FIRST MIDTERM

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Discussion Instructor (Circle One): Brown Chung Pollard Rothman
Discussion Section #: Schweizer Soderberg Vaseghi Viohl

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) Calculate magnitude of the electric field 12.5 m from a point charge of

\[ E = 1.077 \times 10^5 \text{ N/C} \]

(b) Calculate the electric force between a proton and an electron that are

\[ 0.50 \times 10^{-10} \text{ m} \text{ apart. (The proton charge is equal in magnitude to the} \]

the electron charge, but opposite in sign.)

\[ F = -9.22 \times 10^{-9} \text{ N} \text{ attractive} \]

(c) Calculate the first three terms of binomial expansion for the expression

\[ \frac{1}{(x^2 + a^2)^{7/2}} \approx \frac{1}{x^7} - \frac{2}{x^5} \frac{a^2}{x^2} + \frac{6}{x} \frac{a^4}{x^4} \]

(d) Calculate \( g \) at a point \( 1/4 \) the distance from the center of the earth to

the surface. (Assume uniform density.)

\[ g = 2.45 \text{ m/s}^2 \]

(e) Calculate the gravitational force on a 1.00 kg object that is two earth

radii above the earth's surface.

\[ F = 1.08 \text{ N} \text{ attractive} \]
FIRST MIDTERM

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Discussion Instructor (Circle One): Brown Chung Pollard Rothman
Discussion Section #: Schweizer Soderberg Vaseghi Viohl

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Two charges of equal magnitude, +Q and -Q are a distance a apart on the x-axis. Take x = 0 at the midpoint between the two charges.

(a) Write a general expression for the electric field at point P, at the coordinate x.

(b) Using the binomial expansion, calculate the electric field at P, keeping the first two non-zero terms involving a.

(c) If a/x is 0.10, what fractional error is introduced by ignoring the last term in part (b). (That is, what fraction of the total answer is this last term?)

\[ E = \frac{\varepsilon_0}{4\pi\varepsilon_r} \left[ \left( \frac{1}{(x+a)^2} - \frac{1}{(x-a)^2} \right)^{\frac{1}{2}} \right] \]

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\[ E = \frac{\varepsilon_0}{4\pi\varepsilon_r} \frac{1}{x^2} \left[ \left( \frac{1}{1+\frac{a}{x}} \right)^2 - \frac{1}{(1-\frac{a}{x})^2} \right] \]

\[ (1 + \frac{a}{x})^2 = 1 + 2\left( \frac{a}{x} \right) + \left( \frac{a}{x} \right)^2 \]

\[ (1 - \frac{a}{x})^2 = 1 - 2\left( \frac{a}{x} \right) + \left( \frac{a}{x} \right)^2 \]

\[ E = \frac{\varepsilon_0}{4\pi\varepsilon_r} \frac{1}{x^2} \left( \frac{2a}{x} \right)^{\frac{1}{2}} \]

\[ E = \frac{\varepsilon_0}{4\pi\varepsilon_r} \frac{1}{x^2} \left( \frac{2a}{x} \right)^{\frac{1}{2}} \]

\[ \text{Fractional Error} = \frac{\varepsilon_0}{2\varepsilon_r} + a^3 \]

\[ \text{Fractional Error} = \frac{\varepsilon_0}{2\varepsilon_r} + a^3 \]

\[ \text{Fractional Error} = \frac{0.01}{2.01} \]

\[ \text{Fractional Error} = 0.005 \]
a) The electric field of a point charge is given by \[ E = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q}{r^2} \]. Set \( 4\pi \varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N\cdot m^2} \), and use the values given for \( q \) and \( r \).

\[ E = 1.0 \pm 7 \times 10^{-4} \text{ V/m} \]

b) The electrostatic force between two point charges has a magnitude of \( F = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \). Using the charge \( q_1 = 1.6 \times 10^{-19} \text{ C} \) for a proton and \( q_2 = -1.6 \times 10^{-19} \text{ C} \) for an electron, gives us

\[ F = 9.2 \times 10^{-20} \text{ N} \] (negative sign indicates attractive force!)

c) \( \frac{1}{(x^2 + a^2)^{3/2}} = x^{-3} (1 + \frac{x^2}{a^2})^{-3/2} = x^{-3} (1 + \frac{x^2}{a^2})^{3/2} \) where \( y = \frac{x^2}{a^2} \)

Consider a function \( f(y) \). The Taylor series expansion of this function is given as: \( f(y) = f(0) + f'(0) \cdot \frac{y}{1!} + f''(0) \cdot \frac{y^2}{2!} + \ldots \)

In general \( f(y) = \sum_{n=0}^{\infty} f^{(n)}(0) \cdot \frac{y^n}{n!} \) where \( f^{(n)} \) is the \( n \)th derivative with respect to \( y \)

Set \( f(y) = \frac{1}{(1 + y)^{3/2}} = (1 + y)^{-3/2} \), it follows from \( y \)

\( f(y) = 1 - \frac{3}{2} y + \frac{3}{4} y^2 \cdot \ldots \) Combining \( f(1) \) and \( f(4) \)

Gives us \( \frac{1}{(x^2 + a^2)^{3/2}} \approx \frac{1}{x^3} \cdot (1 - \frac{3}{2} x + \frac{3}{4} x^2) \) or \( \frac{\frac{e^4}{x} + \frac{63 \cdot e^4}{8 \cdot x^7}}{} \)

So \( \frac{1}{(x^2 + a^2)^{3/2}} \approx \frac{1}{x^3} - \frac{3}{2} \frac{e}{x^5} + \frac{63 \cdot e^4}{8 \cdot x^7} \)
1) Sketch:

Let the earth have a total mass $M$ and a radius $R$. The mass density, some uniform, is given as:

$$
\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3}
$$

This gives the gravitational field at $r$, given by:

$$
g = G \cdot \frac{M}{r^2}
$$

When $H$ is the mass confined in the sphere of radius $r$. Some uniform density

$$
\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi r^3}
$$

Using the rule $r = \frac{2}{3} R$, we have:

$$
g = G \cdot \frac{M}{\left(\frac{2}{3} R\right)^2}
$$

This is easy to show that $g = \frac{GM}{R^2}$ is the gravitational field at the earth's surface. So from Eq. (5),

$$
g = \frac{GM}{R^2} = \frac{6.67 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^{-2}}{6.37 \times 10^6 \text{ m}} = 9.8 \text{ m/s}^2
$$

2) Sketch:

Consider the earth as a spherical symmetry mass distribution. We then can consider it as a mass point at the center of mass $M$. Then Newton's laws of gravitation gives us for the magnitude:

$$
F = G \cdot \frac{Mm}{r^2}
$$

but $r = 3R$ so

$$
F = G \cdot \frac{Mm}{(3R)^2}
$$

Now $g = \frac{GM}{R^2}$, so

$$
g = \frac{GM}{R^2}
$$

so

$$
F = \frac{1}{3} g \cdot m \cdot 98 \text{ N}
$$

so

$$
F = \frac{1 \times 9.8 \text{ m/s}^2}{3} = 3.26 \text{ N}
$$
A uniformly charged sphere of a non-conductor has radius $R_1$, and charge $Q_1$. It is enclosed in a concentric thin metal spherical shell whose radius is $R_2$. ($R_2 > R_1$). The metal shell has total charge $Q_2$.

(a) Calculate the electric field a distance 37.0 cm from the common center of the two spheres. (Numerical answer including sign.)

(b) Calculate the electric field a distance of 17.5 cm from the common center. (Numerical answer including sign.)

(c) Calculate the electric field a distance 2.00 cm from the common center. (Numerical answer including sign.)

\[
\begin{align*}
Q_1 &= +175 \mu C \\
Q_2 &= -325 \mu C \\
R_1 &= 10.0 \text{ cm} \\
R_2 &= 25.0 \text{ cm}
\end{align*}
\]

\[\mathbf{E} = \frac{\kappa Q_1}{r^2} + \frac{\kappa Q_2}{r^2} = \frac{9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \left[1.75 \times 10^{-4} \text{ C} - 3.25 \times 10^{-4} \text{ C}\right]}{(0.37)^2} = -9.86 \times 10^6 \text{ N}/\text{C} \]

\[\mathbf{E} = \frac{\kappa Q_1}{r^2} = \left[9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right] \left[1.75 \times 10^{-4} \text{ C}\right] = \pm 5.14 \times 10^7 \text{ N}/\text{C} \]

\[\mathbf{E} = \kappa \left[\text{charge enclosed}\right] = \kappa \left[\frac{Q_1}{4 \pi \epsilon_0 (0.1)^3}\right] \left[\frac{4}{3} \pi R^2\right] = \left[9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right] \left[1.75 \times 10^{-4} \text{ C}\right] \left[2 \times 10^{-2} \text{ m}\right] = 3.15 \times 10^7 \text{ N}/\text{C} \]
(a) Calculate the electric force between two positive charges of 6.0 μC and 9.0 μC a distance of 3 centimeters apart. \(5 \times 10^{-4} \text{ N}\)

(b) Calculate the gravitational force between a 100 kg man and a 1.50 \(\times\) 10^3 kg truck, if they are 10.0 meters apart. \(1.00 \times 10^{-2} \text{ N}\)

(c) Find the acceleration of an electron in an electric field of 1.30 \(\times\) 10^3 N/C. \(2.3 \times 10^{14} \text{ m/s}^2\)

