FORMATION OF IMAGES BY SPHERICAL MIRRORS

We now consider the formation of images by mirrors. We will assume that the mirrors are perfect (although this is only true for plane or parabolic mirrors) and characterized by a focal length. We have seen that this is OK for a spherical mirror provided that the aperture of the mirror is small compared to its radius of curvature. In that case we found that the focal length was given by:

\[ f = \frac{R}{2} \]

We also have introduced a sign convention:

- \( f > 0 \) for converging mirrors
- \( s > 0 \) if object is upstream from the mirror
- \( s' > 0 \) if image is upstream from the mirror
- \( h > 0 \) if object is erect
- \( h' > 0 \) if image is erect

There are now several configurations to consider.

CASE 1 CONVERGING MIRROR OBJECT OUTSIDE FOCAL POINT

This case is shown in the following sketch.

Consider triangles ADF and ABC. They are similar because all angles are equal (remember that the angle of reflection equals the angle of incidence). Thus:

\[ \frac{AB}{AD} = \frac{CB}{DF} \]

But
AB = s, AD = s', DF = -h', BC = h

Thus

\[
\frac{s}{s'} = \frac{h}{-h'} \rightarrow h' = -h \frac{s'}{s}
\]

Now consider triangles AEG and DFE. They also are similar for the same reason. Hence:

\[
\frac{AE}{ED} = \frac{AG}{DF}
\]

But

\[
AE = f, \quad ED = s' - f, \quad AG = h, \quad DF = -h'
\]

\[
\therefore \frac{f}{s' - f} = \frac{h}{-h'} = \frac{s}{s'} \rightarrow s'f = ss' - sf
\]

\[
\therefore \frac{1}{s} = \frac{1}{f} - \frac{1}{s'} \rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

**CASE 2 CONVERGING MIRROR OBJECT INSIDE FOCAL POINT**

This case is shown in the following sketch.

Consider triangles ABC and AFE. They have all angles equal and hence are similar. Thus:
\[ \frac{AB}{AF} = \frac{BC}{FE} \]

But

\[ AB = s, \quad AF = -s', \quad BC = h, \quad FE = h' \]

\[ \therefore \frac{s}{-s'} = \frac{h}{h'} \rightarrow h' = -\frac{s'}{s} \]

Now consider triangles ADG and FDE. Again they are similar. Thus:

\[ \frac{AD}{FD} = \frac{AG}{FE} \]

But

\[ AD = f, \quad FD = f - s', \quad AG = h, \quad FE = h' \]

\[ \therefore \frac{f}{f - s'} = \frac{h}{h'} = -\frac{s}{s'} \rightarrow fs' = -sf + ss' \]

\[ \therefore \frac{1}{s} = -\frac{1}{s'} + \frac{1}{f} \rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \]

CASE 3 DIVERGING MIRROR OBJECT OUTSIDE FOCAL POINT

This case is shown in the following sketch.
Consider triangles $ABC$ and $AEH$. They have all angles equal and thus are similar. This gives:

\[
\frac{AB}{AE} = \frac{BC}{EH}
\]

But

\[
AB = s, \quad AE = -s', \quad BC = h, \quad EH = h'
\]

\[
\therefore \frac{s}{-s'} = \frac{h}{h'} \rightarrow h' = -\frac{s'}{s}
\]

Now consider triangles $GAD$ and $GEH$. They are similar giving

\[
\frac{GA}{GE} = \frac{AD}{EH}
\]

But

\[
GA = -f, \quad GE = -f + s', \quad AD = h, \quad EH = h'
\]

\[
\frac{-f}{-f + s'} = \frac{h}{h'} = \frac{s'}{s} \rightarrow -fs' = sf - ss'
\]

\[
\therefore \frac{1}{s} = \frac{1}{s'} - \frac{1}{f} \rightarrow \frac{1}{s + s'} = \frac{1}{s'f}
\]

**CASE 4 DIVERGING MIRROR OBJECT INSIDE FOCAL POINT**

This case is shown in the following sketch.
Consider triangles ABC and AGE. They are similar and hence:

\[
\frac{AB}{BC} = \frac{AG}{GE}
\]

But

\[
AB = s, \quad AG = -s', \quad BC = h, \quad GE = h'
\]

\[
\therefore \frac{s}{-s'} = \frac{h}{h'} \implies h' = -\frac{s'}{s}
\]

Now consider triangles HAD and HGE. Again they are similar with the result:

\[
\frac{HA}{GH} = \frac{AD}{GE}
\]

But

\[
HA = -f, \quad GH = -f + s', \quad AD = h, \quad GE = h'
\]

\[
\therefore \frac{-f}{-f + s'} = \frac{h}{h'} = \frac{s}{s'} \implies -s'f = sf - ss'
\]

\[
\therefore \frac{1}{s} = \frac{1}{s'} - \frac{1}{f} \implies \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

Thus in all possible cases we have exactly the same equations to find the location of the image and its size! All that is necessary is to use our sign convention.

As an example consider the following situation. A 2 cm high object is placed 20cm in front of a converging spherical mirror of radius of curvature 60 cm. Where will the image be, will it be erect or inverted, and what will be its size?

Since the mirror is converging the focal length will be positive. Also, the focal length is half the radius. Thus the \( f = +30\text{cm} \). The object is upstream from the mirror and 20cm from it. Hence \( s = +20 \). Finally it is erect and 2cm high. Thus \( h = +2\text{cm} \). We now use our two equations to get:

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \implies \frac{1}{s} = \frac{1 - \frac{s - f}{sf}}{s'} = \frac{s - f}{sf}
\]

\[
\therefore \frac{s'f}{s - f} = \frac{20 \times 30}{20 - 30} = -60 \text{cm}
\]
\[ h' = -h \frac{s'}{s} = (-2) \left( \frac{-60}{30} \right) = +4 \text{ cm} \]

Thus the image is 60cm behind the mirror, erect, and of height 4 cm. It is a virtual image because the light does not actually pass through it.