Because of reflection and refraction, (at least) two light waves enter the eye when light shines on a thin film of gasoline floating on a thick layer of water.

Because of the extra distance traveled by ray [2] relative to ray [1], there can be interference between the two waves.

For the comparison of the extra path to wavelength, we need to use the wavelength in the medium

\[ \lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{film}}} \]

When light travels through a material with a smaller refractive index towards a material with a larger refractive index, (partial) reflection at the boundary occurs along with a phase change that is equivalent to one-half of a wavelength in the film.

When light travels from a larger towards a smaller refractive index, there is no phase change upon reflection.
27.3 Thin Film Interference

**Example** Color of a Thin Film of Gasoline

A thin film of gasoline floats on a puddle of water. Sunlight falls perpendicularly on the film and reflects into your eyes. The film has a yellow hue because destructive interference eliminates the color of blue (469 nm) from the reflected light. The refractive indices of the blue light in gasoline and water are 1.40 and 1.33. Determine the minimum non-zero thickness of the film.
27.3 Thin Film Interference

**Example  Color of a Thin Film of Gasoline**

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Ray [1]: reflection at point A from lower to higher index: phase shift equivalent to $\frac{1}{2} \lambda_{\text{film}}$

$\ell_1 = \frac{1}{2} \lambda_{\text{film}}$

Ray [2]: reflection at point B from higher to lower no phase shift; but roundtrip through film:

$\ell_2 = 2t$

Effective path difference between Ray [1] and Ray [2] (including phase shift for reflection)

$\Delta \ell = \ell_2 - \ell_1 = 2t - \frac{1}{2} \lambda_{\text{film}}$

Destructive interference between Ray [1] and Ray [2]: difference of half-integer wavelengths

$2t = \lambda_{\text{film}}, 2\lambda_{\text{film}}, 3\lambda_{\text{film}}...$

Minimum non-zero thickness $t$

$t = \frac{\lambda_{\text{film}}}{2} = \frac{1}{2} \cdot \frac{469 \text{ nm}}{1.40} = 168 \text{ nm}$
27.3 Thin Film Interference

The *wedge of air* formed between two glass plates causes an interference pattern of **alternating** dark and bright fringes.

\[ \Delta \ell = 2y - \frac{1}{2} \lambda \]

**Dark Fringes**  \[ \Delta \ell = (m + \frac{1}{2})\lambda \]

**Bright Fringes**  \[ \Delta \ell = m\lambda \quad m = 0,1,2,3... \]

*Thin-film band-pass optical filter*

As long as the film has the highest refractive index of the three regions, then **neither of the two reflections** incur a phase shift.

\[ \rightarrow \Delta \ell = 2t \]

Constructive interference between transmitted ray [1] and ray [2] occurs for the minimum thickness of

\[ t = \frac{1}{2} \lambda_{\text{film}} \]
27.5 Diffraction

In the 2-slit interference pattern, the alternating bright and dark fringes appears to be **modulated by a larger scale structure**

These are the result of the **finite width** of the slits.

Using a **single slit of the same width**, we can produce a pattern that consists just of the **modulating envelope only**.

This **single-slit pattern** is due to the **diffr action** of light.

**Diffraction** refers to the **bending of waves** around obstacles or the edges of an opening.

**Huygens’ principle**

Every point on a wave front acts as a source of tiny wavelets that move forward with the same speed as the wave; the wave front at a latter instant is the surface that is tangent to the wavelets.
27.5 Diffraction

This top view shows the rays for five sources of Huygens’ wavelets.

Wave fronts from all five arrive in phase at the central maximum (bright fringe)

First Dark Fringe: wavelets from 1,3,5 arrive successively separated by half wave-length: Destructive interference

Wavelets from 2,4 also arrive separated by half wave-length.

