Collab #4 Solutions

Part 1: This can be approximated as a parallel plate capacitor.

\[ C = \varepsilon_0 A \frac{d}{L} = \frac{8.85 \times 10^{-12} \pi (4)^2}{10^{-3}} = 2.78 \times 10^{-8} \text{ F} \]

Part 2: \[ U = \frac{1}{2} C Q^2 = \frac{1}{2} \cdot \frac{1}{2.78 \times 10^{-8}} (10^{-9})^2 = 1.8 \times 10^{-4} \text{ J} \]

Part 3: To rough order of magnitude the answer should be the same as for spheres of radius R=1m. Consider change \( \pm Q \) on such spheres.

\[ \pm Q \quad \text{o} \quad \text{o} \quad \Delta V = \frac{2kQ}{R} \Rightarrow C = \frac{Q}{\Delta V} = \frac{Q}{2k} = \frac{1}{18 \times 10^9} \]

\[ \therefore C = 5.56 \times 10^{-11} \text{ F} \]

Part 4:

\[ \text{Equivalent circuit with } C_0 = 2.78 \times 10^{-8} \text{ F} \]

\[ C = \frac{2}{3} C_0 = \frac{2}{3} \left( \frac{2.78 \times 10^{-8}}{3} \right) = 1.85 \times 10^{-8} \text{ F} \]

Part 5: View ckt as \[ \begin{array}{c}
\text{C} \\
\text{C}
\end{array} \]

The voltage distribution is 667 volts and 333 volts, so

\[ Q = CV = 2.78 \times 10^{-8} \times 333 = 0.927 \times 10^{-5} \text{ C} \]

\[ \text{cylinder f (positive)} \]

\[ Q_f = 1.85 \times 10^{-5} \text{ C} \]

\[ Q_f = 9.27 \times 10^{-6} \text{ C} \]

Part 6: View as \[ \frac{2C_0}{4} \]

\[ U = \frac{1}{2} CV^2 = \frac{1}{2} \times 2.78 \times 10^{-8} \times (10^3)^2 = 9.27 \times 10^{-3} \text{ J} \]

Part 7: Without h, we have

\[ C = \frac{1}{2} C_0 \quad \text{The change on the system is the same as in part 7} \]

\[ Q = \frac{2}{3} C_0 (1000 \text{ V}) = 1.85 \times 10^{-5} \text{ C} \]

\[ \therefore U = \frac{1}{2} C Q^2 = \frac{1}{2} \left( \frac{1.85 \times 10^{-5}}{C_0} \right)^2 = 1.24 \times 10^{-2} \text{ J} \]

Thus, work required = \[ \frac{1.24 \times 10^{-2} \text{ J}}{U \text{ without}} - \frac{9.27 \times 10^{-3} \text{ J}}{U \text{ with h}} = 3.09 \times 10^{-3} \text{ J} \]