Name: ________________________________

Unid: ________________________________

Discussion TA: ________________________________

---

**Physics 2220**

**Spring 2013**

**John DeFord**

---

**Exam 2**

---

**Show all work!**

[20 pts.] For the circuit shown, determine the direction and magnitude of the current in the horizontal wire between a and e.

R = 1.00 kΩ
E = 250 V

---

*Step 1:*

Assign: (see drawing)
- loop direction
- current direction

---

*Step 2:*

Determine equation (note that $i_4 = i_1 + i_2 - i_3$)

For each loop:

**Loop 1:**

$250 - 1000i_1 - 4000i_3 = 0$

$-1000i_1 - 4000i_3 = -250$

**Loop 2:**

$-3000i_4 + 4000i_3 = 0$

$-3000(i_1 + i_2 - i_3) + 4000i_2 = 0$

$-3000i_1 - 3000i_2 + 7000i_3 = 0$

**Loop 3:**

$-300i_4 + 500 - 2000i_2 = 0$

$-300(i_1 + i_2 - i_3) + 500 - 2000i_2 = 0$

$-3000i_1 - 5000i_2 + 3000i_3 = -500$

Determine $i_1$, $i_2$, and $i_3$:

\[
\begin{bmatrix}
-1000 & -4000 & -250 \\
-3000 & -3000 & 7000 \\
-3000 & -5000 & 3000 \\
\end{bmatrix}
\]

Using calculator:

\[
\begin{align*}
&i_1 = 10 \text{ mA} \\
&i_2 = 130 \text{ mA} \\
&i_3 = 60 \text{ mA}
\end{align*}
\]

---

*Step 4:*

Determine the current in the wire AE:

$\dot{I}_{ae} = i_3 - i_1 = 60 - 10 = 50 \text{ mA}$ (from a to e)
The circuit shown consists of two resistors, $R_1$ and $R_2$ and two capacitors, $C_1$ and $C_2$. They are connected to battery. The switch is initially open with no charge on the capacitors. The switch is then closed.

(a) [10 pts.] Determine the charge on capacitor $C_1$ as a function of time.

(b) [10 pts.] Determine the charge on capacitor $C_2$ as a function of time.

$R_1 = 2.00 \text{ k}\Omega$

$R_2 = 3.00 \text{ k}\Omega$

$C_1 = 2.00 \text{ \mu F}$

$C_2 = 3.00 \text{ \mu F}$

$e = 120 \text{ V}$

**Step 1:** Determine equivalent variables:

$C_{eq} = C_1 + C_2 = 2 + 3 = 5 \text{ \mu F}$

$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{2(3)}{2 + 3} = 1.2 \Omega$

$\tau = R_{eq} C_{eq} = (1.2)(15 \times 10^{-6}) = 0.006 \text{ s}$

**Step 2:** Derive equation for $V_c$:

$E - IR_{eq} - V_c = 0$

$V_c = E - IR_{eq}$

$I = I_0 e^{t/\tau} = \frac{E}{R_{eq}} e^{t/\tau}$

Thus,

$V_c = V - \left(\frac{E}{R_{eq}} e^{t/\tau}\right) R_{eq}$

$V_c = V \left(1 - \frac{e^{t/\tau}}{\tau}\right)$

**Step 3:** Determine $q_1(t)$ and $q_2(t)$:

$q = C_1 V_c$

Thus,

$q_1 = C_1 V_c = C_1 V \left(1 - \frac{e^{t/\tau}}{\tau}\right)$
A proton, moving in the plane of the page has a kinetic energy of 6.00 MeV. A magnetic field of \( B = 1.00 \, \text{T} \) is directed into the page. The proton enters the magnetic field with its velocity vector at angle \( \theta = 45.0^\circ \) to the linear boundary of the field as shown.

(a) [10 pts.] Find \( x \), the distance from the point of entry to where the proton will leave the field.

(b) [10 pts.] Calculate the angle \( \theta' \) which the proton leaves the field.

\[
\theta = \theta' = 45^\circ
\]

\[
\begin{align*}
\theta & = \theta' \\
\text{determine } x & = 2 R \sin \frac{\theta}{2} \\
& = 2 \times 0.51 \, \text{m}
\end{align*}
\]
[20 pts.] An infinitely long, straight wire carries a current $I_1$ and is partially surrounded by a loop of length $L$ and radius $R$ carrying a current $I_2$. The axis of the loop coincides with the wire. Calculate the magnetic force exerted on the loop.

The magnetic field is parallel with Section 1 and Section 2. Thus, the force is zero. $F = I L B \sin \theta$.

Thus, the problem becomes:

$$F_{\text{net}} = F_2 + F_4$$

where $F_2$ and $F_4$ are the forces that the outer wire causes on wire 2 and wire 4, respectively.

Thus,

$$F_{\text{net}} = F_2 + F_4$$

$$= I_2 L B + I_2 L B$$

$$= 2 I_2 L B$$

since $B = \frac{\mu_0 I_1}{2 \pi R}$.
A long cylindrical conductor of radius $a$ has two cylindrical cavities each of diameter $a$ through the entire length as shown in the end view. A current $I$ is directed out of the page and is uniform through a cross section of the conducting material.

(a) [10 pts.] Find the magnitude and direction of the magnetic field in terms of $\mu_0$, $I$, $r$ and $a$ at point $P_1$.

(b) [10 pts.] Find the magnitude and direction of the magnetic field in terms of $\mu_0$, $I$, $r$ and $a$ at point $P_2$.

Finding currents:

\[ I = J \pi a^2 \]
\[ I_1 = I_2 = \frac{J \pi a^2}{4} = -\frac{I}{4} \]

**Part A:**

\[ B = \frac{\mu_0 I}{2\pi r} - \frac{\mu_0 I}{8\pi (r-a)^2} - \frac{\mu_0 I}{8\pi (r+a)^2} \]
\[ = \frac{\mu_0 I}{2\pi r} - \frac{\mu_0 I}{8\pi (2r-a)^2} - \frac{\mu_0 I}{8\pi (2r+a)^2} \]
\[ = \frac{\mu_0 I}{2\pi} \left[ \frac{1}{r} - \frac{1}{2(2r-a)^2} - \frac{1}{2(2r+a)^2} \right] = \frac{\mu_0 I}{2\pi r} \left( \frac{2r^2-a^2}{4r^2-a^2} \right) \]

**Part B:**

\[ d = \left( r^2 + \frac{a^2}{4} \right)^{1/2} \]

\[ B = \frac{\mu_0 I}{2\pi r} - \frac{\mu_0 I}{8\pi (r^2+\frac{a^2}{4})^{1/2}} - \frac{\mu_0 I}{8\pi (r^2+\frac{a^2}{4})^{1/2}} \]
\[ = \frac{\mu_0 I}{2\pi r} - 2 \frac{\mu_0 I}{8\pi (r^2+\frac{a^2}{4})^{1/2}} = \frac{\mu_0 I}{2\pi r} - \frac{\mu_0 I}{4\pi (r^2+\frac{a^2}{4})^{1/2}} \]
\[ = \frac{\mu_0 I}{2\pi} \left[ \frac{1}{r} - \frac{1}{2 \left( r^2 + \frac{a^2}{4} \right)^{1/2}} \right] = \frac{\mu_0 I}{2\pi} \left[ \frac{1}{r} - \frac{1}{(4r^2+a^2)^{1/2}} \right] \]