This occurs at \( W\sin\theta = \lambda \)
27.5 Diffraction

Second Dark Fringe: wavelets from 1, 2, 3, 4, and 5 arrive successively separated by half wavelength: *Destructive interference*

This occurs at $W \sin \theta = 2\lambda$

For higher order dark fringes, we need to break the slit up into more sources...

Dark fringes for single slit diffraction

$$\sin \theta = m \frac{\lambda}{W} \quad m = 1, 2, 3, \ldots$$

Note that the $m=0$ case here corresponds to the central maximum.
An arrangement consisting of a large number of closely spaced, parallel slits is called a **diffraction grating**.

The multiple slits enhances the constructive interference at the maxima (bright fringes) of the 2-slit case (with the same slit spacing $d$)
27.7 The Diffraction Grating

The bright fringes produced by a diffraction grating are much narrower than those produced by a double slit.

**Principal maxima of a diffraction grating**

\[
\sin \theta = m \frac{\lambda}{d} \quad m = 0, 1, 2, 3, \ldots
\]

distance between adjacent slits

Reversing this equation

\[
\lambda = \frac{d}{m} \sin \theta
\]

This technique allows one to make very precise measurements of the wavelength

Heated atoms emit characteristic wavelengths.

The Emission (absorption) wavelength pattern is called the emission (absorption) **spectrum**

The element helium (He) was originally discovered by its spectrum in sunlight.
Example: Separating Colors With a Diffraction Grating

A mixture of violet (410 nm) light and red (660 nm) light falls onto a grating that contains $1.00 \times 10^4$ lines/cm. For each wavelength, find the angle that locates the first-order maximum.
**Example: Separating Colors With a Diffraction Grating**

A mixture of violet (410 nm) light and red (660 nm) light falls onto a grating that contains $1.00 \times 10^4$ lines/cm. For each wavelength, find the angle that locates the first-order maximum.

$$d = \frac{1}{1.00 \times 10^4 \text{ cm}^{-1}} = 1.00 \times 10^{-4} \text{ cm} = 1.00 \times 10^{-6} \text{ m}$$

**violet (410 nm)**

$$\theta_1 = \sin^{-1} \left( \frac{\lambda_{\text{violet}}}{d} \right) = \sin^{-1} \left( \frac{410 \times 10^{-9} \text{ m}}{1.00 \times 10^{-6} \text{ m}} \right) = 24.2^\circ$$

**red (660 nm)**

$$\theta_1 = \sin^{-1} \left( \frac{\lambda_{\text{red}}}{d} \right) = \sin^{-1} \left( \frac{660 \times 10^{-9} \text{ m}}{1.00 \times 10^{-6} \text{ m}} \right) = 41.3^\circ$$
Chapter 28

Special Relativity

Two arbitrary, inertial frames of reference (i.e. Newton’s Laws apply in both)

Frame F’ is moving at speed v in the +x direction relative to frame F (F is at rest to us)

(1) The speed of light is the same in both frames (inconsistent with Newtonian kinematics):

   Light emitted by a source at rest in F travels at c=0.300 m/ns in both F and F’

(2) A time interval \(\Delta t_0\) (between 2 events) measured by a clock at rest in the F frame is dilated to a longer time interval \(\Delta t\) in F’ (remember F and F’ are arbitrary frames, but here F is special because it is the rest frame of the clock): Time Dilation

\[
\Delta t = \gamma \Delta t_0, \quad \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \rightarrow 1
\]

\(\gamma\) is the famous Relativistic Fudge Factor

(3) The length of an object (along the direction of relative motion between F and F’) at rest in the F frame is contracted to a shorter value in F’ (frame F is special because it is the rest frame of the object): Length Dilation

\[ L = L_0/\gamma \]

(4) Relativistic momentum:

\[ p = \gamma mv \rightarrow mv \]

(5) Total energy = KE + Mass Energy: KE = \((\gamma - 1)mc^2\), Mass Energy = \(mc^2\)
Michelson-Morley Experiment (1888): Trying to find the medium ("aether") in which light propagates, and through which the Earth is moving.

