EXAM 3

PLEASE FILL IN THE INFORMATION BELOW:

Name (printed): Solution

Name (signed):

Student ID Number (unid):

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\[ \mu_0 = 4\pi \times 10^{-7} \text{ Tm/A} \]

\[ \varepsilon_0 = 8.55 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2) \]

\[ c = 3 \times 10^8 \text{ m/s} \]
A gamma-ray telescope intercepts a pulse of gamma radiation from a magnetar, a type of star with a spectacularly large magnetic field. The pulse lasts 0.24 s and delivers $8.4 \times 10^6$ J of energy perpendicularly to the 75-m$^2$ surface area of the telescope's detector. The magnetar is thought to be $4.5 \times 10^{20}$ m (about 50,000 light-years) from earth, and to have a radius of $9.0 \times 10^3$ m. Find the magnitude of the rms magnetic field of the gamma-ray pulse at the surface of the magnetar, assuming that the pulse radiates uniformly outward in all directions.

\[
\Delta t = 0.24 \text{ s}
\]
\[
E = 8.4 \times 10^6 \text{ J}
\]
\[
d = 4.5 \times 10^{20} \text{ m}
\]
\[
R = 9 \times 10^3 \text{ m}
\]
\[
A_T = 75 \text{ m}^2
\]

Power radiated by magnetar is related to intensity as
\[
P_M = 5A \cos \theta = 5 \cdot 4\pi r^2
\]

\[\Rightarrow \text{Relating the intensity at the earth and at the surface of the magnetar: } S_e \cdot 4\pi d^2 = S_M \cdot 4\pi R^2 \quad (\ast)\]

The intensity at the earth is
\[
S_e = \frac{P}{A \cos \theta} = \frac{E \Delta t}{\Delta t A_T}
\]

The intensity at the surface of the magnetar is
\[
S_M = 4\pi R^2 = C \frac{B^2}{\mu_0}
\]

Putting everything back into (\ast): \[
\frac{E \cdot 4\pi d^2}{\Delta t A_T} = \frac{C \cdot B^2 \cdot 4\pi R^2}{\mu_0}
\]

\[\Rightarrow B = \frac{d}{R} \left( \frac{E \mu_0}{c \Delta t A_T} \right)^{\frac{1}{2}} = \frac{4.5 \times 10^{20}}{9 \times 10^3} \left( \frac{8.4 \times 10^6 \cdot 4\pi 10^{-7} \text{ Tm}^2}{3 \times 10^8 \text{ s} \cdot (0.24) \text{ s} \cdot 75 \text{ m}} \right)^{\frac{1}{2}}\]

\[= 2.2 \times 10^6 \text{T}\]
Show all work!

[20 pts.] The drawing shows two plane mirrors that intersect at an angle of 50°. An incident light ray reflects from one mirror and then the other. What is the angle $\theta$ between the incident and outgoing rays?

From the law of reflection,

$\theta_i = \theta_f$,

Triangle $ABC$: $(90 - \alpha) + 50 + (90 - \phi) = 180$

$\Rightarrow \alpha + \phi = 50$.

Triangle $ACD$: $2\alpha + 2\phi + 180 - \theta = 180$

$\Rightarrow 2(\alpha + \phi) = \theta$

Combining the two equations:

$\theta = 2(50) = \boxed{100}$
Show all work!

[20 pts.] An object is located 14.0 cm in front of a convex mirror, the image being 7.00 cm behind the mirror. A second object, twice as tall as the first one, is placed in front of the mirror, but at a different location. The image of this second object has the same height as the other image. How far in front of the mirror is the second object located?

