FINAL EXAM
December 14, 2011

PLEASE FILL IN THE INFORMATION BELOW:

Name (printed):

Name (signed):

Student ID Number (unid):

Discussion Instructor: Marc Lindley     JonPaul Lundquist     Peter Peroncik     Rhett Zollinger

Useful Information:

1 ft = 12 in (exact)
1 mile = 5280 ft (exact)
1 day = 24 hr (exact)
g_{moon} = 1.67 m/s^2 = 5.48 ft/s^2
1 kg = 0.0685 slug
1 horsepower = 550 ft·pounds/s (exact)
R_{earth} = 6.37 \times 10^3 \text{ km}
R_{sun} = 6.96 \times 10^8 \text{ m}
R_{moon} = 1.74 \times 10^3 \text{ km}
k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2
e_{electron \ charge} = -1.60 \times 10^{-19} \text{ C}
Mean earth-moon distance = 3.84 \times 10^5 \text{ km}
1 m = 3.28 \text{ ft}
1 hour = 3600 sec = 60 min (exact)
g_{earth} = 9.80 \text{ m/s}^2 = 32.2 \text{ ft/s}^2
1 year = 365.25 days
1 N = 0.225 pound
M_{earth} = 5.97 \times 10^{24} \text{ kg}
M_{sun} = 1.99 \times 10^{30} \text{ kg}
M_{moon} = 7.35 \times 10^{22} \text{ kg}
G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2
\varepsilon_\circ = 8.85 \times 10^{-12} \text{ F/m}
m_{electron} = 9.11 \times 10^{-31} \text{ kg}
### Table 10.2
Moments of Inertia of Homogeneous Rigid Objects with Different Geometries

<table>
<thead>
<tr>
<th>Object Description</th>
<th>Inertia Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hoop or thin cylindrical shell  $I_{CM} = MR^2$</td>
<td></td>
</tr>
<tr>
<td>Hollow cylinder  $I_{CM} = \frac{1}{2} M(R_1^2 + R_2^2)$</td>
<td></td>
</tr>
<tr>
<td>Solid cylinder or disk  $I_{CM} = \frac{1}{2} MR^2$</td>
<td></td>
</tr>
<tr>
<td>Rectangular plate  $I_{CM} = \frac{1}{12} M(a^2 + b^2)$</td>
<td></td>
</tr>
<tr>
<td>Long thin rod with rotation axis through center  $I_{CM} = \frac{1}{12} ML^2$</td>
<td></td>
</tr>
<tr>
<td>Long thin rod with rotation axis through end     $I = \frac{1}{3} ML^2$</td>
<td></td>
</tr>
<tr>
<td>Solid sphere  $I_{CM} = \frac{2}{5} MR^2$</td>
<td></td>
</tr>
<tr>
<td>Thin spherical shell  $I_{CM} = \frac{2}{3} MR^2$</td>
<td></td>
</tr>
</tbody>
</table>
SHOW ALL WORK!!

Place a circle or box around each answer. Specify units for each answer.
Report all numbers to two significant figures.

A. [5 pts.] Which of the following are always true. Circle all that are correct.

1. The center of mass of a single rigid body is at the geometrical center.
2. The center of mass of a single rigid body is inside the body.
3. The position vector of the center of mass depends on one’s choice for the origin of the coordinate system.
4. When calculating the linear acceleration of a rigid body, the entire mass can be viewed as concentrated at its center of mass.
5. When calculating the angular acceleration of a rigid body, the entire mass can be viewed as concentrated at the center of the mass.

B. [5 pts.] Two equal masses are raised the same distance at constant velocity by pulling on ropes that run over identical pulleys. Mass B is raised twice as quickly as mass A. The magnitude of the forces are $F_A$ and $F_B$, while the power supplied is, respectively, $P_A$ and $P_B$. Circle the correct answers. Assume friction and air resistance can be neglected.

