SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A long cylinder shaped charge distribution of radius \( R_c \) has a charge density given by \( \rho = BR^2 \) (not physically sensible, but it keeps the math simple). \( B \) is a constant.

(a) Find the total charge for a length \( l \) of this cylinder.

(b) Calculate the electric field at an arbitrary value of \( R \) where \( R < R_c \).

(c) Calculate the magnitude of the electric potential difference between \( R = 0 \) and \( R = \frac{2}{3} R_c \).

\[
\begin{align*}
0 & \quad 10^3 & \quad 10 \\
(a) & \quad \text{Find the total charge for a length } l \text{ of this cylinder.} & \\
(b) & \quad \text{Calculate the electric field at an arbitrary value of } R \text{ where } R < R_c. & \\
(c) & \quad \text{Calculate the magnitude of the electric potential difference between } R = 0 \text{ and } R = \frac{2}{3} R_c. & \\
\end{align*}
\]

\[
\begin{align*}
Q & = \int \rho \, dV = \int \rho \, r \, dr \, dz \\
& = \int_0^{2\pi} \, d\theta \int_0^{R_c} \, B \, R^2 \, r \, dr \\
& = 2\pi \, l \, B \int_0^{R_c} \, R^4 \, dr \\
& = \frac{2\pi \, l \, B}{5} \, R_c^5. \\
\end{align*}
\]

b) \[
\begin{align*}
\oint B \cdot dr & = \frac{1}{\epsilon_0} \sum Q.
\end{align*}
\]

At arbitrary \( R \), the enclosed charge is \( \frac{2\pi \, l \, B}{5} \, R_c^5 \), and \[
\begin{align*}
\oint \vec{E} \cdot ds & = \vec{E} \cdot \vec{r} \cdot l = \frac{2\pi \, l \, B}{5} \, R_c^5.
\end{align*}
\]

\[
\Rightarrow \quad \vec{E} = \frac{B}{5\epsilon_0} \, R_c^4.
\]

\[
\begin{align*}
U & = -\int \vec{E} \cdot dr = -\int_0^{2\pi} \frac{1}{\epsilon_0} \, \frac{B}{5} \, R_c^4 \, dr \\
& = -\frac{B}{5\epsilon_0} \, R_c^4 \bigg|_0^{2\pi} = -B \, \frac{1}{75\epsilon_0} \, \left( \frac{2\pi}{3} \right) \, R_c^4.
\end{align*}
\]