(d) Find the value of \(g\) at a distance of 8000 miles above the earth’s surface. \(1.1 \text{ N/kg}\)

(e) The distance from earth to the sun is 93,000,000 miles. Find the mass of the sun. \(1.9 \times 10^{30} \text{ kg}\)
Problem 7

a) Force is: \( F = \frac{1}{4 \pi \epsilon_0} \frac{q_1 q_2}{r^2} \quad \mu = 10^{-6} \)

\[
F = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(6.0 \times 10^{-6} \text{ C})(9.0 \times 10^{-6} \text{ C})}{(3 \times 10^{-2} \text{ m})^2} = 5 \times 10^3 \text{ N}
\]

2 significant figures since 3 cm has only 2 significant figures.

b) Force is: \( F = \frac{6 \cdot 10^4 \text{ N} \cdot \text{m} \cdot \text{m}}{r^2} \)

\[
F = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.0 \text{ kg})(1.5 \times 10^3 \text{ kg})}{(10.0 \text{ m})^2} = 1.00 \times 10^{-9} \text{ N}
\]

either \( 1.00 \times 10^{-9} \text{ N} \) or \( 1.000 \times 10^{-9} \text{ N} \) is correct.

c) Forces: \( F = ma \) and \( F = Eq \).

\[
a = \frac{E}{m} = \frac{7.3 \times 10^4 \text{ N} \cdot \text{C}}{2 \text{ C}} = 3.65 \times 10^7 \text{ N/C}^2
\]

\[
a = \frac{(1.30 \times 10^3 \text{ N} \cdot \text{C}^2)(1.6 \times 10^{-19} \text{ C})}{(9.1 \times 10^{-31} \text{ kg})} = 7.3 \times 10^4 \text{ m/s}^2
\]

2 significant figures since \( 1.6 \times 10^{-19} \text{ C} \) has 2 significant figures.

\( a \) in \( 8000 \) miles above surface, then \( 6000 + 4000 \) miles from earth's center.

\( 12000 \) miles \( \cdot \frac{5280 \text{ ft}}{\text{ mile}} \cdot \frac{0.3048 \text{ m}}{\text{ ft}} = 1.90 \times 10^7 \) meters

\[
\text{grav. } g = \frac{6 \cdot \text{ Earth}}{r^2} \quad \text{or} \quad g = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(6.0 \times 10^4 \text{ kg})}{(1.90 \times 10^7 \text{ m})^2}
\]

\[
g = [1.1 \text{ N/kg}] \quad \text{or} \quad [1.1 \text{ m/s}^2] \quad \text{2 significant figures since } 0.30 \text{ m} = 1 \text{ ft}
\]

\( \text{has } 2 \text{ significant figures} \)
c) You could solve this problem with Kepler's law:

\[ T^2 = \frac{4 \pi^2}{C^3} \] if you know what C is.

Here is the way to solve it without Kepler's law:

The force needed to keep the earth in circular orbit about the sun is

\[ F = \frac{M_e w^2 r}{r} \]

Where w is the angular velocity.

The force that holds the earth in orbit is gravity

\[ F = \frac{G M_e M_s}{r^2} \]

Setting these forces equal we have

\[ M_e w^2 r = \frac{G M_e M_s}{r^2} \]

and solving for \( M_s \)

we get

\[ M_s = \frac{w^2 r^3}{5} \]

\[ w = \frac{2\pi}{T} \]

\[ T = 365 \text{ Days} \cdot \frac{24 \text{ hours}}{\text{Day}} \cdot \frac{3600 \text{ sec}}{\text{hour}} = 31536000 \text{ Sec} \]

Hence \( w = 1.9923849 \times 10^{-7} \text{ rad per sec} \)

Now we know \( r = 93,000,000 \) miles, we need \( r \) in meters

\( 93,000,000 \text{ miles} \cdot \frac{5280 \text{ ft}}{\text{mile}} \cdot \frac{12 \text{ in}}{\text{ft}} = 1.47 \times 10^{11} \text{ m} \)

So

\[ M_s = \frac{w^2 r^3}{5} = \frac{\left(1.9923849 \times 10^{-7} \text{ rad/sec}\right)^2(1.47 \times 10^{11} \text{ m})^3}{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)} \]

Hence

\[ M_s = 1.9 \times 10^{30} \text{ kg} \]

with 2 significant figures since in converting 93,000,000 miles to meters we used 0.30 m = 1 ft and 0.30 has only 2 significant figures.
on parts d and e, a few people used the English units, the correct answers are

d) 3.0 ft/sec^2

e) 1.3 \times 10^{26} slugs

Point breakdown:
5 points for each part

Having 2/1 extra significant figure: -1

One unit: -1

Wrong units: -2

Wrong exponent: -1

Wrong exponent: -2

Statistics

Total tests: 455
Mean score: 17
Std dev: 4.5
Problem 1A

1. Find the value of \( g \) 8000 miles above the earth's surface? \( 1.1 \text{ m/s}^2 \)

2. Find the force between two electrons 1.0 \( \times 10^9 \) m apart. \( 2.3 \times 10^{-46} \text{ N} \)

3. Find the value of \( g \) 2000 miles from the center of the earth. \( 4.9 \text{ m/s}^2 \)

4. Find the electric field 2.0 \( \times 10^{10} \) m away from a helium nucleus with charge +2e. \( 7.2 \times 10^{-30} \text{ N/C} \)

5. A spinning top is rotating clockwise as seen from above. An attempt is made to push the top of the top towards the east. What direction will it move? South

\[
g = \frac{6.67 \times 10^{-11} \left( 6.6 \times 10^{24} \right)}{\left( 8000 + 4000 \right) \times 2 \times 0.30}^2 = 1.1 \text{ m/s}^2
\]

\[
F = \frac{1}{4 \pi \epsilon_0} \left( -1.6 \times 10^{-19} \right)^2 = 2.3 \times 10^{-46} \text{ N}
\]

\[
g = \frac{GM}{r^2} = \frac{4}{3} \pi G \rho r
\]

\[
p_n = \frac{M}{V} = \frac{3GM}{4\pi R^2}
\]

\[
\frac{F \omega^2}{R^2} = \frac{3 \omega^2 r}{4\pi GR}
\]

\[
\Rightarrow \int = \frac{5}{6} \omega^2 R = 3.8 \left( 2 \times 10^5 \right) \text{ rad/s}
\]
1. (a) Find the value of $g$ 3000 miles above the moon's surface (ignore the earth.)

\[ g = 4.4 \times 10^{-2} \text{ m/s}^2 \]

1. (b) Find the angular momentum of the earth rotating on its axis. \(-7.3 \times 10^{33}\) kg m²/s

\[ \text{force} \]

3. (c) Find the force between an electron and a proton 0.5 x 10¹⁰ m apart.

\[ F = 1.6 \times 10^{-7} N \]

4. (d) Find the value of $g$ 500 miles from the center of the moon.

\[ g = 0.75 \text{ m/s}^2 \]

5. (e) Find the electric field 20.0 m away from a metal sphere which has a charge of $3.3 \times 10^3 e$.

\[ E = 6.7 \times 10^1 \text{ N/C} \]
(a) Calculate the electric force between a lithium nucleus \( (z = 3) \) and an electron a distance \( 1.00 \times 10^{-11} \text{ m} \) away. \(-6.9 \times 10^{-6} \text{ N} \) (attractive)

(b) The earth is \( 9.3 \times 10^7 \text{ miles} \) from the Sun. Calculate the gravitational force between Earth and Sun. \( 3.5 \times 10^{22} \text{ N} \)

(c) Calculate the electric field magnitude at a point midway between a charge of \(+7.0 \text{ pC}\) and one of \(-3.5 \text{ pC}\). The distance is 0.75 m. \(0.67 \text{ N/C} \) (to the right)

(d) The Earth-Sun distance is called 1.00 A.U. (astronomical unit). If Jupiter orbits the Sun in 11.9 years, what is the Jupiter-Sun distance in A.U. (Assume circular orbits.) \(5.2 \text{ A.U.} \)

(e) If the mass of the Earth were doubled, and its radius were doubled, by what fraction would your weight be multiplied? \( \frac{1}{2} \)
a) \( Z = 3 \) means there are 3 protons in the nucleus, each with charge \( +e \). Let nucleus = \( q_1 \), electron = \( q_2 \)

\[ F_e = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{(3e)(-e)}{r^2} = -\frac{1}{4\pi\varepsilon_0} \frac{3e^2}{r^2} \]

\[ = -\left(9.00 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right) \left(3 \times 1.6 \times 10^{-19} \text{ C}\right)^2 / \left(1.00 \times 10^{-11} \text{ m}\right)^2 \]

\[ F_e = -6.9 \times 10^{-6} \text{ N} \]

b) \( F_g = G \frac{M_S M_e}{r^2} \) We need \( M_S \):

\[ T^2 = \frac{4\pi^2 r^3}{GM_S} \Rightarrow M_S = \frac{4\pi^2 r^3}{GT^2} \quad r = 9.3 \times 10^7 \text{ m} \]

Converting to metric units \( \Rightarrow M_S = 1.90 \times 10^{30} \text{ kg} \)

