Problem 1 (20 points)
Problem 2.36 from Griffiths (given below).

Problem 2 (20 points)
For a positive charge $+q$ brought close to a conducting sphere (Fig. 2), the solution of the electrostatic problem by the method of images is given in the lecture notes. Take those results as known while doing this problem.

a) What is the image of a dipole, oriented toward the center of the conducting sphere, if the sphere is not grounded?

b) How would the system of image charges found in (a) change, if the sphere were grounded?

For parts (c)-(e) we are back to the single charge problem:

c) Knowing the electric potential outside the grounded sphere from the solution to the image charge problem for a single external charge, calculate the surface charge density as a function of angle $\theta$ shown in Fig. 2 (due to axial symmetry of the problem, the surface charge density does not depend on the azimuthal spherical angle). *

d) Sketch the dependence of the calculated $\sigma(\theta)$ on $\theta$. The easiest way to do this would be to chose a particular relation between $r$ and $R$, say $R = r/2$, and plot the resulting function, apart from the dimensionful factors like $\frac{q^2}{r^2}$.

e) Calculate the total charge induced on the sphere. The integral over $\theta$ that you encounter is simple, but feel free to do it using some symbolic math software. Relate your answer to the solution of the image problem. **

* to calculate the surface charge density, you will have to calculate the derivative of the electric potential in the direction normal to the surface, which is the radial direction in this case. (If the origin of the spherical coordinates is chosen at the center of the sphere.) Do not confuse the radial distance to the observation point with the radius of the sphere when writing the expression for the electric potential and taking derivatives.

** It is important to write $dS$ properly in $\int dS\sigma(\theta)$ when calculating the total charge. In this case it is $dS = 2\pi R^2 \sin(\theta)d\theta$. You can check that this is the correct expression by calculating $\int_0^{\pi} d\theta 2\pi R^2 \sin(\theta) = 4\pi R^2$ - the total surface area of the sphere.
Problem 2.36 Two spherical cavities, of radii $a$ and $b$, are hollowed out from the interior of a (neutral) conducting sphere of radius $R$ (Fig. 2.49). At the center of each cavity a point charge is placed—call these charges $q_a$ and $q_b$.

(a) Find the surface charges $\sigma_a$, $\sigma_b$, and $\sigma_R$.

(b) What is the field outside the conductor?

(c) What is the field within each cavity?

(d) What is the force on $q_a$ and $q_b$?

(e) Which of these answers would change if a third charge, $q_c$, were brought near the conductor?