We now turn to image formation by lenses. We have seen that for “thin” lenses there will be a unique focal point. Again we need a sign convention. We will use the same one we used for mirrors with one change. Since the light passes through a lens we will take the image distance positive if it is behind (downstream) the lens. There are again four configurations to consider.

CASE 1 CONVERGING LENS OBJECT OUTSIDE FOCAL POINT

The situation is shown in the following sketch.

Consider triangles ABC and AGH. They are similar with the result:

$$\frac{AB}{AG} = \frac{CB}{GH}$$

But

$$AB = s, \quad AG = s', \quad CB = h, \quad GH = -h'$$

$$\therefore \frac{s}{s'} = \frac{h}{-h'} \rightarrow h' = -h \frac{s'}{s}$$

Now consider triangles AED and EGH. Again they are similar. Thus:

$$\frac{AE}{EG} = \frac{AD}{GH}$$

But
AE = f,  \quad EG = s' - f,  \quad AD = h,  \quad GH = -h'  

\therefore \frac{f}{s' - f} = \frac{h}{-h'} = \frac{s}{s'} \rightarrow fs' = ss' - sf  

\therefore \frac{1}{s} - \frac{1}{f} = \frac{1}{s'} \rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}  

CASE 2 CONVERGING LENS OBJECT INSIDE FOCAL POINT  

The situation is shown in the following sketch.  

Consider triangles ABH and ACD. Since they are similar we have:  

\[
\frac{AB}{AC} = \frac{BH}{CD}
\]

But  

\[
AB = s, \quad AC = -s', \quad BH = h, \quad CD = h'
\]

\therefore \frac{s}{-s'} = \frac{h}{h'} \rightarrow h' = -\frac{s'}{s}

Now consider triangles GCD and GAE. They are similar so:  

\[
\frac{GC}{GA} = \frac{CD}{AE}
\]
But

\[ \begin{align*}
GC &= f - s', \quad GA = f, \quad CD = h', \quad AE = h \\
\frac{f - s'}{f} &= \frac{h'}{h} = -\frac{s'}{s} \\
\therefore sf - ss' &= -s'f \rightarrow \frac{1}{s'} - \frac{1}{f} = -\frac{1}{s} \\
\therefore \frac{1 + \frac{1}{s}}{s' \frac{1}{f}} &= \frac{1}{f}
\end{align*} \]

CASE 3 DIVERGING LENS OBJECT OUTSIDE FOCAL POINT

The situation is shown below:

Consider triangles ACD and AHE. Since they are similar we get:

\[ \frac{AC}{AH} = \frac{CD}{HE} \]

But

\[ AC = s, \quad AH = -s', \quad CD = h, \quad HE = h' \]
Now consider triangles BHE and BAG. They are similar giving:

\[
\frac{BH}{BA} = \frac{HE}{AG}
\]

But

\[
BH = -f + s', \quad BA = -f, \quad HE = h', \quad AG = h
\]

\[
\therefore \frac{-f + s'}{-f} = \frac{h'}{h} = \frac{s'}{s} \rightarrow -sf + ss' = s'f
\]

\[
\therefore -\frac{1}{s'} + \frac{1}{f} = \frac{1}{s} \rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

CASE 4 DIVERGING LENS OBJECT INSIDE FOCAL POINT

The situation is shown in the following sketch:

Consider triangles ABH and ACE. They are similar giving:

\[
\frac{AB}{AC} = \frac{BH}{CE}
\]

But
Now consider triangles ADG and BDH. Since they are similar we get:

\[
\frac{AD}{BD} = \frac{AG}{BH}
\]

But

\[
\frac{-f}{-f + s'} = \frac{h}{h'} = -\frac{s}{s'} \rightarrow -fs' = sf - ss'
\]

\[
\frac{1}{s} = \frac{1}{s'} - \frac{1}{f} \rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

Thus in all cases (mirrors or lenses) we have the same two equations to determine image location, size, and orientation. All we need to do is use the sign convention consistently.

As an example of the use of these equations for a lens, consider a 2cm high object placed 40cm in front of a diverging lens of focal length 30cm. Since the lens is diverging the focal length will be negative. Hence \( f = -30 \text{cm} \). The object is upstream by 40cm. Thus \( s = +40 \text{cm} \). It is erect and 2cm high. Hence \( h = +2 \text{cm} \). The equations then give:

\[
s' = \frac{sf}{s - f} = \frac{(+40)(-30)}{(+40) - (-30)} = -\frac{1200}{70} = -17.1 \text{ cm}
\]

\[
h' = -h \frac{s'}{s} = -(2)\left(\frac{-17.1}{40}\right) = +.855 \text{ cm}
\]

Therefore the image is 17.1cm in front of the lens, erect, .855cm high, and virtual.