Then \( F_g = (6.67 \times 10^{-11}) \left(\frac{6.0 \times 10^{24} \text{ kg}}{1.90 \times 10^{30} \text{ kg}}\right) \left[\frac{(9.3 \times 10^7 \text{ m}) \times (5.28 \times 10^6 \text{ ft})}{\text{mass}}\right]^2 \]

\[ F_g = 3.5 \times 10^{22} \text{ N} \]

c) Both fields (one from \( \oplus \) and one from \( \ominus \)) are to the right:

\[ E = E_\oplus + E_\ominus = \frac{1}{4\pi\varepsilon_0} \left[\frac{7.0 \times 10^{-12} \text{ C}}{(d/2)^2} + \frac{3.5 \times 10^{-22} \text{ C}}{(d/2)^2}\right] \]

\[ E = 0.67 \text{ N/C} \]
d) Use $T_s^2 = \frac{4\pi R_e^3}{GM}$ and form a ratio:

$$T_s^2 = \frac{4\pi R_e^3}{GM_e} \quad ; \quad T_s^2 = \frac{4\pi R_s^3}{GM_s}$$

$$\frac{T_s^2}{T_e^2} = \frac{\frac{4\pi R_e^3}{GM_e}}{\frac{4\pi R_s^3}{GM_s}} = \frac{R_s^3}{R_e^3}$$

$$R_s = \left[ \frac{(R_e^3)(T_s^2)}{T_e^2} \right]^{\frac{1}{3}} = \left[ \frac{(1\text{ A.u.})^3(11.9\text{ yr})^2}{(1\text{ yr})^2} \right]^{\frac{1}{3}}$$

$$R_s = 5.2 \text{ A.u.}$$

d) $F_g = G \frac{M_e m}{R_e^2}$

If $M_e \rightarrow 2M_e$ and $R_e \rightarrow 2R_e$,

$$F_g' = G \frac{(2M_e)m}{(2R_e)^2} = \frac{2}{4} G \frac{M_e m}{R_e^2} = \frac{1}{2} F_g$$

The fraction is $\frac{1}{2}$
Two charges $q_1$ and $q_2$ when combined give a total charge of $6 \mu C$. When they are separated by $3 \text{ m}$, the force exerted by one charge on the other has magnitude $8 \times 10^{-3} \text{ N}$. Find $q_1$ and $q_2$ if (a) both are positive so that they repel each other; (b) one is positive and the other negative so that they attract each other.

\[ q_1 + q_2 = Q = 6 \times 10^{-6} \mu C \]

\[ F = k \frac{q_1 q_2}{r^2} = 8 \times 10^{-3} \text{ N} \quad r = 3 \text{ m} \]

\[ q_1 q_2 = \frac{r^2 F}{k} = q_1 (Q - q_1) \Rightarrow q_1^2 - q_1 Q + \frac{r^2 F}{k} = 0 \]

\[ q_1 = \frac{6 \times 10^{-6} \pm \sqrt{36 \times 10^{-12} - 4 \times 6 \times 10^{-12}}}{2} = \frac{6 \times 10^{-6} + 2 \times 10^{-6}}{2} \]

\[ q_1 = 4 \mu C, \quad q_2 = 2 \mu C \]

\[ q_1 - q_2 = Q, \quad -k q_1 q_2 = F \Rightarrow q_1 q_2 = \frac{r^2 F}{k} = q_2 (Q + q_2) = q_1^2 + q_2^2 \]

\[ q_1 + q_2 + \frac{r^2 F}{k} = 0 \Rightarrow q_1^2 + 6 \times 10^{-6} q_2 - 8 \times 10^{-12} = 0 \]

\[ q_1 = -6 \times 10^{-6} \pm \sqrt{36 \times 10^{-12} + 4 \times (6 \times 10^{-12})} = -6 \times 10^{-6} \pm \sqrt{36 \times 10^{-12}} = -3 \pm \sqrt{3} \mu C \]

$q_2$ assumed $> 0$ in calculation. $q_1 = -3 \sqrt{3} \mu C, \quad q_2 = \sqrt{3} \mu C$.

\[ q_1 - (\sqrt{3} - 3) \mu C = -1.12 \mu C \]

\[ q_2 + (\sqrt{3} + 3) \mu C = 7.12 \mu C \]
4. Given the square charge distribution pictured.

(a) Find the force on the charge at A (magnitude and direction).

(b) Find the electrostatic potential energy of this charge arrangement.
\[ |\overrightarrow{F}| = \sqrt{F_x^2 + F_y^2} = \frac{3q^2}{4\pi \varepsilon_0 a^2} \left( \frac{3}{2} \right) = \frac{9q^2}{8\pi \varepsilon_0 a^2} \]

\[ \theta = \tan^{-1} \left( \frac{F_y}{F_x} \right) = \tan^{-1} \left( \frac{1-2\sqrt{2}}{1-2\sqrt{2}} \right) \]

\[ \theta = 64.5^\circ \]

\[ \alpha = 180^\circ - 64.5^\circ \]

\[ \alpha = 115.5^\circ \]

\[ \text{Point breakdown} \]

a) Correct form of Coulomb's law: 3/2
   Indicating \( \overrightarrow{F} = \overrightarrow{F_x} + \overrightarrow{F} + \overrightarrow{F} \): 3/2
   Proper vector addition of forces: 2
   Correct form of \( F_x \) and \( F_y \) (cos 45° term): 2
   Correct value of \( F_x \) and \( F_y \): 2
   Sketch of direction and value of angle: 2
   Correct value of \( |\overrightarrow{F}| \): 1

b) Not graded - all exams given: 10
PROBLEM 4A

Three charges of equal magnitude are arranged at the corners of an equilateral triangle, of side $a$, as shown.

(a) Find the electric field at point $A$, the midpoint of the top side. Use the coordinate system shown.

(b) Find the force on a charge $-q$ placed at $A$. 

\[
\begin{align*}
\vec{E} &= \frac{k \frac{q}{(r_{\text{top}})^2}}{3a^2} \vec{a} \quad \text{for } a^2 = \left(\frac{a}{2}\right)^2 + r^2 \\
&= \frac{kq}{3a^2} \vec{a} \quad \text{for } h = \frac{a}{4\sqrt{3}} \\
&= \frac{kq}{3\varepsilon_0 a^2} \vec{a} \\
\end{align*}
\]

\[
\begin{align*}
\vec{F} &= -q\vec{E} = -\frac{kq^2}{3a^2} \vec{a} \\
&= -\frac{kh^2}{3\varepsilon_0 a^2} \vec{a}
\end{align*}
\]
Given the charge arrangement shown:
(a) Find the electric field at point P. Express this using the unit vectors \( \hat{\mathbf{u}} \) and \( \hat{\mathbf{j}} \).
(b) Find the magnitude of the force on a charge of +3q placed at P.

\[
\mathbf{E} = E_x \hat{\mathbf{u}} + E_y \hat{\mathbf{j}}
\]

\[
E_x = \frac{q}{4\pi \varepsilon_0 a^2} - \frac{2q}{4\pi \varepsilon_0 (a^2 + \frac{1}{2} a^2 + \frac{3}{5} a^2)}
\]

\[
= \frac{q}{4\pi \varepsilon_0 a^2} \left[ 1 - \frac{1}{2} + \frac{2}{5\sqrt{5}} \right]
\]

\[
E_y = \frac{q}{4\pi \varepsilon_0 a^2} \left[ \frac{1}{2} + \frac{2}{5\sqrt{5}} \right] = (6.7 \times 10^{-9}) \frac{\mathbf{N}}{\mathbf{C}}
\]

\[
\mathbf{F} = q \times \mathbf{E} = 3q \left( \frac{q}{4\pi \varepsilon_0 a^2} \left[ \frac{1}{2} + \frac{2}{5\sqrt{5}} \right] \right)
\]

\[
|\mathbf{F}| = \sqrt{F_x^2 + F_y^2} = \frac{3q^2}{4\pi \varepsilon_0 a^2} \sqrt{\left( \frac{1}{2} + \frac{2}{5\sqrt{5}} \right)^2 + \left( \frac{1}{\sqrt{5}} \right)^2}
\]

\[
= \frac{3q^2}{4\pi \varepsilon_0 a^2} \left( 1.92 \times 10^{-9} \mathbf{N} \right)
\]
1. An electron is placed at each corner of an equilateral triangle having sides of length 20 cm.

(a) What is the electric field at the midpoint of one of the sides?

(b) What force would another electron placed there experience?

\[ E = E_1 = \frac{(1.6 \times 10^{-19}) (9 \times 10^9) m^2}{(0.1)(55) m^2} = 4.8 \times 10^{-8} \text{ V/m} \]

6. \[ F = -eE = -(1.6 \times 10^{-19}) (4.8 \times 10^{-8} \text{ V/m}) \]
\[ = -7.7 \times 10^{-27} \text{ N, } \uparrow \text{ in direction away from the triangle, down using the diagrams above.} \]
A thin wire carries a uniform charge density $\lambda$, and is bent into a circular arc that subtends an angle $2\theta_0$ as shown in the figure.

(a) Find the electric field at the center of the arc.

