Collab #3 Solutions

Part 1. Since \( E = 0 \) outside \( V_a = 0 \)

Part 2. \[ \int_{a}^{b} \frac{d\phi}{r} = 0 \Rightarrow V_b = 0 \]

Part 3. \[ V_d = k \int \frac{d\phi}{r} = k\left(-\frac{Q}{2R}\right) + \frac{k}{R} \int_{r}^{2R} \frac{d\rho}{r} \]

Outer surface contribution

\[ = -\frac{kQ}{2R} + 4\pi k \rho \frac{1}{2} \left[R^2 - R^2\right] = -\frac{kQ}{2R} \]

Thick shell contribution

\[ = -\frac{kQ}{2R} + 4\pi k \rho \frac{3}{2} R^2 \]

But \( Q \) = charge on thick shell = \[ \frac{4\pi}{3} [2R^3 - R^3] \rho = \frac{7}{3} 4\pi \rho R^3 \]

Thus \( V_d = \frac{-kQ}{2R} + 4\pi k \frac{3}{2} R^2 \left[\frac{3}{7} \frac{Q}{4\pi \rho R^3}\right] \]

\[ = \frac{kQ}{7R} \]

Part 4. Add \( \frac{kQ}{R} \) to the potential at all points

Part 5. \( E = 0 \) between \( d \) and \( c \), hence \( V_c = V_d = \frac{kQ}{7R} \)

Part 6. \[ V_b - V_c = -\int_{c}^{b} E \cdot dr = -\int_{c}^{2R} \frac{kQ}{7} \left[\frac{r}{R^3} - \frac{1}{r^2}\right] dr = -\frac{kQ}{7R} \]

But from parts 2 and 5

\[ V_b - V_c = 0 - \frac{kQ}{7R} = -\frac{kQ}{7R} \]

Part 9. \[ \text{E}^r \]

Part 10. Since \( E^r \) is negative at \( r = 0 \) (i.e., just outside) \( r = 0 \), there must be an infinite density of negative charge at \( r = 0 \). By comparison with the example done in class we can conclude \( \rho = \frac{const}{r} \) for \( r < R \). Since \( E^r \) jumps discontinuously at \( r = R \) there must be a positively charged surface distribution at \( r = R \).