1. The work done on mass A is the same as that done on mass B.
2. $P_A = 2P_B$
3. $P_A = P_B$
4. $F_A \neq F_B$
5. $P_B = 2P_A$

C. [5 pts.] The acceleration due to gravity at the surface of a distant planet is 1/12 that of the surface of the Earth. A projectile is launched from the surface of this planet with an initial speed $v_0$ at an angle $\theta$ with respect to the horizontal. An identical projectile is launched from the surface of the Earth at the same angle $\theta$ but with an initial speed of $6v_0$. Ignoring air resistance, which of the following statements are true?

1. The Earth-launched projectile has the greater range.
2. The planet-launched projectile has the greater range.
3. The two projectiles have equal range.
4. The Earth-launched projectile has the greater time of flight.
5. The planet-launched projectile has the greater time of flight.
6. The two projectiles have equal times of flight.

D. [5 pts.] Consider two bowling ball pendulums. Pendulum A has length $L$ and a bowling ball of mass $M$. Pendulum B has length $2L$ and a mass $2M$. Neglecting air resistance and friction, which of the following are true?

1. The angular frequency ($\omega$) of A is greater than that of B.
2. The angular frequency of B is greater than that of A.
3. Both pendulums have the same period.
4. In the small angle approximation the period depends on the angle $\theta$ at which a pendulum is released.
5. The torque about the pivot is greater for A than for B.
6. The torque about the pivot is greater for B than for A.
7. The torques about the pivots are the same for the two pendulums.
In a lecture demonstration, Adam fired a projectile of initial speed $v_o$ at an object (a cougar doll) of mass $M$ and horizontal distance $D$ from the gun. The cougar doll was released at the same moment that the projectile (of mass $m$) was released. We proved in class that the projectile would hit the cougar doll for any firing angle. Neglect air resistance in the questions below. All answers in parts (a) and (b) should be expressed algebraically in terms of given quantities and $g$.

(a) **[6 pts.]** Calculate the time of flight of the projectile.

(b) **[6 pts.]** How far does the cougar doll fall before it is hit by the projectile?

(c) **[8 pts.]** What is the y-component of the velocity of the projectile immediately before it hits the cougar? Obtain a numerical answer for this part. Assume $\theta = 30^\circ$, $D = 15$ m, $v_o = 30$ m/s, $m = 0.50$ kg, $M = 0.25$ kg.
A pendulum of length L and mass M has a horizontal spring of force constant κ connected to it and to a fixed wall at a vertical distance h below its point of suspension. Assume that the vertical suspension rod of length L is rigid, but ignore its mass. Also, assume that θ remains small and that the spring can be treated as horizontal at all times. Pick the origin at the pivot point.

(a) [5 pts.] Calculate the magnitude of the gravitational torque on the pendulum as a function of θ.

(a) [8 pts.] Calculate the magnitude of the spring torque on the pendulum as a function of θ.

(b) [7 pts.] Calculate the period of vibration of the system.
SHOW ALL WORK!!
Place a circle or box around each answer. Specify units for each answer.
Report all numbers to two significant figures.

A solid cylinder of mass $M$ and radius $R$ starts sliding down a frictionless plane. After sliding down a distance $d$, it hits a rough region where the coefficient of static friction is $\mu_s$ and the coefficient of kinetic friction is $\mu_k$. Soon after it hits the rough region it starts to roll. Express answers to all parts in terms of given quantities and $g$, except for part (c) where a numerical answer is required.

(a) [5 pts.] Calculate the speed of the cylinder just before it first encounters the rough region.

(b) [5 pts.] Calculate the magnitude of the acceleration of the center of mass of the cylinder while sliding in the rough region.

(c) [3 pts.] If $\theta = 30^\circ$, calculate the value of $\mu_k$ for which the acceleration of the cylinder is zero, while it is sliding in the rough region.

(d) [7 pts.] Calculate the magnitude of the frictional force between the cylinder and the plane, when it is rolling without slipping.
A block of mass $m = 2.0$ kg hangs in equilibrium from a vertical spring. When a second identical block is added, the two-block system is stretched compared to that for the one-block system and is observed to have an oscillation frequency of $1.58$ Hz (oscillations per second).

(a) $[8 \text{ pts.}]$ Determine the spring constant $k$.