Due to symmetry, the electric field vector will be parallel (or antiparallel - depending upon the sign of $\lambda$) to the $y$ direction, defined above.

\[
dE_y = \frac{k\lambda ds}{R^2} \cos \theta
\]

\[
d\phi = \lambda ds = \lambda R \, d\theta
\]

\[
dE_y = \frac{k\lambda}{R} \cos \theta \, d\theta
\]

\[
E_y = \frac{k\lambda}{R} \int_{-\theta_0}^{\theta_0} \cos \theta \, d\theta = \frac{k\lambda}{R} \left[ \sin \theta_0 - \sin (-\theta_0) \right]
\]

\[
\sin (-\theta_0) = -\sin \theta_0
\]

\[
E_y = \frac{2k\lambda}{R} \sin \theta_0
\]
3. Given a uniformly charged, non-conducting rod of length 2a. The total charge on the rod is Q. Find the electric field (magnitude and direction) at the point P, a distance a from the left end of the rod. Use the coordinate system shown.

FIRST MIDTERM

Name (Print)  KEY
Name (Sign)  GRADE: JOHN GEHRKE  S.S. No.  AVE = 14.7
Discussion Instructor: Abbott Allen
Brumbaugh Bruno Ho Gehrke
Kaipa Rino B. Wheeler Sewell
Problem No. 3

BE SURE TO SHOW ALL WORK!
Solution to problem 3 exam I

Consider the electric field due to some small length of charged wire $dy$

$$\lambda = \frac{Q}{2a} \quad dy = \lambda dx$$

We must add vectors like vectors so consider the $x$-component

$$dE_x = \frac{d\Phi}{4\pi \varepsilon_0 R^2} \sin \theta = \frac{\lambda dx \sin \theta}{4\pi \varepsilon_0 R^2}$$

From the triangle we have $\sin \theta = \frac{x}{(x^2 + a^2)^{1/2}}$

and $R^2 = (x^2 + a^2)$

$$dE_x = \frac{\lambda dx}{4\pi \varepsilon_0} \frac{1}{(x^2 + a^2)} \frac{x}{(x^2 + a^2)^{3/2}}$$

Adding up all the $x$-components gives

$$E_x = \frac{\lambda}{4\pi \varepsilon_0} \int_0^{2a} \frac{x dx}{(x^2 + a^2)^{3/2}}$$

From the co-axis sheet the integral is

$$E_x = \frac{\lambda}{4\pi \varepsilon_0} \left[ \frac{-1}{(x^2 + a^2)^{1/2}} \right]_0^{2a} = \frac{\lambda}{4\pi \varepsilon_0} \left[ \frac{1}{a} - \frac{1}{\sqrt{5}a} \right]$$

$$E_x = \frac{Q}{8\pi a^2 \varepsilon_0} \left[ 1 - \frac{1}{\sqrt{5}} \right]^2$$

It is in the minus $x$ direction.
In a similar manner we can do the y-component.

\[ dE_y = \frac{d\theta}{4\pi\varepsilon_0 R^2} \cos \theta \quad d\theta = \lambda dy \]

Again from the triangle we have \( \cos \theta = \frac{a}{(x^2 + a^2)^{1/2}} \) and \( R^2 = (x^2 + a^2) \)

\[ dE_y = \frac{\lambda dy}{4\pi\varepsilon_0} \frac{1}{(x^2 + a^2)} \frac{a}{(x^2 + a^2)^{3/2}} \]

Integrating this

\[ E_y = \frac{\lambda a}{4\pi\varepsilon_0} \int_0^{2a} \frac{dx}{a^2 (x^2 + a^2)^{3/2}} \]

From the cover sheet

\[ = \frac{\lambda a}{4\pi\varepsilon_0} \left[ \frac{x}{a^2 (x^2 + a^2)^{1/2}} \right]_0 \]

\[ = \frac{\lambda a}{4\pi\varepsilon_0} \frac{2a}{a^2 (\sqrt{5} a)} = \frac{G}{4\pi\varepsilon_0 \sqrt{5} a^2} \]

\[ E = \frac{\Omega}{8\pi\varepsilon_0} (1 - \frac{1}{5}) \frac{1}{2} + \frac{\Omega}{4\pi\varepsilon_0 \sqrt{5} a^2} \]

\[ |E| = \frac{\Omega}{4\pi\varepsilon_0 a^2} \left[ (2 - \frac{2}{\sqrt{5}})^2 + (\frac{1}{\sqrt{5}})^2 \right] = \frac{\Omega}{4\pi\varepsilon_0 a^2} \left[ 4 - \frac{8}{\sqrt{5}} + \frac{4}{5} + \frac{1}{5} \right] \]

\[ \theta = \tan^{-1} \left( \frac{2 - \frac{2}{\sqrt{5}}}{\frac{1}{\sqrt{5}}} \right) \]

\[ \theta = 58^\circ \]

\[ \Theta = \frac{\lambda a}{4\pi\varepsilon_0} (1 + 22) \]

\[ \Theta = 158^\circ \]

25
4. A spherically symmetric charge distribution of total charge \( Q \) can be described by the relation:

\[
p(\text{charge density}) = \frac{A}{r^2}
\]

for \( R_1 < r < R_2 \).

\( p = 0 \) everywhere else.

(a) Find the value of the constant \( A \).

(b) Find the value of the electric field at any value of \( r \) between \( R_1 \) and \( R_2 \).
A cylindrically symmetric charge distribution in a nonconductor is described by the relation

\[ \rho = \frac{A}{R^2} \quad \text{for} \quad R_1 < R < R_2 \]

and is zero everywhere else. \((A)\) is a constant.\)

(a) Find the total charge on a length \(L\) of the cylinder.

(b) Find the electric field at a point \(P\), where \(P\) is between \(R_1\) and \(R_2\).

13 pts. a) \(Q = \oint \sigma \, ds = \frac{A}{R^2} \oint r \, d\theta = \frac{A}{R^2} \int_0^{2\pi} r \, d\theta = \frac{A}{R^2} \int_0^{2\pi} \frac{L}{2} r \, d\theta = \frac{AL}{2} \theta_2 - \frac{AL}{2} \theta_1 \)

12 pts. b) \( \int \vec{E} \cdot d\vec{A} = \frac{\sigma}{\varepsilon_0} \); \( q = \frac{\sigma}{\varepsilon_0} \left( \frac{L}{R_1} - \frac{L}{R_2} \right) \) from \( \sigma \)

\( E \) is normal & constant if a cylindrical Gaussian surface is chosen:

\[ E = \frac{\sigma}{\varepsilon_0} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

\[ S\theta = 2\pi R L \]

\[ E / \pi r^2 = \frac{\sigma}{\varepsilon_0} \left( \frac{1}{R_1} - \frac{1}{r} \right) \]

\[ E = \frac{\sigma}{\varepsilon_0} \left( \frac{1}{R_1} - \frac{1}{r} \right) \]
Physics 172  
Winter Quarter 1987  
January 23, 1987  
George Williams  

\[
\begin{align*}
X &= 14.2/2.5 \\
Y &= 5.44 \\
N &= 331
\end{align*}
\]

Grader: Pollard  
Name (Signed):  

Discussion Instructor (Circle One): Bertolino  
Hari  
Jaw  
Krantz  
Lakner  
McDonald  
Pollard  

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!  
Use the conversion constants and data given on the front page.

(a) Use the binomial expansion to calculate the coefficient of \(x^3\) for \((1 + x)^{-3/2}\).

\[ -26.8 \]

(b) Assume the planet Mercury orbits the Sun in 90.0 days at a distance of 30,000,000 mi. Use this data to calculate the mass of the sun in kg. It will not be the same as on the data sheet.

\[ 1.102 \times 10^{30} \text{ kg} \]

(c) Calculate the magnitude of the electric field at a distance of 1.00 × 10^{-10} m from a proton. (The charge in the proton is \(q = +1.6 \times 10^{-19} \text{ C}\).)

\[ 1.440 \times 10^{-4} \text{ N/m} \]

(d) Take the Earth-Moon distance as 240,000 mi. Calculate the gravitational force in Newtons between the Earth and the Moon.

\[ 1.965 \times 10^{20} \text{ N} \]

(e) A fine wire 12.0 m long has a total charge of \(+4.25 \times 10^{-8} \text{ C}\). Calculate the electric field and direction 1.00 m away from the center of the wire at a point halfway between the ends.

\[ 6.37 \times 10^4 \text{ N/m} \text{, \text{1 m away from wire}} \]
a) \((1 + x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \ldots\)

here \(n = -\frac{9}{2}\)

then \(a_2 = \frac{(-\frac{9}{2})(-\frac{9}{2} - 1)(-\frac{9}{2} - 2)}{2!} = \frac{9 \times 10 \times 13}{8 \times 6} = -26.81\)

Common error:

The coefficient of \(x^2\) is not the first 4 terms

Grading:
- 2 points for number and (+) wrong sign
- 5 points for number and correct sign

b) \(T = 90.0\) days = \(7.776 \times 10^6\) sec.

\(R = 38,000,000\) miles = \(4.829 \times 10^{10}\) m

\(T = \frac{G M m}{R^2} = M \left(\frac{2 \pi R}{T}\right)^2\)

\(\implies M = \frac{2 \pi^2 R^3}{G T^2} = 1.1023 \times 10^{30}\) kg

c) \(E = k \frac{G}{r^2} = 1.440 \times 10^{11}\) N/m

d) \(F = \frac{G M m}{r^2} = 1.9646 \times 10^{26}\) N

e) \(\begin{align*}
\downarrow^{1/2} & \quad \downarrow^{1/2} \\
\leftarrow r & \quad \leftarrow r \\
\uparrow & \quad \uparrow \\
29 & \quad 29
\end{align*}\)

\(L = 12.0\) m

\(r = 10^{-2}\) m

40
Note that $\sigma L$, so "intuitively" you can see Gauss's law is ok. In that case

\[ E_{2 \pi r l} = \frac{Q}{\varepsilon_0} \]

\[ E = \frac{Q}{2 \pi \varepsilon_0 r L} = 6.36 \times 10^{-4} \text{ N/C} \]

How good is this? Do it exactly.

