Problem 1.

a. A parallel-plate capacitor with circular plates of radius R is fed with long straight wires running along the capacitor's axis, as shown. Calculate the magnetic field resulting from the buildup of charge Q in the capacitor for points which are on the capacitor's midplane but outside of the capacitor itself (r > R). Show that Ampère's law for steady currents gives the same result, provided that r > R.

\[
\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi}{dt}
\]

Using Ampère's law for steady currents:

\[
\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I
\]

\[
2\pi r B = \mu_0 I_0 \Rightarrow B = \frac{\mu_0 I_0}{2\pi r}
\]

b. An electromagnetic planewave travels in a vacuum. The magnetic field varies with distance y (meters) and t (seconds) as \( \mathbf{B} = 4.4 \sin(1.7y - 8.4 \times 10^7 t) \hat{y} \). What is the electric field \( \mathbf{E} \) and the mean intensity \( \overline{S} \) of the radiation?

\[
\mathbf{E} = \mathbf{E}_0 \sin(1.7y - 8.4 \times 10^7 t) \hat{x}
\]

\[
\overline{S} = \frac{E_0 B_0}{2} \Rightarrow \overline{S} = 1.32 \times 10^{15} \text{ W/m}^2
\]

c. Find the wavelength \( \lambda \) and frequency \( f \) of the plane wave in part (b).

\[
\lambda = \frac{\omega}{k} = \frac{\lambda}{1.7} \Rightarrow \lambda = 3.7 \text{ m}
\]

\[
f = \frac{c}{\lambda} = \frac{3 \times 10^8}{3.7} \Rightarrow f = 8.1 \times 10^7 \text{ Hz}
\]
Problem 2.

a. A satellite measures the average intensity of light from the Sun to be 9.8 kW/m² near the orbit of Mercury \((R = 5.8 \times 10^{10} \text{ m})\). The satellite absorbs all of the Sun’s radiation through a surface of area 2.4 m² facing the Sun. What is the force from Solar radiation on the satellite?

\[
P = \frac{F}{c} = \frac{F \cdot A}{c} = \frac{5A}{c} = \frac{9.8 \times 10^{12} \times 2.4 \times 10^{8}}{3.0 \times 10^{8}} = 7.8 \times 10^{-5} \text{ N}
\]

\[
P_w = \frac{F}{c} \cdot \frac{A}{c} = \frac{7.8 \times 10^{-5} \text{ N}}{1} = 7.8 \times 10^{-5} \text{ N}
\]

b. Using the intensity of sunlight measured by the satellite in part (a) at Mercury’s orbit, find the total luminosity of the Sun.

\[
P = \frac{5A}{3} = \frac{9.8 \times 10^{12} \times 2.4 \times 10^{8}}{3.0 \times 10^{8}} = 4.1 \times 10^{12} \text{ W}
\]

C. A 10 kg satellite accelerates by sending off a steady burst of laser light. If the laser power is 950 MW and the burst lasts for 30 s, what is the change in the satellite’s speed?

\[
\Delta \rho = \frac{U}{c} = \frac{9.5 \times 10^{12} \times 30}{3.0 \times 10^{8}} \text{ m/s} \quad \Delta \rho = \frac{\Delta P}{m} = \frac{9.5 \times 10^{12} \times 30}{3.0 \times 10^{8}} \text{ m/s}
\]

\[
\Delta V = \frac{U}{c} = \frac{9.5 \times 10^{12} \times 30}{3.0 \times 10^{8}} \text{ m/s} = 9.5 \text{ m/s}
\]

\[
f = \frac{F}{c} = \frac{9.5 \times 10^{12}}{3.0 \times 10^{8}} \text{ m/s}
\]

\[
a = \frac{F}{m} = \frac{9.5 \times 10^{12} \times 30}{3.0 \times 10^{8}} \text{ m/s} \quad V = a \cdot t = \frac{9.5 \times 10^{12} \times 30}{3.0 \times 10^{8}} = 9.5 \text{ m/s}
\]

d. A beam of unpolarized light travels through three polarized filters as shown. If the average intensity of light in the beam after having passed through all three filters is \(S_f = 1.34 \text{ W/m}^2\), what is the average intensity \(S_f'\) after the middle filter is removed?

\[
S_f = \frac{1}{3} \cdot 1.34 \text{ W/m}^2 \quad S_{30^\circ} = \frac{1}{3} \cdot 1.34 \text{ W/m}^2
\]

\[
S_{60^\circ} = \frac{1}{3} \cdot 1.34 \text{ W/m}^2
\]

\[
S_f' = S_{30^\circ} + \gamma = 0.60 \text{ W/m}^2 + \gamma
\]
Problem 3.

a. A ray enters a prism of index of refraction \( n = 1.53 \), as shown. Sketch the path of the ray. Label all relevant angles of incidence, reflection and/or refraction, and give their numerical values.

\[
\begin{align*}
n \sin \theta_c &= 1 \\
\theta_c &= \sin^{-1} \left( \frac{1}{n} \right) = 40.8^\circ + 1 \\
n \sin \theta_f &= \sin \theta_f \\
\theta_f &= 50^\circ + 2
\end{align*}
\]

b. Two light beams, one red and the other blue, travel through a slab of glass. How much faster is the red light if the indices of refraction are \( n_{\text{red}} = 1.51 \) and \( n_{\text{blue}} = 1.54 \)?

\[
V = \frac{C}{n} + 2
\]

\[
V_r - V_b = C \left( \frac{1}{n_r} - \frac{1}{n_b} \right) = \frac{C (n_b - n_r)}{n_r n_b} = 3.86 \times 10^6 \text{m/s}
\]

\( + 2 \)

c. In the diagram below, the values of the indices of refraction are \( n_1 = 1.33 \) and \( n_2 = 1.9 \), and the light rays are all polarized in the plane of the page. Sketch the direction of oscillation of the radiating charges (labelled \( q \)) along the incident and refracted rays. State in a short sentence why there is no radiation along the reflected ray. Finally, calculate the angle of incidence, \( \theta_1 \).

\[
\tan \theta_1 = \frac{n_2}{n_1} + 2
\]

\[
\theta_1 = \arctan \left( \frac{n_2}{n_1} \right) = 55^\circ
\]

\( + 2 \)

There is no radiation in the direction of acceleration (same as the direction of oscillation of the radiating charges).