**Bound currents**

We saw earlier that the appearance of electric polarization in dielectrics led to the appearance of bound charges, both bulk and surface ones.

An analogous situation appears when a particular body acquires magnetic polarization; this leads to bound currents flowing around that body.

Such bound currents (or magnetization currents) inside a medium should be contrasted with free (or transport) currents, the latter can cause a net charge flow through a cross section of conductor ("charge transport") while bound currents cannot do that (see below). "Bound currents" are by "bound electrons".

Bound currents also get bulk and boundary contributions. Let us consider the boundary ones first. To this end, imagine we have a sample with uniform magnetization:

[Diagram of a sample with uniform magnetization]

...and represent the uniform magnetization as being due to a bunch of current loops stacked next to each other, each having area $A_j$ and...
the height of $d$ (see the fig). We notice immediately that all currents inside the sample cancel as the adjacent sides of the loops carry opposite currents. However, the sides of loops that "touch" the boundary do not have neighbors and their currents are not cancelled. Thus there is a giant loop of uncompensated current flowing around the sample. To determine the current, we write

\[ \| \vec{M} \| = \frac{I \cdot dS}{\partial x} = \frac{I}{d} = K_0 - \text{surface current density}. \]

The direction of the current is such that it is $\perp$ to both $\vec{M}$ ($\perp$ to the plane) and $\vec{n}$ (obviously), thus we can write $K_0 = \alpha \vec{M} \times \vec{n}$, and it is obvious from the figure that $\alpha = 1$, thus

\[ K_0 = \vec{M} \times \vec{n} \]

The component of $\vec{M} \times \vec{n}$ on a surface, thus along $\vec{n}$, does not contribute to $K_0$.

Now we treat bulk contributions: when $\vec{M} = \vec{M}(x)$ changes in space, the cancellation between the currents of loops inside the sample do not cancel.

\[ \frac{d}{dx} = -M_z(x+\Delta x) + M_z(x) = -\frac{M_z}{\partial x} \Delta x \]

or

\[ \Delta y = \frac{K_y}{\partial x} = -\frac{M_z}{\partial x}. \]
This is not the only contribution to $j_y$. If there is $M_x$ that varies in the $z$-direction, we get an additional contribution from the same kind of argument, the total $j_y$ is

$$j_y = \frac{\partial M_x}{\partial z} - \frac{\partial M_y}{\partial x}$$

-- this is nothing but the $y$-component of $\nabla \times \mathbf{M}$. We can repeat these considerations for $j_x$ and $j_z$ to conclude that indeed

$$\mathbf{j} = \nabla \times \mathbf{M}$$

Relation between bulk and boundary currents:

As usual, we have to separate the surface currents explicitly if we had to consider rapid variation of $\mathbf{M}$ on atomic scales near a sample boundary. If we allow such rapid variation at the expense of getting singularities in the derivatives of $\mathbf{M}$, we can combine both kinds of currents into a single expression

$$\mathbf{j}_b = \nabla \times \mathbf{M}$$. Let us see where boundary currents are hidden:
If we have a sample with a uniform \( \vec{M} = (0, 0, M) \), we get \( \nabla \times \vec{M} = \hat{y} \cdot \left( -\frac{\partial M_z}{\partial x} \right) = M \cdot \hat{y} \cdot (8(x-x_k) - 8(x-x_L)) \).

That is, if we consider all currents as bulk ones, there are \( \delta \)-function-like spikes of \( \vec{j} \) at the boundaries:

- We interpret the coefficients in front of the \( \delta \)-functions as surface current densities:

\[ k_y(x_R) = M, \quad k_y(x_L) = -M \quad \text{as expected} \]

from \( \vec{K} = \vec{M} \times \hat{n} \).

Overall, we conclude that we can treat all bound currents as bulk only if the surface current term appears as a part of the bulk current tied to the boundary:

\[ \int_{-\delta}^{\delta} \vec{J} \cdot d\vec{s} \rightarrow \text{this integral vanishes if there is no surface current.} \]

This is the point of view we take from now on.

Why "bound":

\[ \text{Bound} = \int_{\text{loop}} \vec{J} \cdot d\vec{s} = \int_{\text{loop}} (\nabla \times \vec{M}) = \oint_{\text{surface}} \vec{M} \cdot d\vec{n} = 0. \]
The **H** field (aka "the very messy place")

The current elements we have just considered are completely analogous to any other current as far as their ability to produce the magnetic field is concerned. Thus

\[
\nabla \times \mathbf{B} = \mu_0 (\mathbf{J}_f + \mathbf{J}_b) = \mu_0 (\mathbf{J}_f + \nabla \times \mathbf{M})
\]

free current

Thus if we introduce \( \mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M} \) we see that

\[
\nabla \times \mathbf{H} = \mathbf{J}_f
\]

- curl of \( \mathbf{H} \) is determined by \( \mathbf{J}_f \) only.

To summarize:

\[
\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \quad \text{or} \quad \mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}, \quad \text{and}
\]

\[
\nabla \times \mathbf{H} = \mathbf{J}_f \quad \text{or} \quad \oint \mathbf{H} \times d\mathbf{l} = I_{\text{free}}.
\]

\( \mathbf{H} \) is analogous to \( \mathbf{D} \) on electostatics, however, \( \mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M} \) looks like \( \mathbf{B} = \varepsilon \mathbf{E} + \mathbf{P} \), which causes a lot of confusion. Just remember that \( \mathbf{B} \) is the true magnetic field, just like \( \mathbf{E} \) while \( \mathbf{H} \) is an auxiliary quantity, which, nevertheless, is what often controlled in experiments.
As in the case with $\mathcal{D}$, the fact that $\partial_x \mathbf{H} = \mathbf{J}_p$ does not imply that we can simply solve the vacuum problem for $\mathbf{B}$ with the same current, since $\partial \cdot \mathbf{H} \neq 0$ in general:
\[
\partial \cdot \mathbf{H} = \partial \cdot (\rho_0 \mathbf{B} - \mathbf{M}) = -\partial \cdot \mathbf{M}, \quad \text{and} \quad \partial \cdot \mathbf{M} \text{ need not be zero:} \quad \partial \cdot \mathbf{M} = \partial \mathbf{M}(z) \Rightarrow \frac{\partial \mathbf{M}}{\partial z} \neq 0.
\]
Thus $\mathbf{H}$ would be particularly useful when it is clear from symmetry that $\partial \cdot \mathbf{M} = 0 = \partial \cdot \mathbf{P}$. 

\[\text{\textsuperscript{6}}\]