By symmetry, we can see that the components parallel to the wire cancel.

Then

\[ E = \int dE \perp = \int dE \cos \theta = \int \frac{k d\theta}{x^2 + r^2} \]

Let $d\theta = 2a dx$, $\lambda = Q/L$

Then,

\[ E = k \lambda \int L^2 \frac{d x}{(x^2 + r^2)^{3/2}} \]

\[ = \frac{k \lambda}{r} \left[ \frac{1}{(x^2 + r^2)^{1/2}} \right] \]

\[ = \frac{Q}{2 \pi \varepsilon_0 r L} \left[ 1 + \left( \frac{2r}{x} \right)^2 \right]^{-1/2} \]

\[ \approx \frac{Q}{2 \pi \varepsilon_0 r L} \] Gauss's law result

Relative error \( \approx \frac{1}{2} \left( \frac{2r}{x} \right)^2 = \frac{1}{2} \left( \frac{2 \times 10^{-3}}{12} \right) = 1.39 \times 10^{-8} \)
Second Midterm

Name (Print) Hyung Tong Kim    Name (Signed)   Average: 17/25

Discussion Instructor (Circle One): Bertolina Hari Jaw Krantz
Discussion Section #: Lakner McDonald Pollard

SHOW ALL WORK!!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

In the diagram shown, \(|q| = 3.00 \times 10^{-12} \text{ C}\). The point \(P\) is directly below the negative charge. Take \(a = 2.00 \times 10^{-4} \text{ m}\).

(a) Calculate the electric field, magnitude and direction, at point \(P\).
(b) Calculate the magnitude of the force on a charge \(Q = 4.00 \times 10^{-7} \text{ C}\), placed at \(P\).

\[
\begin{align*}
\text{(a) } |E_1| &= \frac{k(3 \times 10^{-10})}{2a^2} \text{ N/C} \quad (2) \\
|E_2| &= \frac{k(3 \times 10^{-10})}{a^2} \text{ N/C} \quad (2) \\
|E_3| &= \frac{k(3 \times 10^{-10})}{a^2} \text{ N/C} \quad (2)
\end{align*}
\]

i) \(x\)-component: \(E_x = E_1 \cos 45^\circ - E_3 \cos 45^\circ = 0 \quad (3)\)
ii) \(y\)-component: \(E_y = E_3 - (E_1 + E_2) \cos 45^\circ \quad (3)\)
\[
= \frac{k(3 \times 10^{-10})}{(2 \times 10^{-4})^2} (1 - \cos 45^\circ)
\]
\[
= 1.98 \times 10^5 \text{ N/C} \quad (4)
\]
Therefore, the magnitude at \(P\) \(\rightarrow E_p = 1.98 \times 10^5 \text{ N/C}\)
the direction at \(P\) \(\rightarrow\) toward (-) charge \(\square\)

\[
(b) \quad F_p = QE_p = (4 \times 10^{-7}) \times (1.98 \times 10^5) = 7.91 \times 10^{-2} \text{ N}.
\]

Total: 26
A thick, hollow sphere is constructed with inner radius $R_1$ and outer radius $R_2$. Between $R_1$ and $R_2$ there is a charge density given by $\rho = A/R$, where $A$ is constant. Everywhere else there is no charge.

(a) Calculate the electric field at point $P$, where $R_P > R_2$.
(b) Calculate the electric field at point $Q$, where $R_1 < R_Q < R_2$.

View Points to solve this problem

By using Gauss's law you can solve this problem, which gives you how to consider the charge inside the sphere given by each point.

$R > R_2$

By Gauss's law

$$\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{\Phi}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int_V \rho dV$$

$$4\pi R^2 E(R) = \frac{QA}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int_0^R 4\pi r^2 dr$$

$$= \frac{4\pi A}{\varepsilon_0} \int_0^{R_2} r dr = \frac{2\pi A}{\varepsilon_0} \left( R_2^2 - R_1^2 \right)$$

$$E(R) = \frac{A}{2\varepsilon_0} \frac{R_2^2 - R_1^2}{R^2}$$

(b) Similarly, $R < R < R_2$

$$4\pi R^2 E(R) = \frac{Q(R)}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int_0^R 4\pi r^2 dr$$

$$= \frac{4\pi A}{\varepsilon_0} \int_0^{R_2} r dr = \frac{2\pi A}{\varepsilon_0} \left( R_2^2 - R_1^2 \right)$$

$$E(R) = \frac{A}{2\varepsilon_0} \left( 1 - \frac{R_1^2}{R^2} \right)$$
Three charges are at the corners of an equilateral triangle as shown.

(a) Calculate the electric field (magnitude and direction) at point P.
(b) Calculate the force on a charge $-Q$ (magnitude and direction) placed at P.

\[
\begin{align*}
E_1^x &= -\frac{kQ}{a^2}\cos 30^\circ = \frac{\sqrt{3}}{2} \frac{kQ}{a^2} \\
E_1^y &= -\frac{kQ}{a^2}\sin 30^\circ = -\frac{1}{2} \frac{kQ}{a^2} \\
E_2^x &= -\frac{kQ}{a^2}\cos 30^\circ = -\frac{\sqrt{3}}{2} \frac{kQ}{a^2} \\
E_2^y &= -\frac{kQ}{a^2}\sin 30^\circ = -\frac{1}{2} \frac{kQ}{a^2} \\
E_3 &= \frac{k(-2Q)}{(\sqrt{3}a)^2} = \frac{2}{3} \frac{kQ}{a^2} \\
E_3^y &= 0
\end{align*}
\]

\[
\begin{align*}
E_x &= E_1^x + E_2^x + E_3^x = \frac{kQ}{a^2} \left( \frac{\sqrt{3}}{3} - \frac{1}{3} \right) = 1.065 \frac{kQ}{a^2} \\
E_y &= E_1^y + E_2^y + E_3^y = 0
\end{align*}
\]

magnitude: \( E = \sqrt{E_x^2 + E_y^2} = 1.065 \frac{kQ}{a^2} \)

direction: Along negative x-axis, (\( \theta = 180^\circ \)).

(b)
\[
\begin{align*}
F_x &= \pm \frac{kQ^2}{a^2} \left( \sqrt{3} - \frac{2}{3} \right) = 4.26 \frac{kQ^2}{a^2} \\
F_y &= 0
\end{align*}
\]

magnitude: \( F = \sqrt{F_x^2 + F_y^2} = 4.26 \frac{kQ^2}{a^2} \)

direction: Along positive x-axis, (\( \theta = 0^\circ \)).
Consider a sphere of non-conductor which has a charge density given by 

\[ \rho = \rho_0 \left(1 - \frac{9R^2}{R_0^2}\right) \]

where \(\rho_0\) and \(\alpha\) are constants. The sphere has a radius \(R_0\).

(a) Calculate the magnitude of the electric field an arbitrary distance \(R\) from the center of the sphere where \(R < R_0\). (Leave this answer in symbolic form in terms of \(\rho_0\), \(\alpha\), \(R_0\), \(R\), \(k\) as needed.) [15 pts.]

(b) If the sphere has a radius of 1.25 cm, a total charge of \(1.75 \times 10^{-6}\) C, and it is known that the charge density goes to zero at the surface, calculate the numerical value of \(\alpha\). [5 pts.]

(c) With the assumptions the same as in (b), calculate the numerical value for \(\rho_0\). [5 pts.]

For Gauss's law, \(E\) in a sphere of radius \(R\):

\[ E \propto \frac{Q}{4\pi R^2} \]

Now

\[ \Phi = \int \rho dV = \int \left[\rho(1 - \frac{9R^2}{R_0^2})\right] dV \]

\[ = 4\pi \rho_0 \int \left(\frac{R^3}{3} - \frac{9R^5}{5R_0^2}\right) dR \]

\[ = 4\pi \rho_0 \left[ \frac{R^4}{3} - \frac{9R^6}{5R_0^2} \right] \]

\[ \therefore E = \frac{\rho_0}{3} \left(\frac{R^4}{R_0^4}\right) \]

At \( R = R_0 \), \( \Phi = 0 \)

\[ \rho(R_0) = 0 = \rho_0 \left(1 - \alpha R_0^2\right) \]

or \( \alpha = \frac{1}{2} \)

One can write \(\Phi\) directly from eqn. (2) with \( R = R_0 \)

\[ \rho_0 = \frac{\Phi}{4\pi R_0^3} = \frac{\Phi}{4\pi} \alpha = \frac{0.42 \frac{C}{m^3}}{\rho_0} \]

\[ = \frac{5.12 \times 10^{-6}}{m^3} \]
FIRST EXAM

Name (print) ___________________________ Name (signed) ___________________________

Discussion Instructor (circle one): Emerson Gaughan Iguchi Stoops Zhang

Discussion Section #: ____________________

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) Calculate the magnitude of the electric force between two electrons that are 0.72 \times 10^{-10} \text{ m} apart.

\[ F = k \frac{e^2}{r^2} = 9 \times 10^9 \times \frac{(-1.6 \times 10^{-19})^2}{(0.72 \times 10^{-10})^2} = 4.4 \times 10^{-8} \text{ N} \]