Using a semi-silvered mirror, the apparatus splits the beam from a coherent light source into two perpendicular arms.

The light reflected back from the plane mirrors in each arm would produce Interference fringes on the semi-silvered mirror.

Assuming the aether to be stationary with respect to the Sun, fringes were expected to shift (by 0.4 fringe) as one rotates the apparatus through 90°.

Sensitivity of apparatus: 1/100 of a fringe

**Result:** no shift detected

**Interpretation:** no aether

From the expectation of EM theory: No medium needed. The speed of EM waves should be invariant with respect to change of frame of reference. This is in conflict with Newtonian kinematics.
Invariance of speed of light in vacuum: Experimental Tests

- Alvaeger F.J.M. Farley, J. Kjellman and I Wallin, Physics Letters 12, 260 (1964). Arkiv foer Fysik, Vol 31, pg 145 (1965). Measured the speed of gamma rays from the decay of fast $\pi^0$ ($\sim 0.99975 \, c$) to be $c$ with a resolution of 400 parts per million. Optical extinction is not a problem for such high-energy gamma rays. The speed of the $\pi^0$ is not measured, but is assumed to be similar to that measured for $\pi^+$ and $\pi^-$.  

- Sadeh, Phys. Rev. Lett. 10 no. 7 (1963), pg 271. Measured the speed of the gammas emitted from $e^+e^-$ annihilation (with center of mass $v/c \sim 0.5$) to be $c$ within 10%.

- This experiment was criticized in Lo Savio, Phys. Lett. A, 1988, Vol 133, pg 176. It is certainly true that at the instant of annihilation the $e^+$ need not be traveling in the same direction it had initially, or have the same speed (most annihilations occur at very low energy as the positrons stop). This experiment is inconclusive at best.

- Babcock and Bergmann, Journal Opt. Soc. Amer. Vol. 54, pg 147 (1964). This repeat of Kantor's experiment in vacuum shows no significant variation in the speed of light affected by moving glass plates. Optical Extinction is not a problem. $k < 0.02$.

- Filipas and Fox, Phys. Rev. 135 no. 4B (1964), pg B1071. Measured the speed of gamma rays from the decay of fast $\pi^0$ ($\sim 0.2 \, c$) in an experiment specifically designed to avoid extinction effects. Their results are in complete disagreement with the assumption $c+v$, and are consistent with SR. $k < 0.5$ with a confidence level of 99.9%.

- Beckmann and Mandics, “Test of the Constancy of the Velocity of Electromagnetic Radiation in High Vacuum”, Radio Science, 69D, no. 4, pg 623 (1965). A direct experiment with coherent light reflected from a moving mirror was performed in vacuum better than $10^{-6}$ torr. Its result is consistent with the constant velocity of light. This experiment is notable because Beckmann was a perennial critic of SR. Optical Extinction is not a problem.

- Operation of FLASH, a free-electron laser, [http://vuv-fel.desy.de/](http://vuv-fel.desy.de/). A free-electron laser generates highly collimated X-rays parallel to the relativistic electron beam that is their source. If the region that generates the X-rays is $L$ meters long, and the speed of light emitted from the moving electrons is $c+kv$ (here $v$ is essentially $c$), then at the downstream end of that region the minimum pulse width is $k(L/c)/(1+k)$, because light emitted at the beginning arrives before light emitted at the downstream end. For FLASH, $L=30$ meters, $v=0.9999997 \, c$ (700 MeV), and the observed X-ray pulse width is as short as 25 fs. This puts an upper limit on $k$ of $2.5\times10^{-7}$. Optical extinction is not present, as the entire process occurs in very high vacuum.
Consequences of the Invariance of the Speed of Light

An optical experiment is at rest in reference frame $F'$.

$F'$ is moving at a constant speed $v$ in the $+x$ direction relative to the Earth frame $F$.

In the experiment, a very short pulse of light is emitted in the $+y$ direction, and reflects off a plane mirror a distance $L_0$ above and back to a sensor next to the light source.

