5.8 (a) For an atom with \( Z \) protons in the nucleus (charge \( Ze \)) and one electron (charge \(-e\)), Coulomb’s law (Eq. 5.9) becomes
\[
F = -\frac{1}{4\pi \varepsilon_0} \frac{Ze^2}{r^2},
\]
and the electrical potential energy becomes
\[
U = -\frac{1}{4\pi \varepsilon_0} \frac{Ze^2}{r}.
\]
This means that the quantity “\( e^2 \)” should be replaced by “\( Ze^2 \)” wherever it appears in an expression for the hydrogen atom. Similarly, “\( e^4 \)” should be replaced by “\( Z^2 e^4 \)” The result will be the corresponding expression for a one-electron atom with \( Z \) protons. From Eq. (5.13), the orbital radii are therefore
\[
r_n = \frac{4\pi \varepsilon_0 \hbar^2}{\mu Z e^2} n^2 = \frac{a_0}{Z} n^2,
\]
where \( a_0 = 0.0529 \) nm. The energies are
\[
E_n = -\frac{\mu Z^2 e^4}{32\pi^2 \varepsilon_0^2 \hbar^2} \frac{1}{n^2} = -13.6 \text{ eV} \frac{Z^2}{n^2}
\]
from Eq. (5.14).

(b) \( Z = 2 \) for singly ionized helium (He II), and so for the ground state \((n = 1)\),
\[
r_1 = \frac{a_0}{2} (1^2) = 0.0265 \text{ nm}
\]
and
\[
E_1 = -13.6 \text{ eV} \frac{Z^2}{1^2} = -54.4 \text{ eV}.
\]
The ionization energy is 54.4 eV.

(c) \( Z = 3 \) for doubly ionized lithium (Li III), and so for the ground state,
\[
r_1 = \frac{a_0}{3} (1^2) = 0.0176 \text{ nm}
\]
and
\[
E_1 = -13.6 \text{ eV} \frac{Z^2}{1^2} = -122 \text{ eV}.
\]
The ionization energy is 122 eV.
5.9 (a) If the hydrogen atom were held together solely by the force of gravity, the force between the proton and electron would be supplied by Newton's law of universal gravitation (Eq. 2.11),

\[ F = -\frac{G m_p m_e}{r^2} \hat{r}, \]

and the potential energy is given by Eq. (2.14),

\[ U = -\frac{G m_p m_e}{r}. \]

This means that the quantity \( \frac{e^2}{4\pi \epsilon_0} \) should be replaced by \( \frac{G m_p m_e}{r} \) wherever it appears in an expression for the hydrogen atom. Similarly, \( \frac{e^4}{16\pi^2 \epsilon_0^2} \) should be replaced by \( G^2 m_p^2 m_e^2 \). The result will be the corresponding expression for the gravitational atom. From Eq. (5.13), the ground-state orbital radius \((n = 1)\) is therefore

\[ r_1 = \frac{\hbar^2}{\mu G m_pm_e} = 1.20 \times 10^{18} \text{ nm} = 8.03 \times 10^{17} \text{ AU}. \]

The ground-state energy is

\[ E_1 = -\frac{\mu G^2 m_p^2 m_e^2}{2\hbar^2} = -2.64 \times 10^{-78} \text{ eV}. \]

from Eq. (5.14).

5.11 Use \( \lambda = \frac{1}{\lambda} \left( \frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2} \right) \) (Eq. 5.8 with \( m = m_{\text{low}} \) and \( n = n_{\text{high}} \)) for the hydrogen wavelengths. The shortest wavelengths emitted for the Lyman series \((n_{\text{low}} = 1)\), Balmer series \((n_{\text{low}} = 2)\), and Paschen series \((n_{\text{low}} = 3)\) occur for \( n_{\text{high}} \to \infty \). The series limits are therefore given by

\[ \lambda_{\infty} = \frac{1}{R_H n_{\text{low}}^2} = (91.18 \text{ nm}) n_{\text{low}}^2. \]

So

\[ \lambda_{\infty} = 91.18 \text{ nm} \quad \text{ (ultraviolet)} \]

for the Lyman series limit,

\[ \lambda_{\infty} = 364.7 \text{ nm} \quad \text{ (near ultraviolet)} \]

for the Balmer series limit, and

\[ \lambda_{\infty} = 820.6 \text{ nm} \quad \text{ (infrared)} \]

for the Paschen series limit.

