Four blocks EACH of mass $m = 10$ kg are arranged as shown in the picture, on top of a frictionless table. A hand touching block 1 applies a force of $F_{h1} = 90$ N to the right. The coefficient of friction between the blocks is sufficient to keep the blocks from moving with respect to each other.

What is the total force exerted by block 2 on block 3?

$$F_{23\text{net}} =$$

To keep the horizontal + vertical components separate we will designate the horizontal component of block $i$ on block $j$ as $F_{ij}$ and we know that $F_{ij} = -F_{ji}$. The vertical component will be denoted as $N_{ij}$.

Start with block 4

$F_{34} = ma$

Next

$F_{23} - F_{34} = ma$

$\Rightarrow F_{23} = F_{34} + ma = 2ma$

$F_{12} = F_{23} + ma = 3ma$

$F_{1h} = F_{12} + ma = 4ma$

$a = \frac{F_{1h}}{4m}$

$F_{23} = 2m \left( \frac{F_{1h}}{4m} \right) = \frac{1}{2} F_{1h}$

Next to consider the vertical
Block 4
\[ N_{3y} - mg = 0 \]
\[ \Rightarrow N_{3y} = mg \]

Block 3
\[ N_{23} - N_{3y} - mg = 0 \]
\[ N_{23} = N_{3y} + mg = 2mg \]

Don't need to continue past this point

\[ F_{23} = \sqrt{f_{23}^2 + N_{23}^2} \]
\[ = \sqrt{(\frac{F_{1h}}{2})^2 + (2mg)^2} \]

Plug in our numbers

\[ F_{23} = \sqrt{(\frac{90}{2})^2 + (2(10)(9.8))^2} \]
\[ F_{23} = 201 N \]
A wooden block of mass \( m = 9 \) kg starts from rest on an inclined plane sloped at an angle \( \theta \) from the horizontal. The block is originally located 5 m from the bottom of the plane.

If the block, undergoing constant acceleration down the ramp, slides to the bottom in \( t = 2 \) s, and \( \theta = 30^\circ \), what is the magnitude of the kinetic frictional force on the block?

\[ f_k = \text{ } \]

\[ \Sigma F_x = mg \sin \theta - F_k = ma \]

\[ F_k = mg \sin \theta - ma \]

Now to determine acceleration using equation of motion

\[ x_f = x_0 + v_0 t + \frac{1}{2} at^2 \quad \text{starts at rest and the origin point} \]

\[ 5 = \frac{1}{2} a (2)^2 \quad \Rightarrow \quad a = \frac{2(5m)}{4.5^2} = \frac{2 x_f}{t^2} \]

\[ F_k = mg \sin \theta - ma = mg \sin \theta - m \left( \frac{2 x_f}{t^2} \right) \]

\[ = (9 \text{ kg}) g \sin(30^\circ) - m \left( \frac{2 x_f}{t^2} \right) \]

\[ = 21.6 \text{ N} \]
In a classic carnival ride, patrons stand against the wall in a cylindrically shaped room. Once the room gets spinning fast enough, the floor drops from the bottom of the room! Friction between the walls of the room and the people on the ride make them the "stick" to the wall so they do not slide down. In one ride, the radius of the cylindrical room is \( R = 7.7 \) m and the room spins with a frequency of 19.9 revolutions per minute.

1) What is the speed of a person "stuck" to the wall?

\[ \text{m/s} \]

2) What is the normal force of the wall on a rider of \( m = 48 \) kg?

\[ \text{N} \]

3) What is the minimum coefficient of friction needed between the wall and the person?

\[ \text{} \]

4) To be safe, the engineers making the ride want to be sure the normal force does not exceed 1.5 times each persons weight - and therefore adjust the frequency of revolution accordingly. What is the minimum coefficient of friction now needed?

The person's speed is given by
\[
\text{v} = \frac{\Delta s}{\Delta t}
\]

where \( \Delta s \) is the distance traveled and can be obtained by
\[
\Delta s = \text{frequency} \times \text{circumference} = 2\pi R \text{of}
\]

so
\[
\text{v} = \frac{\Delta s}{\Delta t} = \frac{2\pi (7.7 \text{m})(19.9)}{60 \text{s}}
\]

\[ v = 16 \text{ m/s} \]
(3 cont'd)

b) \( a_c = \frac{v^2}{R} = \frac{4\pi^2 f^2 R}{\Delta t^2} \)

The only horizontal force is the normal force.