(b) What is the magnitude of the acceleration (in m/s^2) of an electron in an electric field of magnitude 275 N/C?

\[ a = \frac{F}{m} = \frac{1.6 \times 10^{-19} \times 2.75}{9.11 \times 10^{-31}} = 4.83 \times 10^{13} \text{ m/s}^2 \]

(c) Calculate the magnitude of the electric field at a point midway between two charges, one of +5.5 \times 10^{-6} \text{ C} and the other of -7.5 \times 10^{-6} \text{ C}. The distance d is 1.75 m.

\[ E = \frac{k \frac{q_1}{(d_1)^2} + k \frac{q_2}{(d_2)^2}}{d} = \frac{4 \times 9 \times 10^9 \times 10^{-6}}{(1.75)^2} \times 13.3 \times 10^{-6} = 1.5 \times 10^5 \text{ N/C} \]

(d) If the electric field at the surface of the earth is found to be 100 N/C, and pointed downward, and at 1000 m above the surface of the earth it is 25 N/C also pointed downward, calculate the number of elementary charges (e = 1.6 \times 10^{-19} \text{ C}) in a cube 1000 m on a side, with its bottom at the earth's surface.

\[ \Phi = 1000^2 \times (100 - 25), \quad Q = \varepsilon_0 \Phi \quad \text{Number of charges} = \frac{Q}{e} = \frac{8.85 \times 10^{-12} \times 105^2 \times (100 - 25)}{1.6 \times 10^{-19}} = 4.15 \times 10^{17} \]

(e) In the binomial expansion of \((x^2 + a^2)^{-3/2}\), calculate completely the term in \(a^4\).

\[ (x^2 + a^2)^{-3/2} = x^{-3}(1 + \frac{a^2}{x^2})^{-3/2} = x^{-3} \left(1 - \frac{3}{2} \frac{a^2}{x^2} + \frac{\frac{3}{2} \cdot \frac{3}{2} \cdot \frac{1}{2}}{(1)} \frac{a^4}{x^4} + \cdots \right) \]

\[ a^4 \text{ term:} \quad x^{-3} \cdot \frac{\frac{3}{2} \cdot \frac{3}{2} \cdot \frac{1}{2}}{(1)} \frac{a^4}{x^4} = \frac{3}{8} \frac{a^4}{x^4} \]
A long cylinder of non-conductor of radius $R_c = 1.75 \text{ cm}$, has a charge density given by

$$\rho(r) = Br^3 \quad \text{for} \quad 0 < r < R_c$$

The charge density is zero everywhere else.

(a) If the total charge per unit length is $-3.75 \times 10^{-7} \text{ C/m}$, calculate the numerical value of $B$.

(b) Calculate the magnitude of the electric field at a point $r = 0.95 \text{ cm}$. State the direction of the field.

\[\begin{align*}
\lambda &= \frac{Q}{l} = -3.75 \times 10^{-7} \text{ C/m} \\
Q &= \int g(r) \, dV \\
(-3.75 \times 10^{-7} \text{ C/m}) \cdot l &= \int_0^{R_c} Br^3 \cdot 2\pi r \, dr \\
(-3.75 \times 10^{-7} \text{ C/m}) &= 2\pi B \int_0^{R_c} r^4 \, dr \\
(-3.75 \times 10^{-7} \text{ C/m}) &= 2\pi B \left[ \frac{r^5}{5} \right]_0^{0.0175 \text{ m}} \\
(-3.75 \times 10^{-7} \text{ C/m}) &= 2\pi B \left( \frac{0.0175^5}{5} \right) \\
B &= -182 \text{ C/m}^2
\end{align*}\]
\( \oint E \cdot dA = \frac{Qm}{\varepsilon_0} \)

\[ |E| \cdot 2\pi r l = \frac{2\pi l |E| B |r^5}{5 \varepsilon_0} \]

\[ |E| = \frac{|B| r^4}{5 \varepsilon_0} \]

\[ |E| = \frac{182 \, \text{C/m}^2 \, (0.0095 \text{m})^4}{5 \cdot 8.85 \times 10^{-12} \, \text{C}^2/\text{N.m}^2} \]

\[ |E| = 3.35 \times 10^4 \, \text{N/C} \]

Direction radially inward
We have calculated that the field of an electric dipole for points on the axis of the dipole is given by

\[ E = \frac{1}{4 \pi \varepsilon_0} \frac{2a}{x^3} \]

where \( 2a \) is the distance between the two charges \( q \), and here the dipole is aligned along the \( x \) axis. An electric quadrupole can be treated as two dipoles, head to head, or tail to tail, as shown. Using the dimensions given, and the binomial expansion, calculate the first non-zero term for the electric field of the quadrupole shown for a point at \( \pm x \), on the \( x \) axis, where \( x \gg a \).

**NOTE!**
This equation assumes the separation is \( a \), NOT \( 2a \) as stated. No points were taken off for not noticing.

**First**

\[ |E_1| = \frac{1}{4 \pi \varepsilon_0} \frac{2q(2a)}{(x-a)^3} \quad \text{good for } x > a \]

\[ |E_1| = \frac{4g_0}{4 \pi \varepsilon_0} (x-a)^{-3} = \frac{9a}{\pi \varepsilon_0} (1 - \frac{a}{x})^{-3} x^{-3} \]

**Second**

\[ |E_2| = \frac{1}{4 \pi \varepsilon_0} \frac{2q(2a)}{(x+a)^3} \quad \text{good for } x > -a \]

\[ |E_2| = \frac{4g_0}{4 \pi \varepsilon_0} (x+a)^{-3} = \frac{9a}{\pi \varepsilon_0} (1 + \frac{a}{x})^{-3} x^{-3} \]

**Binomial Expansion on Both**

\[ |E_1| = \frac{9a}{\pi \varepsilon_0} x^{-3} \left[ 1 + 3 \left( \frac{a}{x} \right) + \frac{3(3-1)}{2!} \left( \frac{a}{x} \right)^2 + \ldots \right] \]

\[ |E_2| = \frac{9a}{\pi \varepsilon_0} x^{-3} \left[ 1 - 3 \left( \frac{a}{x} \right) + \frac{3(3-1)}{2!} \left( \frac{a}{x} \right)^2 + \ldots \right] \]
Given a line of charge with charge density \( \lambda = +3.75 \, \mu C/m \) (3.75 \( \times \) \( 10^{-6} \) C/m), and length \( l = 1.27 \, m \). Calculate the electric field (magnitude and direction) at point \( P \) located at distance \( a \) from the line, and distance \( l/4 \) from one end of the line. Take \( a = 0.180 \, m \), and calculate a numerical value complete with units.

\[ E = k \int_{r=0}^{r=a} \frac{dq}{r^2} \quad \text{and} \quad dq = d\lambda \, dx \quad \text{also} \quad r^2 = a^2 + x^2 \quad \text{therefore} \quad r = \cos \theta + \sin \theta \]

Note: \( \cos \theta = -\frac{a}{r} \) because \( \theta \) is measured from \( +x \) not \(-x\) and \( \sin \theta = \frac{r}{a} \).

\[ \vec{E}_x \text{ is:} \quad \vec{E}_x = k \int_{a}^{0} \frac{\cos \theta \, dx}{x^2} \quad \text{substituting in} \quad \cos \theta = -\frac{a}{r} \quad \text{and} \quad r^2 = a^2 + x^2 \]

\[ \frac{1}{E_x} = -k \ln \left[ \frac{\frac{1}{r} \left( \frac{1}{(a/2)^2} \right) + \frac{1}{r + (l/4)^2}}{\frac{1}{(a/2)^2}} \right] \quad \text{From the integral table (eq. 123),} \quad \frac{1}{E_x} = -5.47 \times 10^5 \frac{N}{C} \]

\[ \vec{E}_y \text{ is:} \quad \vec{E}_y = k \int_{a}^{0} \frac{\sin \theta \, dx}{x^2} \quad \Rightarrow \quad \vec{E}_y = k \lambda \int_{a}^{0} \frac{dx}{(a^2 + x^2)^{3/2}} \quad \text{From the table (eq. 122),} \quad \vec{E}_y = 3.54 \times 10^5 \frac{N}{C} \]

\[ \frac{1}{E} = \sqrt{\frac{1}{E_x^2} + \frac{1}{E_y^2}} \quad \Rightarrow \quad \frac{1}{E} = 0.80 \quad \Rightarrow \quad \theta = \tan^{-1} \left( \frac{E_y}{E_x} \right) \quad \Rightarrow \quad \theta = 80.6^\circ \text{ from } -x \text{ axis} \]

\[ \frac{E^2}{\mu} = 3.52 \times 10^5 \frac{N}{C} \]
Problem 4

\[ \cos \alpha = \frac{L-x}{r} = \frac{L-x}{\sqrt{(L-x)^2 + L^2}} \]
\[ \sin \alpha = \frac{L}{r} = \frac{L}{\sqrt{(L-x)^2 + L^2}} \]

\[ \vec{E}_{x_2} = \int \vec{d}E_{x_2} = k \int \frac{\lambda \, dx}{r^2} \cos \alpha = k \int_0^L \frac{\lambda(L-x)}{(L-x)^2 + L^2} \frac{dx}{\sqrt{(L-x)^2 + L^2}} \]

\[ \vec{E}_{y_2} = \int \vec{d}E_{y_2} = k \int \frac{\lambda \, dx}{r^2} \sin \alpha = k \int_0^L \frac{\lambda L \, dx}{(L-x)^2 + L^2} \frac{dx}{\sqrt{(L-x)^2 + L^2}} \]

\[ \vec{E} = \vec{E}_{x_1} + \vec{E}_{x_2} + \vec{E}_{y_1} + \vec{E}_{y_2} = k \lambda \int_0^L \frac{(L + L-x)}{(L-x)^2 + L^2} \frac{dx}{\sqrt{(L-x)^2 + L^2}} \]

\[ E = |\vec{E}_{x_1}|^2 + |\vec{E}_{y_2}|^2 = k \lambda \int_0^L \frac{(L + L-x)}{(L-x)^2 + L^2} \frac{dx}{\sqrt{(L-x)^2 + L^2}} \]