5.12 The wavelength of the electron is given by Eq. (5.17),

\[ \lambda = \frac{\hbar}{p} = \frac{\hbar}{m_e v} = 1.45 \times 10^{-11} \text{ m} = 0.0145 \text{ nm}. \]

5.14 As shown in Example 5.4.2, the minimum speed of a particle (in this case, an electron) can be estimated using Heisenberg's uncertainty relation, Eq. (5.19), as

\[ v_{\text{min}} = \frac{p_{\text{min}}}{m_e} \approx \frac{\Delta p}{m_e} \approx \frac{\hbar}{m_e \Delta x}. \]

With \( \Delta x \approx 1.5 \times 10^{-12} \text{ m} \), the minimum speed of the electron is \( v_{\text{min}} \approx 7.7 \times 10^7 \text{ m s}^{-1} \), about 26% of the speed of light. Relativistic effects are important for white dwarf stars.
5.17 From Eq. (5.22), the normal Zeeman effect produces a frequency splitting given by
\[ \Delta v = \frac{eB}{4\pi\mu} = 4.76 \times 10^{16} \text{ Hz}. \]
Using \( v_0 = c/\lambda_0 \) with \( \lambda_0 = 656.281 \text{ nm} \) shows that the frequencies are
\[ v_0 + \Delta v = 4.56853 \times 10^{14} \text{ Hz} \]
\[ v_0 = 4.56806 \times 10^{14} \text{ Hz} \]
\[ v_0 - \Delta v = 4.56758 \times 10^{14} \text{ Hz}. \]
The corresponding wavelengths are
\[ \frac{c}{v_0 + \Delta v} = 656.212 \text{ nm} \]
\[ \frac{c}{v_0} = 656.281 \text{ nm} \]
\[ \frac{c}{v_0 - \Delta v} = 656.349 \text{ nm}. \]

6.7 (a) From Eq. (6.6), \( \theta_{eye} = 1.34 \times 10^{-4} \text{ rad} = 28'' \).

(b) \( \theta_{moon} = 2\pi/d = 9.0 \times 10^{-3} \text{ rad} = 190'' \), implying \( \theta_{moon}/\theta_{eye} = 33.7 \). When Jupiter is at opposition (closest approach to Earth), \( \theta_{planet} = 2.3 \times 10^{-4} \text{ rad} = 47'' \), implying \( \theta_{planet}/\theta_{eye} = 1.7 \). The resolution is worse during all other relative positions of Jupiter and Earth.

(c) It is possible for the disk of the Moon to be resolved with the naked eye. However, the disk of Jupiter is comparable to the eye’s resolution limit; to distinguish any features on the surface of Jupiter, they would need to be nearly the size of the planet’s disk.

6.8 (a) From Eq. (6.6), \( \theta_{max} = 3.4 \times 10^{-6} \text{ rad} = 0.69'' \).

(b) Given the Moon’s mean distance of \( 3.84 \times 10^5 \text{ km} \), the minimum size of a crater that can be resolved is \( h = d\theta = 1.3 \text{ km} \).

(c) Atmospheric turbulence limits the resolution.

6.9 (a) The NTT focal ratio is \( f/2.2 \) and the diameter of the primary mirror is 3.58 m. From Eq. (6.7), \( F = 2.2 = f/3.8 \text{ m} \), giving \( f = 8.36 \text{ m} \).

(b) From Eq. (6.4), \( d\theta/dy = 1/f = 0.120 \text{ rad m}^{-1} \).

(c) \( \Delta \theta = 2.9'' = 1.4 \times 10^{-5} \text{ rad} \), implying \( \Delta y = f \Delta \theta = 11.8 \mu \text{m} \).
Homework #4 Solutions

6.11  
(a) \( \nu_m = 1.43 \) GHz, \( \nu_f = 1.405 \) GHz, and \( \nu_u = 1.455 \) GHz.
(b) Using Eq. (6.10) with \( S(\nu) = S_0 = 2.5 \) mJy and \( D = 100 \) m,

\[
P = \int_d \int_{\nu} S(\nu) f_{\nu} d\nu dA
= \pi S_0 \left( \frac{D}{2} \right)^2 \int_{\nu_f}^{\nu_u} f_{\nu} d\nu.
\]

Substituting the functional form for the filter function and evaluating the integral gives

\[
P = \pi S_0 \left( \frac{D}{2} \right)^2 \left[ \frac{1}{2} (\nu_u - \nu_f) \right] = 4.91 \times 10^{-18} \text{ W}.
\]

(c) At a distance \( d \) from the source, the power at the telescope is given by

\[
P_{\text{rec}} = P_{\text{em}} \frac{\pi (D/2)^2}{4\pi d^2}.
\]

Solving for the emitted power, \( P_{\text{em}} = 7.5 \times 10^{42} \text{ W} \).