A sphere of nonconductor is negatively charged with a charge density given by
\[ \rho = AR^2, \] where \( A \) is a constant. The radius of the sphere is \( R_0 \).

(a) [15 pts.] Calculate the magnitude of the electric potential difference between the surface of the sphere and a point \( P \), a distance \( R \) from the center. \( (R < R_0) \).

(b) [5 pts.] State the sign of the potential difference \( V(R) - V(R_0) \) and give a clear physical (not mathematical) reason for this sign.

(c) [5 pts.] If the total charge on the sphere is \( Q \), calculate \( A \).

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\begin{align*}
\text{Charge within the sphere of radius } R & \leq Q(R) = \int_0^R \rho \, dV = \int_0^R 2\pi r^2 \rho \, dr = \int_0^R 2\pi A r^2 \, dr = \frac{2\pi A R^3}{3} \\
\text{Electric field at } R & \sim \text{Electrical field at } R, \text{ by Gauss' law.} \\
E(R) & = \frac{Q(R)}{4\pi \epsilon_0 R^2} = k \cdot \frac{2\pi A R^3}{3 R^2} = \left(\frac{4\pi k A}{3}\right) R = \left(\frac{4\pi k A}{3}\right) R^4 \\
\text{Potential difference } V(R) - V(R_0) & = -\int_R^{R_0} E(R) \, dR = \left[ -\frac{A}{\epsilon_0} \cdot \frac{R^3}{R} \right]_R^{R_0} = \frac{A (R_0^3 - R^3)}{3 \epsilon_0} = \frac{4\pi A (R_0^3 - R^3)}{3 \epsilon_0} \\
\text{From } (a), & : A < 0 \text{ (negatively charged)} \& R < R_0, (R < R_0) \\
& : A (R_0^3 - R^3) > 0, \therefore V(R) - V(R_0) > 0, \therefore V(R) - V(R_0) < 0 \\
\text{Or: } & V(R) - V(R_0) \text{ is negative.} \\
\therefore & \text{The sphere is negatively charged}, \text{ } E \text{ pointed towards the center} \\
& \text{V(R) should increase with } R, \therefore V(R) - V(R_0) < 0 \Rightarrow V(R) - V(R_0) \text{ is negative.} \\
(c) \quad Q(R_0) = \frac{2\pi A R_0^3}{3} = Q \Rightarrow A = \frac{3Q}{2\pi R_0^3}
\end{align*}
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