Direction

\[ \tan \beta = \frac{\vec{E}_{y_2}}{\vec{E}_{x_1}} = \tan 45^\circ \Rightarrow \beta = 45^\circ \]
Since \( E_1 \) is in \(-x\) and \( E_2 \) is in \(+x\)

\[
|E_T| = |E_2| - |E_1|
\]

\[
E_T = \frac{q_0}{\pi \varepsilon_0} x^{-3} \left[ -\frac{2}{3} x (a/x) + \ldots \right]
\]

Taking first non-zero term

\[
E_T = \frac{q_0}{\pi \varepsilon_0} x^{-3} \left[ \frac{-6a}{x} \right]
\]

\[
E_T = -\frac{6 q_0 a^2}{\pi \varepsilon_0} x^{-4}
\]

Minus sign indicates
\( \vec{E}_T \) points in \(-x\) direction.
FIRST MIDTERM

Name (print) _____________________________ Name (signed) _____________________________

Discussion Instructor (circle one): Baseligia Morrill Reeve Stoops Zhang

Discussion Section # ________________

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) If a satellite circles the Earth in a circular orbit 500 km above the surface of the Earth in exactly 90 min, what is the mass of the Earth (not quite the same as on the data sheet)?

\[
M = \frac{4\pi^2 r^3}{T^2} \quad r = R_E + h = 6.88 \times 10^3 \text{ km} \quad T = 90 \times 60 = 5400 \text{ sec}
\]

\[6.61 \times 10^{24} \text{ kg}\]

(b) Calculate the total energy of the Earth in its orbit around the Sun. Assume the radius of the orbit is 1.48 \times 10^8 \text{ km}.

\[T_{\text{total}} = T_k + T_r = -\frac{1}{2} G \frac{Mm}{r^2} = -\frac{1}{2} \times 6.67 \times 10^{-11} \times \frac{1.99 \times 10^{30} \times 5.98 \times 10^{24}}{1.48 \times 10^8^2} \]

\[-2.68 \times 10^{33} \text{ J}\]

(c) Calculate the magnitude of the electric force between two electrons that are 0.2 \times 10^{-10} \text{ m} apart.

\[F = k \frac{e^2}{r^2} = 9.00 \times 10^9 \times \frac{(-1.6 \times 10^{-19})^2}{(0.2 \times 10^{-10})^2} \]

\[5.76 \times 10^{-7} \text{ N}\]

(d) Using the binomial expansion on \((1 + x^3)^{-4/3}\) where \(x \ll 1\), calculate the coefficient of the term in \(x^9\).

\[-1.73\]

\[(1 + x^3)^{-4/3} = 1 + (-\frac{4}{3}) x^3 + \frac{1}{2!} \left(-\frac{4}{3}\right) \left(-\frac{4}{3} - 1\right) x^6 + \frac{1}{3!} \left(-\frac{4}{3}\right) \left(-\frac{4}{3} - 1\right) \left(-\frac{4}{3} - 2\right) x^9 + \cdots\]

(e) If a cube has a total charge inside of +3.20 nC, calculate the total electric flux, with units, crossing the faces of the cube.

\[3.62 \times 10^2 \text{ N m}^2 / \text{C}\]

\[\Phi = \frac{Q}{\varepsilon_0} = \frac{3.20 \times 10^{-9}}{8.85 \times 10^{-12}} = 3.62 \times 10^2\]
FIRST MIDTERM

Name (print)  Ludi Baselgia  Name (signed)  Solution

Discussion Instructor (circle one): Baselgia  Morrill  Reeve  Stoops  Zhang

Discussion Section #

SHOW ALL WORK!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constant and data given on the front page.

Three charges are placed at the vertices of an equilateral triangle as shown.

(a) Calculate the force, magnitude and direction on a charge \( q_0 \) placed at \( P \) which is at the midpoint of the base.

(b) Calculate the electric field at \( P \) due to the three charges at the corners of the triangle.

\[ Q = 9.0 \, \mu C = 9.0 \times 10^{-6} \text{C} \]
\[ a = 1.25 \, \text{cm} = 0.0125 \, \text{m} \]
\[ q_0 = -6.25 \, \mu C = -6.25 \times 10^{-6} \text{C} \]

1. Use Coulomb's law: \( F = \frac{kq_1q_2}{r^2} \)
   Calculate the magnitude of forces on \( q_0 \):

   \[ F_1 = \frac{k \cdot 2Q \cdot 19}{(a/2)^2} = 9.0 \times 10^9 \frac{\text{N} \cdot \text{C}^2}{\text{m}^2} \cdot \frac{(2 \cdot 3.0 \times 10^{-6})(6.25 \times 10^{-6})}{(0.0125/2)^2} = 2.592 \times 10^4 \text{ N} \]

   \[ F_2 = \frac{k \cdot 3Q \cdot 19}{a^2} = 9.0 \times 10^9 \frac{\text{N} \cdot \text{C}^2}{\text{m}^2} \cdot \frac{15 \times 3.0 \times 10^{-6}(6.25 \times 10^{-6})}{(0.0125)^2} = 3.888 \times 10^4 \text{ N} \]

   \[ F_3 = \frac{k \cdot 4Q \cdot 19}{3a^2/2} = 9.0 \times 10^9 \frac{\text{N} \cdot \text{C}^2}{\text{m}^2} \cdot \frac{(4.5 \times 10^{-6})(6.25 \times 10^{-6})}{(0.0125)^2} \frac{2}{3} = 1.728 \times 10^4 \text{ N} \]

   Total force on \( q_0 \):

   \[ \vec{F}_t = (F_1 + F_2 + F_3) \cdot \hat{F} = (2.592 \times 10^4 + 3.888 \times 10^4 + 1.728 \times 10^4) \cdot \hat{F} \]

   \[ = -6.408 \times 10^4 \text{ N} \cdot \hat{F} + 1.728 \times 10^4 \text{ N} \cdot \hat{F} \]

   Magnitude and direction of \( \vec{F}_t \):

   \[ |\vec{F}_t| = \sqrt{F_x^2 + F_y^2} = \sqrt{(6.408 \times 10^4)^2 + (1.728 \times 10^4)^2} \]

   \[ \alpha = \tan^{-1} \left( \frac{F_y}{F_x} \right) = \tan^{-1} \left( \frac{1.728 \times 10^4}{6.408 \times 10^4} \right) = 14.9^\circ \Rightarrow \text{Direction at } 165^\circ \text{ from } +x \text{-axis} \]

2. \[ E = \frac{F}{q_0} \]

   \[ E = \frac{F_1}{q_0} = \frac{6.71 \times 10^4 \text{ N/C}}{1.073 \times 10^2 \text{ C}} = 6.29 \times 10^2 \text{ N/C} \]

   \[ \text{and direction of } E \text{ is opposite to } \vec{F} \]

3. \[ E = \frac{F_2}{q_0} = \frac{3.888 \times 10^4 \text{ N/C}}{1.073 \times 10^2 \text{ C}} = 3.62 \times 10^2 \text{ N/C} \]

   \[ \text{and direction of } E \text{ is opposite to } \vec{F} \]
Two rods of nonconductor are placed at right angles. The rods each have a total charge of \(3.25 \times 10^{-6}\) C that is uniformly distributed and their length is \(L = 1.25\) m. Calculate the electric field, magnitude and direction, at point \(P\). \(P\), together with the ends of the rods, form a square of side \(L\).

