Physics 5020: Final examination.
(Dated: Friday, April 29, 2011)

Warning: please be aware that this exam must be done individually, no team-work and/or consultations with peers are allowed.
The exam is due on Monday, May, by 5 p.m.

Problem 1.
(a) Two point charges, $-5q$ and $+2q$, are located at points $x = 0$ and $x = a$, correspondingly. See Figure 1. Find the point, or points, at which the electric field due to the two charges is zero. Sketch the electric field lines qualitatively.
[10 points]
(b) Consider now the system of two positive charges, of magnitude $+5q$ and $+2q$, located at points $x = 0$ and $x = a$ as in part (a). Find the point, or points, at which the electric field due to the two charges is zero. Sketch the electric field lines qualitatively. [10 points]
Hint: this simple problem is surprisingly tricky. Be sure to make a good sketch of it when solving.

![Figure 1: Two charges on the line.](image)

Problem 2.
(a) A cylindrical conductor (wire) of radius $R$ carries a uniform current $I$, the current flows into the page. Find the magnetic field (both the magnitude and the direction) at distance $r < R$ inside the wire. [10 points]
(b) Now suppose that a cylindrical hole is of radius $a$ is bored parallel to the cylindrical axis out of the wire. The center of the hole is at a distance $d$ from the axis of the wire such that $d + a < R$, as shown in Figure 2 below. A uniform current $I$ (directed as before into the page) passes through the cylindrical wire. Find the magnetic field (both the magnitude and the direction) inside the bore hole. [15 points]

![Figure 2: Cross-section of a uniform current-carrying wire with a bore hole.](image)
Problem 3.
Consider electric circuit with two resistors, \( R_1 \) and \( R_2 \), and an inductor, \( L \), connected to a battery of e.m.f. \( V \) as shown in Figure 3. The switch is initially closed and a steady, time-independent current is flowing through the circuit.
(a) Find the magnitude of the steady current \( I \) in this situation. [5 points]
(b) Suppose now that at time \( t = 0 \) the switch is abruptly opened so that the battery is instantly disconnected from the circuit. Write the circuit equation for the current \( I(t) \) in this situation and solve it. Be sure to use correct initial condition for the current. [5 points]
(c) Calculate how much energy has dissipated in the resistors in the time interval from \( t = 0 \) to \( t = T \). Where did this energy come from? [5 points]

![Figure 3: Circuit with the switch. The switch is set open at \( t = 0 \).](image)

Problem 4.
(a) Consider a horizontal slab of air of thickness (height) \( dz \). If this slab is at rest, the pressure holding it up from below must balance both the pressure from above and the weight of the slab. Use this fact to find an expression for \( dP/dz \), the variation of pressure with altitude \( z \), in terms of the density \( \rho \) of air. [3 points]
(b) Use the ideal gas law to write the density of air in terms of pressure \( P \), temperature \( T \), and the average mass \( m \) of the air molecules. Derive the differential equation for \( P \) and solve it, assuming that the temperature of the atmosphere is independent of height \( z \) (which is not too bad an assumption at low attitudes). [5 points]
(c) How does the density of air \( \rho(z) \) depend on \( z \)? [2 point]