Consider a positively charged sphere whose charge density can be expressed as 
\[ \rho = \rho_0 (1 - aR), \]
where the radius of the sphere is \( R_0 \). If the charge density goes to zero at \( R = R_0 \), and \( Q \) is the total charge, calculate:

(a) the value of \( a \) (in terms of \( R_0 \));
(b) the value of \( \rho_0 \) in terms of \( Q \), \( R_0 \) and numbers only;
(c) the electric field at \( R = R_0/3 \) (eliminate \( \rho_0 \) and \( a \) from your final expression).

\[ S \]
(a). \[ \rho(R) = \rho_0 (1 - aR). \]
\[ \rho(R_0) = \rho_0 (1 - aR_0) = 0 \quad \Rightarrow \quad 1 - aR_0 = 0, \quad \Rightarrow \quad a = 1/R_0 \]

\[ 10 \]  
(b). Total charge \( Q = \int \rho \, dV = \int_0^{R_0} \rho_0 (1 - aR) \cdot 4\pi R^2 \, dR = 4\pi \rho_0 \int_0^{R_0} (R^2 - aR^3) \, dR \]
\[ = 4\pi \rho_0 \left( \frac{R_0^3}{3} - \frac{aR_0^4}{4} \right) \quad \text{with} \quad a = \frac{1}{R_0}, \quad 4\pi \rho_0 \left( \frac{R_0^{3/2}}{3} - \frac{R_0^{3/4}}{4} \right) = \frac{\pi \rho_0}{3} \]
\[ \Rightarrow \rho_0 = \frac{3Q}{\pi R_0^3} \]

\[ 15 \]  
(c). By Gauss' Law.

at \( R = \frac{R_0}{3} \): \[ E = \frac{\rho e_0}{4\pi R^2} = \frac{1}{4\pi \frac{1}{3} R_0^2} \cdot \frac{Q}{9\epsilon_0} = \frac{Q}{4\pi \frac{1}{9} R_0^2} \]