Problem 1
A homogeneous cylinder with a length of 15 cm and a radius of 1 cm has a voltage of 4 volts across it and carries a uniform current of 12 Amps.

6 points (a) What is the current density at the center of the cylinder?
\[ J = \frac{I}{A} = \frac{12 \text{ A}}{\pi\left(10^{-2} \text{ m}\right)^2} = \]

6 points (b) What is the electrical power being dissipated in the cylinder?
\[ P = IV = (12 \text{ A})(4 \text{ V}) = \]

7 points (c) What is the resistivity (not the resistance) of the material of which the cylinder is made?
\[ R = \frac{V}{I} = \frac{4 \text{ volts}}{12 \text{ A}} = \frac{1}{3} \Omega \]
But \[ R = \rho \frac{l}{A} \]
\[ \rho = \frac{RA}{l} = \left(\frac{1}{3} \Omega\right)\left(\pi \times 10^{-4} \text{ m}^2\right) \]
\[ \frac{0.15 \text{ m}}{} = 6.98 \times 10^{-4} \]

6 points (d) Electrons (charge = \(-1.6 \times 10^{-19} \text{ C}\)) are the carriers in the cylinder. The density of mobile electrons is \(10^{20}\) electrons per m\(^3\). What is the drift speed of the electrons?
\[ \overline{V} = \frac{n|e| V_d}{} = 3.82 \times 10^4 \]
\[ V_d = \frac{\overline{J}}{n|e|} = \frac{3.82 \times 10^4}{10^{20} \times 1.6 \times 10^{-19}} = 2.39 \times 10^3 \]
**Problem 2 Short Explanations** (Answer in complete coherent sentences. Answers without explanations will receive little or no credit. Use back of sheet if necessary.)

**8 points**

(a) Two capacitor plates carry charge \( \pm Q \). Does the voltage between the plates increase or decrease when a dielectric slab is introduced in the space between the plates? Explain why this happens from a microscopic (i.e., atomistic) point of view.

Dipoles in the dielectric material align with the \( \mathbf{E} \) field. The field produced by the rotated dipoles is uniform and in the direction opposite to that produced by the \( \pm Q \) charges. Thus the \( \mathbf{E} \) field is smaller than that produced by the \( \pm Q \) charges alone, and hence the voltage difference is reduced between the plates.

**9 points**

(b) The following two formulas are given in your book for the power dissipated in a resistor: (A) \( P = I^2 R \), and (B) \( P = V^2 / R \). The resistance \( R \) of a superconductor is zero; is the power dissipated in a superconductor zero or infinite? Explain why one formula applies and the other does not.

Current can circulate in a superconductor. Once a current is set up it goes on forever, hence it cannot be dissipating power and we have \( P = 0 \). Since \( I \neq 0 \), we conclude \( V = 0 \). Thus both formulas are correct but \( P = V^2 / R \) should be interpreted as \( P = 0 \) and hence as useless.

**8 points**

(c) The top figure at the right shows an electric field between parallel plates with the electric field completely confined to the region between the plates. Explain why the field cannot be completely confined, why there must be “fringing fields” as shown in the bottom figure. Hint: Consider the potential difference between points A and B as evaluated on the dotted path in the top figure.

Consider the path from A to B along the dotted line. If there were no fringing fields outside, then along the dotted line we would have \( \int \mathbf{E} \cdot d\mathbf{l} = 0 \), since \( \mathbf{E} = 0 \) along the dotted line. But \( |\int \mathbf{E} \cdot d\mathbf{l}| \) is the voltage difference and cannot be \( E_0 V_0 \) [Clearly \( \Delta V = |\int \mathbf{E} \cdot d\mathbf{l}| \) along an internal path is not zero.]

Problem 3
A spherically symmetric, but nonuniform, charge distribution is confined to the region between radii \( R_1 \) and \( R_2 \) as shown. The radial electric field is zero for \( r < R_1 \) and is zero for \( r > R_2 \). For \( R_1 < r < R_2 \) the radial electric field has the value \( E_r = E_0 \), where \( E_0 \) is a positive constant. Define the electrostatic potential to be zero at infinity.

2 points (a) What is the potential at point \( A \), just outside the charge distribution?

\[ \text{Since } E = 0 \text{ for } r > R_2 \text{ the potential at } A \]
\[ \text{must be the same as it is at } \therefore V_A = 0 \]

3 points (b) What is the potential at point \( B \) just inside the outer surface of the charge distribution?

\[ V_A - V_B = \int_A^B E_r \, dl = 0 \]
\[ \text{since } E \text{ is finite and the path length is zero (i.e., negligible).} \]

7 points (c) On the graph below, sketch the potential \( V \) as a function of radius \( r \).

\[ E = 0 \]
\[ \text{hence } V \]
\[ \text{is constant} \]

\[ \frac{dV}{dr} = -E_0 \]

7 points (d) What is the potential at point \( C \)?

\[ V_B - V_C = \int_C^B E_r \, dl = -\int_{R_1}^{R_2} E_0 \, dr = -E_0 (R_2 - R_1) \]

Thus \( V_C = E_0 (R_2 - R_1) \)

6 points (e) What is the electrostatic energy stored in this charge configuration?

Energy density \( = \frac{1}{2} \varepsilon_0 \varepsilon_r \varepsilon = \frac{1}{2} \varepsilon_0 E_0^2 \)

Volume \( = \frac{4\pi}{3} (R_2^3 - R_1^3) \)

Energy = energy density \times volume

\[ \frac{4\pi \varepsilon_0 R_2^3 (R_2^3 - R_1^3) E_0^4}{3} \]
Problem 4

This problem involves parallel plate capacitors with plates that measure 3 cm by 3 cm and are separated by 1 millimeter. The capacitors each have a capacitance of 47 pF.

5 points  (a) If the voltage across one of these capacitors is 5 Volts, what is the magnitude of charge on its plates?

\[ Q = CV = 47 \times 10^{-12} \times 5 \]

\[ \boxed{2.35 \times 10^{-10} \, C} \]

7 points  (b) If the energy stored in one of these capacitors is 1 Joule, what is the voltage across the capacitor?

\[ E_{\text{energy}} = 1 \, \text{Joule} = \frac{1}{2} CV^2 = 23.5 \times 10^{-12} \times V^2 \]

\[ V = \sqrt{\frac{1}{23.5 \times 10^{-12}}} = 2.06 \times 10^5 \, V \]

\[ \boxed{2.06 \times 10^5 \, V} \]

6 points  (c) What is the dielectric constant of the material that fills the space between the plates of the capacitor?

\[ C = \kappa \varepsilon_0 \frac{A}{\ell} \Rightarrow \kappa = \frac{C \ell}{\varepsilon_0 A} = \frac{(47 \times 10^{-12})(10^{-3})}{8.85 \times 10^{-12} \left( \frac{3 \times 10^{-6}}{3} \right)^2} \]

\[ = 5.9 \]

\[ \boxed{5.9} \]

7 points  (d) Three of the 47 pF capacitors are connected as shown. What is the effective capacitance between points A and B?

\[ \frac{2\sqrt{5}}{3} \times 47 \, \text{pF} = 31.3 \, \text{pF} \]