So 

\[ N = m a_c \quad \text{m = 48 kg} \]

\[ N = 1605 \text{ N} \]

c) We are looking to obtain \( M_s \) in this part.

\[ f_s = M_s \cdot N \]

\[ f_s - mg = 0 \quad \text{since there is no vertical acceleration} \]

\[ N M_s = mg \Rightarrow M_s = \frac{mg}{N} \]

So we obtain a value of \( M_s \) is

\[ M_s = 0.293 \]

d) Now we set \( N = 1.5mg \)

\[ \frac{4\pi^2 f^2 m R}{\Delta t^2} = 1.5mg \quad \text{solve for } f \]

So our frequency is now

\[ f = \pm \frac{\Delta t \sqrt{\frac{3g}{2 \pi^2 R}}} {2 \sqrt[3]{\pi}} \]
Safe frequency is

\[ f = \frac{\Delta t \sqrt{\frac{3g}{R}}}{2^{3/2} \pi} \]

We obtain \( f = 13.2 \)

which gets us to

\[ M_s = 0.667 \]
A block with mass $m_1 = 8.8 \text{ kg}$ is on an incline with an angle $\theta = 39^\circ$ with respect to the horizontal. For the first question there is no friction, but for the rest of this problem the coefficients of friction are: $\mu_k = 0.37$ and $\mu_s = 0.407$.

1) When there is no friction, what is the magnitude of the acceleration of the block?

$$\text{____ m/s}^2$$

2) Now with friction, what is the magnitude of the acceleration of the block after it begins to slide down the plane?

$$\text{____ m/s}^2$$

3) To keep the mass from accelerating, a spring is attached. What is the minimum spring constant of the spring to keep the block from sliding if it extends $x = 0.12 \text{ m}$ from its unstretched length.

$$\text{____ N/m}$$

4) Now a new block with mass $m_2 = 15.8 \text{ kg}$ is attached to the first block. The new block is made of a different material and has a greater coefficient of static friction. What minimum value for the coefficient of static friction is needed between the new block and the plane to keep the system from accelerating?

$$\text{____}$$
$F_y = N - mg \cos(\theta) = 0$

$N = mg \cos \theta$

$F_x \Rightarrow \text{frictionless}$

so $F_x = mg \sin \theta = ma$

$a_x = \frac{mg \sin \theta}{m} \quad \theta = 39^\circ \quad \therefore$

$a_x = 6.17 \text{ m/s}^2$

b) Now we include kinetic friction

$F_x = mg \sin \theta - \mu_k N = ma$

determined from prior part

$N = mg \cos \theta$

$mg \sin \theta - \mu_k (mg \cos \theta) = \frac{ma_x}{m}$

therefore $a_x = g \sin \theta - \mu_k g \cos \theta$

$a_x = 3.35 \text{ m/s}^2$
c) Now we add in spring force, leave friction, but now is static

\[ F_s = k \cdot D_1 \]
\[ F_{fs} = M_3 \cdot N \]

\[ F_x = mg \sin \theta - F_{fs} - F_s = ma_x = 0 \]

\[ M_3 \sin \theta - M_3 (mg \cos \theta) - k \cdot D_1 = 0 \]

\[ \frac{k \cdot D_1}{D_1} = \frac{mg \sin \theta - M_3 mg \cos \theta}{D_1} \]
\[ k = \frac{mg \sin \theta - M_3 mg \cos \theta}{D_1} \]

minimum spring constant \( D_1 = 0.12 \)
\[ k = 225 \text{ N/m} \]

\[ m = 8.8 \text{ kg} \]
\[ M_3 = 0.407 \]

d) with the new block the normal force is still given by

\[ N = mg \cos \theta, \text{ with its respective weight} \]

**Block 1**

\[ F_{x_1} = M_1 g \sin \theta - F_{f_{x_1}} - T = 0 \]
\[ T = -F_{f_{x_1}} + m_1 g \sin \theta = mg \sin \theta - M_3 g \cos \theta \]

\[ T \text{ is now in terms of known quantities} \]

**Block 2**

\[ F_{x_2} = T + m_2 g \sin \theta - F_{f_{x_2}} = 0 \]
\[ m_2 g \sin \theta - m_2 g \cos \theta M_3 + m_2 g \sin \theta = M_3 \left( \frac{m_2 g \cos \theta}{m_2 g \cos \theta} \right) \]
(cont'd)

Now we have $\mu_2$ in terms of known quantities so we have

$$\mu_2 = \frac{m_1 g \sin \theta - m_2 g \cos \theta + m_2 g \sin \theta}{m_2 g \cos \theta}$$

$$= \frac{\sin \theta (m_1 + m_2) - m_1 M_2 \cos \theta}{m_2 \cos \theta}$$

minimum $\mu_2 = 1.03$
A 4.3 kg block is held at rest against a vertical wall by a horizontal force of 147 N.

