Part 1 Since \( \overrightarrow{E} \) is parallel to the curve on which we are integrating, and since \( \overrightarrow{E} \) is constant on that curve:

\[
emf = \oint E \cdot dl = E \oint dl = E(2\pi a)
\]

But \( E \cdot dl = E^2 = K^2 (x^2 + y^2) e^{-2t/\tau} \tau = K^2 a^2 e^{-2t/\tau} \Rightarrow E = K a e^{-t/\tau}
\]

and hence \( emf = K 2\pi a^2 e^{-t/\tau} \)

Part 2 If \( B \propto e^{-t/\tau} \) then \( -\frac{dB}{dt} = \frac{1}{\tau} B \) and \( -\frac{d\phi}{dt} = \frac{1}{\tau} \phi \)

Thus \( -\frac{d\phi}{dt} = \frac{1}{\tau} \phi = emf = K 2\pi a^2 e^{-t/\tau} \)

Hence \( \phi = KT 2\pi a^2 e^{-t/\tau} \)

Part 3 The flux inside a circle of radius \( a \) is proportional to \( a^2 \). The area inside the circle is also proportional to \( a^2 \). Thus when we change the radius, the change in area accounts for all of the change in \( \phi \). If \( B \) were different at different radii, this could not be the case.

Since \( B \) is uniform (independent of \( r \)) we have

\[
\phi = B \cdot A = KT 2\pi a^2 e^{-t/\tau} \Rightarrow B = 2KT e^{-t/\tau}
\]

Part 4 A static \( B \) field (i.e., one that is unchanging in time) would make no contribution to the induced \( \overrightarrow{E} \) field, hence we could infer nothing about it.

Part 5 Divide the square into the 4 legs shown.

On legs 1 and 4 the legs are \( \perp \) to \( \overrightarrow{E} \), so the contribution to \( \oint \overrightarrow{E} \cdot dl \) is zero.

For Leg 1: \( x = a \cdot \overrightarrow{E} = K(a \hat{x} - y \hat{y}) e^{-t/\tau} \)

\[
\int_{E} dl = \int_0^a Kae^{-t/\tau} \ dy = K a^2 e^{-t/\tau}
\]

For Leg 2: \( y = a \cdot \overrightarrow{E} = K(x \hat{x} - a \hat{y}) e^{-t/\tau} \)

\[
\int_{E} dl = \int_0^a Kae^{-t/\tau} \ dx = + K a^2 e^{-t/\tau}
\]
Thus the total emf is $2K\alpha^2 e^{-t/T}$

But in the square $\Phi = AB = (a^2)\left(2KTe^{-t/T}\right)$

and by Faraday's law,

$$\text{emf} = -\frac{d\Phi}{dt} = 2\alpha^2 K e^{-t/T} \leq \text{agrees with answer above}$$

**Part 6** First, lets find the emf on a circle of radius $\alpha$.

Since $r = \alpha$, constant on a circle, the factor $\frac{\alpha}{r} - \frac{\beta}{r^3}$ just modifies $|\vec{E}|$ and we can write down the emf immediately:

$$\text{emf} = \left(\frac{\alpha}{\alpha^2} - \frac{\beta}{\alpha^3}\right) \frac{K 2\pi \alpha^2 e^{-t/T}}{\text{answer to part 1}}$$

It follows that the flux inside the circle is

$$\Phi = \left(\frac{\alpha}{\alpha^2} - \frac{\beta}{\alpha^3}\right) \frac{K 2\pi \alpha^2 e^{-t/T}}{\text{answer to part 2}} = 2\pi KT (\alpha - \frac{\beta}{\alpha}) e^{-t/T}$$

To infer $B(r)$ realize that

$$\Phi = \frac{K 2\pi \alpha^2 e^{-t/T}}{\text{answer to part 2}} = \alpha \int \frac{dA}{\vec{B} \cdot dA} = \int B(r) 2\pi r dr$$

What function $B(r)$ could result in an answer for the integral that has the form on the left? The only possibility is

$$B(r) = \frac{\beta KTe^{-t/T}}{r^3}$$

(Try it in the integral!)

**Part 7** What current source could produce a $B$ field that is perpendicular to the plane and dies off as $1/r^3$.

We know that a ring of radius $R$, lying in the plane will produce a $B$ field $\propto$ to the plane that dies off as $1/r^2$ for $r > R$. So imagine a ring of current centered on the origin of infinite current and zero size, and time dependence $e^{-t/T}$. This does the job.