Problem 1

a) Without loss of generality we can let the line connecting the two ends of the wire be lying on the x-axis.

\[ \text{l} = \text{Total length} \]
\[ \text{d} = \text{distance between ends} \]
\[ I = \text{current (const)} \]

Any segment \( dl \) of wire can be written in its x and y components. \( dl = dx \hat{x} + dy \hat{y} \).

The magnetic force is

\[
\begin{align*}
\text{d}F_x &= \int_{\hat{x}}^{\hat{y}} I \, dl \times \vec{B} \\
&= I \int_{\hat{x}}^{\hat{y}} (dx \hat{x} + dy \hat{y}) \times \vec{B} \\
&= I \left[ -\int_{\hat{x}}^{\hat{y}} dy \, B_x \hat{y} + \int_{\hat{x}}^{\hat{y}} dx \, B_y \hat{x} \right] \\
&= -I \, dB \, \hat{y} \\
&= 0
\end{align*}
\]

which does not depend on the total length of the wire... only the distance between its ends. \( \square \)
\[ b) \quad \sum f_y = T \cos \theta - mg = 0 \implies T = \frac{mg}{\cos \theta} \]

\[ \sum f_x = IlB - T \sin \theta \implies IlB = mg \tan \theta \]

\[ \Rightarrow \theta = \tan^{-1} \left( \frac{IlB}{mg} \right) = \tan^{-1} \left( \frac{(1.2A)(4910^{-2} m)(0.5T)}{(30 \times 10^{-3} kg)(9.8 \frac{m}{s^2})} \right) \]

\[ \theta = 45^\circ \]
Problem 2

Charge density \( \lambda \).

\[
\mathbf{F} = \lambda \mathbf{r} \cdot \mathbf{E}
\]

where

\[
E = \frac{\lambda l}{2\pi \varepsilon_0 r}
\]

(\text{Gauss's Law})

\[
\frac{F_e}{l} = \frac{\lambda^2}{2\pi \varepsilon_0 r}
\]

Magnetic force per unit length experienced by the same beam:

\[
F_B = e \left| \mathbf{v} \times \mathbf{B} \right| = \lambda e v B
\]

where

\[
B = \frac{\lambda v \mu_0}{2\pi r}
\]

(Ampere's Law)

We can compare their strengths by taking the ratio:

\[
\frac{F_B}{F_e} = \frac{\lambda^2 v^2 \mu_0}{2\pi \varepsilon_0 \lambda^2} = \frac{v^2}{\varepsilon_0 c^2}
\]

Since electrons are massive particles, they have

\( v < c \)

so

\[
\frac{F_B}{F_e} < 1
\]

which means that the beams repel.

b) The beams neither repel nor attract when \( F_B = F_e \) so

\( v = c \)

which cannot happen.
Problem 3

\[ \vec{V}_0 = V_0 \cos \alpha \hat{z} + V_0 \sin \alpha \hat{x} \]

\[ \vec{E} = E \hat{z} \]

\[ \vec{B} = B \hat{z} \]

The charge \( q \) experiences the Lorentz force

\[ \vec{F} = q (\vec{E} + \vec{v} \times \vec{B}) = m \vec{a} \]

where

\[ \vec{v} = x \hat{x} + y \hat{y} + z \hat{z} \]

\[ \vec{a} = x \hat{x} + y \hat{y} + z \hat{z} \]

Breaking this into components:

\[ m \ddot{x} = q \dot{y} B \]

\[ m \ddot{y} = -q \dot{x} B \]

\[ m \ddot{z} = q E \]

The electric field accelerates the particle in the \( z \) direction.

The magnetic field creates uniform circular motion in the \( xy \)-plane. The frequency is the cyclotron frequency, \( \omega = \frac{qB}{m} \)

Notice that the \( z \) equation is not coupled to the \( x \) and \( y \) equations.

So the trajectory looks like a spiral, where the distance between subsequent loops increases.