**Collab Solution 13**

**Part 1**

One way is to add all charge divided by distance

\[
V_c = \text{potential at center} = \frac{kQ}{b} + \frac{kQ}{c} + k \int_{\text{charge at } r = c}^{\text{charge at } r = b} \frac{4\pi r^2 dr}{\rho} \]

\[
= \frac{kQ}{c} - \frac{kQ}{b} + k \frac{4\pi a^2}{\rho} = \frac{kQ}{b} \left[ \frac{1}{c} - \frac{1}{b} + \frac{2}{2a} \right]
\]

**Part 2**

\[
J = \text{current density} = \frac{E}{\rho}
\]

\[
\vec{J} \cdot d\vec{l} \text{ around outside of bar} = M_0 I = M_0 \frac{Ea^2}{\rho}
\]

The integral from \(P_1 \) to \(P_2 \) can be split into leg I and leg II as shown. Leg II is \( \perp \) to \( \vec{B} \) field... so

Leg I contributes \( \frac{1}{8} \) of \( \vec{J} \cdot d\vec{l} \) as answer is

\[
\frac{M_0 E a^2}{\rho}
\]

**Part 3**

First consider

\[
\frac{1}{\rho a^2} \int\frac{dx}{\sqrt{x^2+a^2}}
\]

\[
E_y = k \lambda a \int_{\frac{a}{\sqrt{x^2+a^2}}}^{x} \frac{dx}{\sqrt{x^2+a^2}}
\]

\[
E_x = -k \lambda a \int_{\frac{a}{\sqrt{x^2+a^2}}}^{x} \frac{x dx}{\sqrt{x^2+a^2}}
\]

Thus

\[
\frac{k\lambda a}{\rho a} \left( \frac{1}{x} - \frac{1}{a} \right)
\]

The two bars, by symmetry, give

\[
\int_{\frac{a}{\sqrt{x^2+a^2}}}^{x} \frac{dx}{\sqrt{x^2+a^2}} = k\lambda a
\]

**Part 4**

Answer must be same as answer to the question: What is the torque by the ring on the solenoid?
Part 4 (cont'd) The field due to the ring at its center is
\[ B = \frac{\mu_0 I_1 N_1}{2 R_1} \]

The magnetic dipole moment of the solenoid is
\[ M = N_2 I_2 \pi R_2^2 \] see eq. 29-10

The torque has magnitude
\[ T = |\mathbf{M} \times \mathbf{B}| = |\mathbf{M}||\mathbf{B}| \cos \theta = \frac{M^2}{2} \]

Part 5 Compute the impedance
\[ X_L = \frac{L}{2\pi f(10)} = 18.85 \Omega \]
\[ C = \varepsilon_0 \frac{A}{d} = 8.85 \times 10^{-12} \cdot \frac{10^2}{10^{-6}} \]

\[ X_C = \frac{1}{\omega C} = \frac{1}{\pi \times 10^{-4} \times 8.85 \times 10^{-9}} = 17.98 \Omega \]

\[ Z = \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{87.2^2 + 1^2} = 1.32 \Omega \]

\[ \text{peak current} = \frac{800V}{1.32\Omega} = 605A \]

If this is the peak current, then the peak charge on the capacitor must be
\[ Q = 605A \times 2\pi f(10) \]

The corresponding E field is
\[ E = \frac{Q}{\varepsilon_0 A} \]

\[ E = \frac{9.63}{8.85 \times 10^{-12}} = 1.09 \times 10^{10} \text{ V/m} \]

Part 6 First image
\[ l = 23\frac{1}{2} \]

\[ \frac{1}{l'} = \frac{1}{5} - \frac{1}{2} = \frac{1}{10}cm = \frac{3}{20}cm \]

\[ l = \frac{10}{23\frac{1}{2}} \times 1cm = .429 \text{ cm} \]

Second image
\[ l' = 10cm \]

\[ \frac{1}{l' + \frac{1}{f}} = \frac{1}{5} \]

\[ f = -7cm \]

\[ l = 10cm \]

\[ \frac{1}{l'} = \frac{1}{f} - \frac{1}{l} = -\frac{1}{7} - \frac{1}{10} \]

\[ l' = \frac{70}{17}cm = 4.12 \text{ cm} \]

Final image size = \[ \frac{4.12cm}{10cm} \times .429 \text{ cm} = .176 \text{ cm} \]