A clock measures the time $\Delta t_0$ elapsed between the departure of the light pulse (event 1) and the arrival of the reflected pulse at the sensor (event 2).

We added a subscript “0” to the time interval $\Delta t$ because the two events occur at the same location in $F'$ (with clock).

And to the length $L$ because the apparatus is at rest in $F'$.
As seen in frame F':

$$\Delta t_0 = \frac{2L_0}{c} \quad \rightarrow \quad L_0 = \frac{1}{2} c \Delta t_0$$

What about in Earth frame F? \(\Delta t = ?\)

Total path traveled:

$$D = 2s \quad \Rightarrow \quad s = \sqrt{L_0^2 + (\frac{1}{2} v \Delta t)^2} \quad \Rightarrow \quad D = 2\sqrt{L_0^2 + (\frac{1}{2} v \Delta t)^2}$$

Observation in F':

$$L_0 = \frac{1}{2} c \Delta t_0 \quad \Rightarrow \quad D = \sqrt{c^2 \Delta t_0^2 + v^2 \Delta t^2}$$

Invariance of speed of light:

$$D = c \Delta t \quad \Rightarrow \quad c \Delta t = \sqrt{c^2 \Delta t_0^2 + v^2 \Delta t^2} \quad \Rightarrow \quad c^2 \Delta t^2 = c^2 \Delta t_0^2 + v^2 \Delta t^2$$

$$(c^2 - v^2)\Delta t^2 = c^2 \Delta t_0^2 \quad \Rightarrow \quad (1 - \frac{v^2}{c^2})\Delta t^2 = \Delta t_0^2 \quad \Rightarrow \quad \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \equiv \gamma \Delta t_0$$

Time Dilation

Time intervals between events are shortest in the frame in which they occur at the same location ("proper time"). In other frames:

$$\Delta t = \gamma \Delta t_0, \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
**Length Contraction:** A truck of length $L_0$ in its rest frame $F'$ passes a stationary observer on the ground at speed $v = 0.180 \text{ m/ns} = 0.600c$: Front at $t_1 = 0$, back at $t_2 = 20 \text{ ns}$. Find $L$ & $L_0$.

### Ground based observer:

- Event 1: $t_1 = 0$
- Event 2: $t_2 = \frac{L}{v} = \Delta t_0 = 20.0 \text{ ns}$

$$L = v\Delta t_0 = (0.180 \text{ m/ns})(20.0 \text{ ns}) = 3.60 \text{ m}$$

### Observer in the truck:

- Event 1: $t_1' = 0$
- Event 2: $t_2' = \frac{L_0}{v} = \Delta t$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2 / c^2}}$$

$$v = \frac{L_0}{\Delta t} = \frac{L}{\Delta t_0}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}} = \frac{1}{\sqrt{1 - (0.6)^2}} = 1.25$$

$$L = \frac{\Delta t_0}{\Delta t} L_0$$

$$L = \sqrt{1 - v^2 / c^2} L_0 = L_0 / \gamma$$

$$L = L_0 / \gamma \rightarrow 3.60 \text{ m} = L_0 / 1.25 \rightarrow L_0 = 4.50 \text{ m}$$
28.5 Relativistic Momentum

Relativistic momentum

\[ p = \frac{mv}{\sqrt{1 - v^2/c^2}} \equiv \gamma mv \]

**Note:** Objects with finite mass can never attain or exceed speed \( v = c \)

Newton’s 2nd Law becomes

\[ F = \frac{\Delta p}{\Delta t} = \Delta \left( \frac{mv}{\sqrt{1 - v^2/c^2}} \right) / \Delta t \]

Total energy of an object

\[ E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \equiv \gamma mc^2 \]

Rest (mass) energy of an object

\[ E_o = mc^2 \]

Kinetic energy of an object

\[ KE = E - E_o = (\gamma - 1)mc^2 \]