(a) What is the magnitude of the frictional force exerted by the wall on the block?

\[ N \]

(b) What is the magnitude of the minimum horizontal force needed to prevent the block from falling if the coefficient of friction between the wall and the block is \( \mu_s = 0.4 \)?

\[ N \]

\[ F_y = F_{fs} - mg = 0 \]

\[ F_{fs} = mg \quad m = 4.3 \text{ kg} \]

\[ F_{fs} = 42.2 \text{ N} \]

b) \( F_{fs} = \mu_s N \)

\[ F_{fs} = \mu_s N = mg \Rightarrow N = \frac{mg}{\mu_s} \]

\[ F_{\text{applied}} - N = 0 \]

The minimum applied force to keep object at rest is

\[ F_s = 105 \text{ N} \]

*Note: the given 147 N was not used in either part.*
A 12-kg turtle rests on the bed of a zookeeper's truck, which is traveling down a country road at 63 mi/h. The zookeeper spots a deer in the road, and slows to a stop in 13 s. Assuming constant acceleration, what is the minimum coefficient of static friction between the turtle and the truck bed surface needed to prevent the turtle from sliding?

Normed force is determined first

\[ N - mg = 0 \quad N = m \]

\[ \frac{F_s}{N} = \mu_s \quad N = mg \]

Both the turtle and the truck experience the same acceleration since the turtle does not slip.

The only force on the turtle in the x-direction is the friction force so

\[ ma = F_s \]

\[ ma = mg \mu_s \Rightarrow \mu_s = \frac{a}{g} \]

Now to determine acceleration

\[ V_0 = 63 \text{ mi/hr} \cdot \frac{1600 \text{ mi}}{1 \text{ hr}} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} \text{ to get } \frac{m}{s} \]

\[ t = 13 \text{ s} \]

\[ V_0 = 28 \text{ m/s} \]

\[ a = \frac{V_0}{t} = \frac{28 \text{ m/s}}{13 \text{ s}} \]

\[ \mu_s = \frac{a}{g} = 0.22 \]
A block of mass $M = 10.8 \text{ kg}$ rests on a bracket of mass $m = 4.8 \text{ kg}$, as shown in the figure below. The bracket sits on a frictionless surface. The coefficients of friction between the block and the bracket on which it rests are $\mu_s = 0.40$ and $\mu_k = 0.30$.

(a) What is the maximum force $F$ that can be applied if the block is not to slide on the bracket?

(b) What is the corresponding acceleration of the bracket?

For the block we have

\[
F_{x_1} = F_3 - T = Ma_{x_1} \quad \Rightarrow \quad \frac{F_3 - T}{M} = a_{x_1}
\]

Now for the bracket we have

\[
F_{x_2} = F + T - F_3 = Ma_{x_2} \quad \text{but} \quad a_{x_1} = a_{x_2}
\]

\[
\frac{F + T - F_3}{M} = \frac{F_3 - T}{M}
\]
Now assuming massless, frictionless pulley we have $T = F$
so now we have
\[
\frac{2F - F_3}{m} = \frac{F_3 - F}{M}
\]
Solve for $F$ to get
\[
F = \frac{F_3 M + F_3 m}{2M + m}
\]
\
$F_3 = \mu N$
where $N = Mg$ when no vertical acceleration on horizontal surface
$F_3 = Mg_m$
\[
F_{max} = \frac{M^2 g M + Mm g M}{2M + m}
\]
\[
F_{max} = 25N \text{ for not-slippping}
\]
\[b) \text{ Now to find acceleration }
\]
\[
a_x = \frac{2F - F_3}{m} = \frac{2\left(F_3 M + F_3 m\right)}{2M + m} - \frac{F_3}{m}
\]
\[
a_x = 1.605 \text{ m/s}^2
\]