SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A bolt of lightning has a current density distribution given by \( j = AR^4 \) for \( R < R_o \) and 0 for \( R > R_o \) (not physical, but keeps the math simple).

(a) Calculate the magnetic field at a radius \( R \), where \( R < R_o \).

(b) Find the energy in the magnetic field for a length \( l \) and between \( R = 0 \) and \( R = \frac{3}{4} R_o \).

\[
15(a) \quad \oint B \cdot dl = \mu_0 I_{\text{enclose}}
\]

\[
2\pi RB = \int_0^R 2\pi R^4 dR = \int_0^R 2\pi R^6 dR = 2\pi R^6
\]

\[
\therefore \quad B = \frac{\mu_0 A R^5}{6}
\]

\[
15(b) \quad \text{Energy density} \quad \frac{B^2}{2\mu_0} = \frac{\mu_0 A^2 R^6}{24}
\]

\[
E = \int \frac{B^2}{2\mu_0} dV = \int_0^{\frac{3}{4}R_o} \frac{\mu_0 A^2 R^6}{72} 2\pi R dR
\]

\[
= \frac{\pi \mu_0 A^2}{36} \left[ \frac{3}{4} R^8 \right]_0^{\frac{3}{4}R_o} = \frac{\pi \mu_0 A^2 l}{432} \left( \frac{3}{4} R_o \right)^2
\]

\[
= 7.83 \times 10^{-5} \pi A^2 \mu_0 R_o^2
\]