(b) $[12 \text{ pts.}]$ Determine the change in length $\Delta L$ of the stretched spring due to the addition of the second mass when it is not oscillating.
A satellite of mass \( m = 2.80 \text{ kg} \), in an elliptical orbit with the Earth at a focus, has the following parameters. At the point of nearest approach (perigee) it is 465 km above the surface of the Earth (assumed to be spherical), whereas at the most distant point (apogee) it is 2370 km above the Earth’s surface.

(a) \([10 \text{ pts.}]\) Find the ratio \( v_p/v_a \) of the speed at perigee to that of apogee.

(b) \([10 \text{ pts.}]\) Calculate the period of the satellite’s orbit. (Hint: The formula for the period of an object in an elliptical orbit is similar to that for a circular orbit except that the radius \( r \) of the circle is replaced by \( a \), the semi-major axis of the ellipse.)
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\[
\text{a) } v_0 \cos \theta t = D \quad t = \frac{D}{v_0 \cos \theta}
\]

\[
\text{b) } h = \frac{v_0^2 \sin 2\theta}{2g} + \frac{1}{2}gt^2 = \frac{1}{2}g \left( \frac{D}{v_0 \cos \theta} \right)^2 = \frac{gD^2}{2v_0^2 \cos^2 \theta}
\]

\[
\text{c) } v_y = v_{0y} + at = -v_0 \sin \theta + g \frac{D}{v_0 \cos \theta}
\]

\[
= -30 \times \frac{1}{2} + 9.8 \times \frac{15}{30 \times 0.5} = -15 + \frac{98}{15} \approx -9.3 \text{ m/s}
\]
A pendulum of length $L$ and mass $M$ has a horizontal spring of force constant $k$ connected to it and to a fixed wall at a vertical distance $h$ below its point of suspension. Assume that the vertical suspension rod of length $L$ is rigid, but ignore its mass. Also, assume that $\theta$ remains small and that the spring can be treated as horizontal at all times. Pick the origin at the pivot point.

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(b) $[8\text{ pts.}]$ Calculate the magnitude of the spring torque on the pendulum as a function of $\theta$.

(b) $[7\text{ pts.}]$ Calculate the period of vibration of the system.

(c) The torques are in direction of decreasing $\theta \Rightarrow$ we should have a $-\text{sign.}$

\[ \tau_{\text{net}} = \tau \alpha \]
Problem 3

(c) \( \tau_{net} = I \alpha \)

\[ I = ml^2 \quad \text{[just treating mass as point]} \]

\[ \alpha = \frac{d^2 \theta}{dt^2} \]

\[ \tau_{net} = -mgl\theta - kh^2\theta \quad \text{[using correct sign]} \]

\[ m \frac{d^2 \theta}{dt^2} = -(mgl + kh^2)\theta \]

\[ \frac{d^2 \theta}{dt^2} = -\frac{mgl + kh^2}{ml^2}\theta \]

Compare this to a spring \( \frac{d^2 x}{dt^2} = -\frac{k}{m} x \) with \( \omega^2 = \frac{k}{m} \)

or a simple pendulum \( \frac{d^2 \theta}{dt^2} = -\frac{g}{L} \theta \) with \( \omega^2 = \frac{g}{L} \)

\[ \Rightarrow \quad \omega^2 = \frac{g}{L} + \frac{kh^2}{ml^2} \]

The angular jepn is related to the period by \( \omega T = 2\pi \)

\[ T = \frac{2\pi}{\omega} = 2\pi \frac{1}{\sqrt{\frac{g}{L} + \frac{kh^2}{ml^2}}} = 2\pi \sqrt{\frac{ml^2}{mgl + kh^2}} \]
A solid cylinder of mass \( M \) and radius \( R \) starts sliding down a frictionless plane. After sliding down a distance \( d \), it hits a rough region where the coefficient of static friction is \( \mu_s \) and the coefficient of kinetic friction is \( \mu_k \). Soon after it hits the rough region it starts to roll. Express answers to all parts in terms of given quantities and \( g \), except for part (c) where a numerical answer is required.

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(d) [7 pts.] Calculate the magnitude of the frictional force between the cylinder and the plane, when it is rolling without slipping.