\[
\frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o} \rightarrow f = \frac{s_i s_o}{s_i + s_o} = \frac{(-7)(14)}{7} = -14\text{ cm}
\]

\[h_{i1} = h_{i2}\]

\[h_{o2} = 2h_{o1}\]

\[M_1 = \frac{h_{i1}}{h_{o1}} = -\frac{s_{i1}}{s_{o1}} = \frac{(-7)}{14} = 0.5\]

\[M_2 = -\frac{s_{i2}}{s_{o2}} = \frac{h_{i2}}{h_{o2}} = \frac{h_{i1}}{2h_{o1}} = 0.25\]

\[\rightarrow s_{o2} = -4s_{i2}\]

\[\frac{1}{f} = \frac{1}{s_{i2}} + \frac{1}{s_{o2}} \rightarrow s_{i2} = \frac{s_{o2} f}{s_{o2} - f}\]

\[\rightarrow s_{o2} = -4 \left( \frac{s_{o2} f}{s_{o2} - f} \right)\]

\[\rightarrow s_{o2} - s_{o2} f = -4 \frac{s_{o2} f}{s_{o2} - f}\]

\[\rightarrow s_{o2} = -3f = \boxed{42 \text{ cm}}\]
Visitors at a science museum are invited to sit in a chair to the right of a full-length diverging lens ($f_1 = -3.00$ m) and observe a friend sitting in a second chair, 2.00 m to the left of the lens. The visitor then presses a button and a converging lens ($f_2 = +4.00$ m) rises from the floor to a position 1.60 m to the right of the diverging lens, allowing the visitor to view the friend through both lenses at once.

(a) Find the magnification of the friend when viewed through the diverging lens only.
(b) Find the overall magnification of the friend when viewed through both lenses. Be sure to include the algebraic signs (+ or -) with your answer.

\[
M = \frac{\frac{S_i}{S_o}}{\frac{h_i}{h_o}}
\]

\[
\frac{1}{f_1} = \frac{1}{S_{i1}} + \frac{1}{S_{o1}} \quad \Rightarrow \quad S_{i1} = \frac{f_1 S_{o1}}{S_{o1} - f_1} = \frac{-6}{2(-3)} = -1.2 \text{ m}
\]

\[
M_1 = -\frac{S_{i1}}{S_{o1}} = \frac{f_1}{f_1 - S_{o1}} = \frac{-3}{2(-3)} = +0.6
\]

6) Use the image from $f_1$ as the object for $f_2$:

\[
S_{o2} = 1.6 \text{ m} - S_{i1} = 1.6 \text{ m} - (-1.2 \text{ m}) = 2.8 \text{ m}
\]

\[
S_{i2} = \frac{f_2 S_{o2}}{S_{o2} - f_2} = -9.33 \text{ m}
\]

\[
M_2 = -\frac{S_{i2}}{S_{o2}} = \frac{f_2}{S_{o2} - f_2} = +3.33
\]

\[
M_{total} = M_1 M_2 = (+2)
\]
Show all work!

[20 pts.] A filmmaker wants to achieve an interesting visual effect by filming a scene through a converging lens with a focal length of 50.0 m. The lens is placed between the camera and a horse, which canters toward the camera at a constant speed of 7.0 m/s. The camera starts rolling when the horse is 40.0 m from the lens.

(a) Find the average speed of the image of the horse during the first 2.0 s after the camera starts rolling.
(b) Find the average speed of the image of the horse during the following 2.0 s.

\[
\frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o} \quad \Rightarrow \quad s_i = \frac{s_o f}{s_o - f}, \quad s_i(t) = \frac{(s_o - vt)f}{(s_o - vt) - f}
\]

(a) \( s_i(0) = \frac{40 \cdot 50}{40 - 50} = -200 \text{ m} \)

\( s_i(2) = \frac{(40 - 14) \cdot 50}{(40 - 14) - 50} = -54.2 \text{ m} \)

\( \rightarrow V_{i(\text{avg})} = \frac{s_i(2) - s_i(0)}{2} = \frac{-54.2 \text{ m} - (-200 \text{ m})}{2} = 72.9 \text{ m/s} \)

(b) \( s_i(4) = \frac{(40 - 28) \cdot 50}{(40 - 28) - 50} = -15.8 \text{ m} \)

\( \rightarrow V_{i(\text{avg})} = \frac{s_i(4) - s_i(2)}{2} = \frac{-15.8 \text{ m} - (-54.2 \text{ m})}{2} = 19.2 \text{ m/s} \)