Inductance

a. Two circular wire loops lie in a plane about a common central point. If the current in the larger loop is clockwise and decreasing in time, which direction is the induced current in the smaller loop? Justify your answer.

Use Lenz’ law that the current induced in the inner loop will try to generate a B-field to oppose change in magnetic flux. Since B from larger loop is inward and weakening with time the current in the smaller loop will try to support the inward direction field. Thus it is clockwise.

b. A square loop with length \( l \) to a side, containing a resistor (resistance \( R \)), is pushed with speed \( v \) into a region of constant magnetic field \( \mathbf{B} \) as shown. What is the voltage across the resistor?

\[
E = -\frac{d\Phi_B}{dt}
\]

Faraday law: \( E = \frac{d\Phi_B}{dt} \) (\( \Phi_B = x \times lB \) (\( x \) is length of loop in B-field))

\[
\frac{d\Phi_B}{dt} = lB \frac{dx}{dt} = lBV
\]

Thus \( V_R = E = lBV \)

c. A time-varying, spatially-uniform magnetic field of strength \( B = b \sin(\gamma t) \) is confined to an infinite cylindrical region of radius \( R \) as shown. Find the electric field strength outside of the cylindrical region as a function of time \( t \) and distance \( r \) from the cylinder’s axis.

\[
\Phi_B = \pi R^2 \mathbf{B}(t), \text{ so}
\]

\[
\frac{d\Phi_B}{dt} = -b\pi R^2 \cos(\gamma t)
\]

The Faraday law here is \( \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \)

The line integral is \( 2\pi r \) \( \mathbf{E} \) (\( \mathbf{E} \) by symmetry), so

\[
E = \frac{b\pi R^2}{2r} \cos(\gamma t)
\]
Problem 2. Inductors and RL circuits.

a. Two long coaxial solenoids have 100 windings per meter and radii \( r_1 = 0.05 \) m and \( r_2 = 0.7 \) m respectively. Find the mutual inductance of the pair of solenoids. \( L = 5 \) m

Define mutual inductance \( M = \frac{\phi_2}{I_1} \)

\[ \phi_2 = B \pi r_1^2 \times \text{# of loops} = \pi r_1^2 500 \]

\[ I_1 = \frac{B}{\mu_0 100} \]

\[ \Rightarrow M = (\pi r_1^2 500) \times (\frac{B}{\mu_0 100}) = \frac{4 \pi^2 (0.05)^2 \times 10^{-3} \times 10 \times 500}{\text{magnetic flux}} = 9.6 \times 10^{-8} \text{ H} \]

b. What is the total current in the circuit shown immediately after the switch is closed?

Immediately after switch is closed \( \frac{\text{current}}{\text{total}} = \frac{\text{current}}{\text{total}} \)

\[ = \frac{\text{current}}{\text{total}} \]

\[ = \frac{\text{current}}{\text{total}} \]

Thus \( I(t=0) = \frac{\varepsilon}{2R} \)

c. The switch in the circuit shown is closed at time \( t = 0 \) s. If \( R = 2.5 \Omega \), \( L = 32 \text{ H} \), and EMF \( \varepsilon = 6 \text{ V} \), what is the potential across the resistor at time \( t = 5 \) s?

From loop equations, know that \( \varepsilon_L = \varepsilon_0 e^{-Rt/L} \)

Also, from loop \( V_R = \varepsilon_0 - \varepsilon_L = \varepsilon_0 (1 - e^{-Rt/L}) \)

with \( \varepsilon_0 = 6 \text{ V} \), \( R = 2.5 \Omega \), \( L = 32 \text{ H} \), \( t = 5 \text{ s} \),

\[ V_R(t=5\text{s}) = 6 \left(1 - e^{-\frac{2.5 \times 5}{32}}\right) = 1.94 \text{ V} \]
Problem 3. LC & RLC-circuits

a. An LC circuit has 5 A running through the inductor \((L = 9.9 \, \text{H})\) at a time \(t = 0\) s when the capacitor \((C = 5.2 \times 10^{-3} \, \text{F})\) is uncharged. How quickly will the current in the inductor fall to zero?

\[
C \quad \begin{array}{c}
\text{natural frequency} \quad \omega_0 = \frac{1}{\sqrt{LC}} = 4.41 \, \text{rads/s} \\
\text{frequency} \quad f = \frac{\omega_0}{2\pi} = \\
\text{current is varying sinusoidally and will hit zero in} \quad \frac{\pi}{2\omega_0} = \\
\end{array}
\]

b. What is the maximum charge on the capacitor in part (a)?

Use Conservation of Energy, for example. \(U_{\text{tot}}(t=0) = \frac{1}{2} \cdot 9.9 \, \text{H} \cdot (5 \, \text{A})^2 = 123 \, \text{J}\)

(from \(U = \frac{1}{2}LI^2\) when there is no charge on \(C\)). This will be the energy in the capacitor when the charge on it has maxed out:

\[U_{\text{tot}} = \frac{1}{2} \frac{1}{C} Q^2 \Rightarrow Q = \sqrt{2CU_{\text{tot}}} = 1.13 \, \text{C} \]

c. An RLC circuit is driven by a sinusoidal AC generator with \(V_{\text{rms}} = 120 \, \text{V}\) at 60 Hz. If \(R = 250 \, \Omega\), \(L = 22 \, \text{H}\), and \(C = 4.4 \, \mu\text{F}\), what is the phase \(\phi\) of the driving potential relative to the current in the circuit?

Driving frequency \(\omega = 2\pi f = 2\pi (60 \, \text{Hz}) = 377 \, \text{rads/s}\)

Capacitive/Inductive reactances are:

\[X_C = \frac{1}{\omega C} = 602 \, \Omega \quad X_L = \omega L = 8.29 \, \Omega\]

\[\tan \phi = \frac{X_L - X_C}{R} = 30.76 \quad \phi = \tan^{-1}(30.76) = 1.54 \, \text{rad} \text{ or } 88.1^\circ\]

d. How much power is drawn by the circuit in part (c)?

\[\text{Avg. Power is} \quad \bar{P} = I_{\text{rms}} V_{\text{rms}} \cos \phi \quad \text{or} \quad \frac{V_{\text{rms}}^2 \cos \phi}{2}\]

Where impedance \(Z\) is \(\sqrt{R^2 + (X_L - X_C)^2} = \)

with \(\phi = 88.1^\circ\) from part c,

\[\bar{P} = 61 \, \text{mW}\]