\( \text{Energy is conserved for this portion} \)

\[
\frac{1}{2} m v^2 = m g h - d \sin \theta
\]

\[
\sqrt{V^2} = \sqrt{2 g d \sin \theta} \implies \sqrt{V^2} = \sqrt{2 g d \sin \theta}
\]

\( b) \)

\[
\begin{align*}
\sum F_x &= m g \sin \theta - f_k = ma \\
\therefore f_k &= M_k N = M_k m g \cos \theta
\end{align*}
\]

\[
\sum F_y = N - m g \cos \theta = 0 \implies N = m g \cos \theta
\]

\[
M_k \sin \theta = m g \cos \theta = ma
\]

\[
a = g \sin \theta - M_k g \cos \theta
\]

\( c) \)

\( a = 0 \) \implies \( 0 = g \sin \theta - M_k g \cos \theta \)

\[
M_k = \frac{\sin(30^\circ)}{\cos(30^\circ)} = 0.58
\]
d) The point of contact is instantaneously at rest when it is rolling without slipping, so \( v_{cm} = R \omega \), and \( a_{cm} = R \alpha \)

\[
mgsin\theta - f_s = ma_{cm}
\]

\[
\tau = f_s R = I\alpha = \frac{Ia_{cm}}{R} \Rightarrow a_{cm} = \frac{f_s R^2}{I}
\]

\[
mgsin\theta - f_s = m \left( \frac{f_s R^2}{I} \right)
\]

\[
mgsin\theta = f_s \left( 1 + \frac{mR^2}{I} \right)
\]

It is a solid cylinder so \( I = \frac{1}{2} mR^2 \)

\[
mgsin\theta = f_s \left( 1 + 2 \right)
\]

\[
\frac{1}{3} mgsin\theta = f_s
\]
A block of mass \( m = 2.0 \) kg hangs in equilibrium from a vertical spring. When a second identical block is added, the two-block system is stretched compared to that for the one-block system and is observed to have an oscillation frequency of 1.58 Hz (oscillations per second).

(a) [8 pts.] Determine the spring constant \( k \).

(b) [12 pts.] Determine the change in length \( \Delta L \) of the stretched spring due to the addition of the second mass when it is not oscillating.

\[
(\alpha) \; \omega = \sqrt{\frac{k}{m}} \implies 2\pi f = \sqrt{\frac{k}{2m}} \implies k = 2m(2\pi f)^2
\]

\[
k = 2 \cdot 2.0 \text{ kg} \cdot (2\pi \cdot 1.58 \text{ Hz})^2 = 3.9 \times 10^2 \text{ kg/s}^2
\]

\[
(\beta) \; F = -kx = mg \quad \text{for one block equilibrium}
\]

\[
k(x+\Delta L) = 2mg
\]

\[
\implies \Delta L = \frac{mg}{k} = \frac{2.0 \text{ kg} \cdot 9.8 \text{ m/s}^2}{3.9 \times 10^2 \text{ kg/s}^2} = 0.5 \text{ cm}
\]
A satellite of mass \( m = 2.80 \) kg, in an elliptical orbit with the Earth at a focus, has the following parameters. At the point of nearest approach (perigee) it is 465 km above the surface of the Earth (assumed to be spherical), whereas at the most distant point (apogee) it is 2370 km above the Earth’s surface.

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**Part a**
There is no external torques, so angular momentum is conserved. At perigee and apogee velocity is perpendicular to position, i.e., \( \theta = 90^\circ \)

\[
L_f = L_0 \
\Rightarrow L_A + I_c w_E = L_p + I_c w_E \\
\Rightarrow R_a m v_a = R_p m v_p
\]

\[
\Rightarrow \frac{v_p}{v_a} = \frac{R_A}{R_p} = \frac{8.74 \times 10^3 \text{ km}}{6.835 \times 10^3 \text{ km}} = 1.279 \Rightarrow 1.3
\]

**Part b**

\[
T^2 = \frac{4\pi^2}{GM_e} a^3
\]

\[
\Rightarrow T = \sqrt[3]{\frac{4\pi^2 \left(7.7875 \times 10^6 \text{ m} \right)^3}{(6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}) \cdot (5.97 \times 10^{24} \text{ kg})}} = 6.8 \times 10^3 \text{ seconds}
\]