For two point charges $Q$ in orbit with angular velocity $\Omega$ and radius $a$, the standing wave solution to the linear problem is

$$\Psi_{\text{stand}} = 2Q \sum_\ell \sum_{m=\text{even}}^{} mY^*_\ell m(\pi/2, 0)Y_\ell m(\theta, \varphi)j_\ell(m\Omega r_<)n_\ell(m\Omega r_>). \quad (1)$$

The static solution (solution for $\Omega = 0$) to the linear problem is

$$\Psi_{\text{stand}} = -2Q \sum_\ell \sum_{m=\text{even}}^{} \frac{1}{2\ell + 1} Y^*_\ell m(\pi/2, 0)Y_\ell m(\theta, \varphi) r_<^\ell r_>^{\ell - 1}. \quad (2)$$

We are interested in the solution outside the sources, that is, a solution for $r_< = a$ and $r_> = r$. A comparison of the static and standing wave solutions amounts to the comparison

$$\text{stat} = \frac{1}{2\ell + 1} \frac{a^\ell}{r^\ell + 1/2} \quad \text{vs.} \quad \text{stat} = -m\Omega j_\ell(m\Omega a)\sqrt{r} n_\ell(m\Omega r). \quad (3)$$

As $m\Omega r \to 0$ these two have the ratio

$$1 + \frac{(m\Omega r)^2}{2(2\ell - 1)} + \cdots \quad (4)$$

This suggests that (for moderate $\ell$) the static approximation stops being good at some value of $m\Omega r$ near unity.

The following graphs show the logs of “stat” and “stand” for several values of $m\Omega$. Very very roughly, we see that for $m\Omega = 0.6$ (our standard case for $m = 2, \Omega = 0.3$) the approximation is good up to around $r = 1$; this suggests that more generally the approximation is good up to $r \approx 0.6/m\Omega$. For $m\Omega = 0.3$ this predicts that the approximation is good up to $r \approx 2$. The $m\Omega = 0.3$ very roughly confirms this. For $m\Omega = 0.1$ we would predict that the approximation is good up to $r \approx 6$. The graph makes it look as if the approximation is only good up to 4 or so, but that’s close enough.
This rule tells us – roughly – that for $r$ less than $0.6/m\Omega$ the solution is accurately approximated by the static solution, and hence is not influenced by boundary conditions. The condition $r = 1$, however, is right at the sources for $m\Omega = 0.6$. It is still a good approximation to say that the solution is approximately static for for $r$ less than 1.3, or – assuming $m = 2$ – that the condition is that $r$ be less than $0.4/\Omega$. A glance at the plots confirms this; in each case the error seems to be no more than 10%.

**We take the inner edge of our matching to be $r = 0.4/\Omega$**

The only reason to have any width to the matching region is to have the extracted solution be smooth. Let us take the outer edge to be at $r = 0.5/\Omega$. Here the “static approximation” is beginning to get into trouble. The matching then spans the range of $r$ over which the static approx goes from good approx to not so good.

**We take the inner edge of our matching to be $r = 0.5/\Omega$**

![Figure 1: The PSW solution is meant to be an approximation to the physical spacetime only in a limited region.](image_url)
Figure 2: The PSW solution is meant to be an approximation to the physical spacetime only in a limited region.

Figure 3: The PSW solution is meant to be an approximation to the physical spacetime only in a limited region.