Choose coordinate system as shown below.

\[
\lambda = \frac{3.25 \times 10^{-6}}{1.25} = 2.6 \times 10^{-6}\ \text{N/m}
\]

\[
\cos \theta = \frac{L}{r} = \frac{L}{\sqrt{(L-y)^2 + L^2}}
\]

\[
\sin \theta = \frac{L-y}{r} = \frac{L-y}{\sqrt{(L-y)^2 + L^2}}
\]

\[
\vec{E}_x = \int dE_x = k \int \frac{\lambda dy}{r^2} \cos \theta = k \int_0^L \frac{\lambda dy}{[(L-y)^2 + L^2]^{3/2}}^2 + 4
\]

\[
\vec{E}_y = \int dE_y = k \int \frac{\lambda dy}{r^2} \sin \theta = k \int_0^L \frac{\lambda (L-y) dy}{[(L-y)^2 + L^2]^{3/2}}^2 + 4
\]
 Physics 302  
 Winter Quarter 1991  
 January 25, 1991  
 George Williams

FIRST MIDTERM

Name (print): ____________________________  Name (signed): ________  MA: ________

Discussion Instructor (circle only): Davis  DeTienne  Hamed  Molina  Paul  Zhang

Discussion Section # ________

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) If g on the surface of Mars is 3.70 m/s² and the mass of Mars is \(6.00 \times 10^{23}\) kg, calculate the radius of Mars.

\[
g = \frac{GM}{R^2} \Rightarrow R = \left(\frac{GM}{g}\right)^{1/2} = \left(\frac{6.00 \times 10^{23}}{3.70}\right)^{1/2} = 3.29 \times 10^6 \text{(m)}
\]

(b) Calculate the magnitude of the electric force between an electron and a proton (the hydrogen nucleus) if they are \(0.75 \times 10^{-10}\) m apart. The charge of the proton is positive and is equal in magnitude to the charge of the electron. (Numerical answer.)

\[
F = \frac{kq^2}{r^2} = 4.11 \times 10^{-8} \text{(N)}
\]

(c) If the Moon has an average density of \(3340 \text{ kg/m}^3\) and a radius of 1738 km, what is the mass of the Moon?

\[
M = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \left(\frac{1738 \times 10^3}{2}ight)^2 = 7.34 \times 10^{22} \text{(kg)}
\]

(d) Calculate the electric field 2.00 m from the surface of a uniformly charged non-conducting sphere, whose radius is 3.00 m and whose total charge is \(5.75 \times 10^3\) C. (Numerical answer.)

\[
E = \frac{kQ}{r^2} = \frac{(9 \times 10^9 \times 5.75 \times 10^3)}{5^2} = 2.07 \times 10^6 \text{(N/C)}
\]

(e) Use the binomial expansion to calculate the complete term in \(a^4\) for

\[
\left(1 - \frac{a^2}{x^2}\right)^{3/2}
\]

\[
\frac{35}{8} x^4
\]
Calculate the electric field, magnitude and direction at point P. The direction should be given as an angle measured from the positive x-axis. (Be sure to include the proper sign for the angle.)

$Q_1 = +125 \ \mu C$
$Q_2 = -300 \ \mu C$
$Q_3 = +175 \ \mu C$
a = 4.25 \ mm

\[ E_x = E_{1x} + E_{2x} + E_{3x} \]
\[ = k \left( \frac{Q_1}{5a^2} + \frac{2Q_2}{3a^2} \right) + \frac{kQ_3}{2a^2} \left( \frac{1}{\sqrt{3}} \right) = 1.11 \times 10^{10} \ \text{N/C} - 5.28 \times 10^{10} \ \text{N/C} \]
\[ = -4.17 \times 10^{10} \ \text{N/C} \]

\[ E_y = -k \left( \frac{Q_1}{5a^2} \frac{1}{\sqrt{3}} + \frac{Q_2}{2} \left( \frac{1}{\sqrt{3}} \right) + Q_3 \right) = -0.557 \times 10^{10} \ \text{N/C} + 5.28 \times 10^{10} \ \text{N/C} \]
\[ = -3.99 \times 10^{10} \ \text{N/C} \]

\[ \| E \| = 5.77 \times 10^{10} \ \text{N/C} \]
\[ \theta = -136^\circ \ \text{from +ve x axis}. \]
FIRST MIDTERM

Name (print): R. Romer  Name (signed): 15.5

Discussion Instructor (circle): Baske Chakhbazian DiCarlo Gundlach Romer Wei

Discussion Section #

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) Calculate the magnitude of the electric field at a distance of \(0.500 \times 10^{-10}\) m (about the size of an atom) from the center of a nucleus with a positive charge of \(+3e\).

\[
E = \frac{k \cdot \frac{3e}{2}}{r^2} = \frac{1.6 \times 10^{-19} \cdot 3 \cdot 1.6 \times 10^{-19}}{(0.5 \times 10^{-10})^2} = \frac{1}{1.3 \times 10^{16}}
\]

(b) If the mass of the sun is \(1.99 \times 10^{30}\) kg, calculate the period, in years, of an asteroid at a distance from the sun twice the distance of the earth from the sun. (Note: It is not necessary to do a lot of messy algebra to solve this.)

\[
\tau = \sqrt{\frac{4\pi^2}{GM}} = \sqrt{\frac{4\pi^2}{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}} = 2.83 \times 10^3
\]

(c) For the expression

\[
\frac{1}{(x^2 - a^2)^{3/2}}
\]

use the binomial expansion to calculate the term in \(a^5\). Don’t lose any pieces.

\[
\frac{1}{x^5} \left(1 + \frac{5}{x^2} + \frac{10}{x^4} + \frac{10}{x^6} + \frac{5}{x^8} + \frac{1}{x^{10}}\right) - 1 = \frac{1}{x^5} - \frac{10}{x^3} + \frac{5}{x} - 1
\]

(d) Calculate the magnitude of the gravitational force between two students of mass 100 kg each if they are 10.0 m apart. (Approximate the students as spheres!)

\[
F = G \frac{m_1 m_2}{r^2} = 6.67 \times 10^{-11} \frac{100 \times 100}{10^2} = 6.67 \times 10^{-9} N
\]

(e) An electron is accelerated from rest by an electric field of \(1.00 \times 10^4\) N/C. What is its speed after traveling 10.0 cm?

\[
s = \frac{1}{2} at^2 \quad \text{and} \quad \vec{s} = \sqrt{\omega \cdot \vec{E}} = \sqrt{2 \cdot 1.6 \times 10^{-19}} \times 9.4 \times 10^{-15} = 1.1 \times 10^{-14} \text{ m}
\]
Three electric charges are placed as shown. Take \( a = 3.20 \times 10^{-3} \, \text{m} \).

(a) Calculate the force, in \( i, j \) notation, on a charge of +9.00 \( \mu \text{C} \) placed at \( P \).

\[
F = \frac{k q q_1}{r^2} \hat{i}
\]

\[
F_1 = k \cdot \frac{-2 \cdot 9 \times 10^{-9} \times 1.58 \times 10^{-9} \hat{i}}{a^2} = -1.58 \times 10^{-9} \, \text{N} \, \hat{i}
\]

\[
F_2 = k \cdot \frac{-3 \times 9 \times 10^{-9}}{(\sqrt{2}a)^2} (\cos 90^\circ \hat{i} + \sin 90^\circ \hat{j})
\]

\[
= -1.12 \times 10^{-9} \, \text{N} \, \hat{i} - 1.12 \times 10^{-9} \, \text{N} \, \hat{j}
\]

\[
F_3 = k \cdot \frac{4 \times 9 \times 10^{-9}}{(\sqrt{2}a)^2} (\sin 45^\circ \hat{i} + \cos 45^\circ \hat{j})
\]

\[
= 8.39 \times 10^{-9} \, \text{N} \, \hat{i} + 8.39 \times 10^{-9} \, \text{N} \, \hat{j}
\]

So, \( \Sigma F = F_1 + F_2 + F_3 = -1.30 \times 10^{-9} \, \hat{i} + 1.96 \times 10^{-9} \, \hat{j} \) (N)

(b) \( \mathbf{E} = \frac{\Sigma F}{q} = \frac{\Sigma F}{9 \times 10^{-6}} = -1.49 \times 10^9 \, \text{N/C} \, \hat{i} - 2.17 \times 10^9 \, \text{N/C} \, \hat{j} \)
FIRST MIDTERM

Name (print): Zhukov
Name (signed) __________________________

Discussion Instructor (circle): Brown Chakhbazian Condell Fortnoi Zhukov

Discussion Section # ________

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) For the expression,

\[
\frac{1}{(1 - x)^{5/2}} = (1 - x)^{5/2} = 1 + \frac{5}{2}(1 - x) + \frac{\frac{5}{2} \cdot \frac{3}{2}}{2} (1 - x)^2 + \frac{\frac{\frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2}}{2^3}}{2} (1 - x)^3 = \]

Calculate the \(x^2\) term in the binomial expansion.

\[= 1 - \frac{5}{2} x + \frac{15}{8} x^2 \]

(b) An electron is accelerated from rest in an electric field of \(2.35 \times 10^2 \text{ V/m}\). Calculate its speed after it has traveled 7.00 cm.

\[\frac{V_1^2}{2a} = s^2 \quad a = \frac{F}{m} = \frac{eE}{m} \quad V = \sqrt{\frac{2eE}{m}} = 2.40 \times 10^6 \text{ m/s} \]

(c) If the radius of the earth were doubled and the mass of the earth increased by a factor of 10.0, what is the weight of a student who now weighs 1200 lb?

\[R_1 = 2R \quad M_1 = 10M \quad W = \frac{GM_1M}{R_1^2} \quad W_1 = \frac{GM_1M_1}{R_1^2} = \frac{Gm10m}{4R^2} = \frac{10W}{4} = 3000 \text{ lb} \]

(d) Calculate the magnitude of the electric force between an electron and the nucleus of a hydrogen atom. The nuclear charge is equal to the charge of the electron. Take the distance between them as \(1.00 \times 10^{-10} \text{ m}\).

\[F = k \frac{e^2}{r^2} = 2.30 \times 10^{-8} \text{ N} \]

(e) Calculate the speed (in m/s) of a satellite in a circular orbit 2000 km above the surface of the earth.

\[\frac{mV^2}{R^2} = \frac{GM}{R^2} \quad V = \sqrt{\frac{GM}{R^2}} = \sqrt{\frac{Gm}{R^2 + R_e}} = 6.30 \times 10^3 \text{ m/s} \]
SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Three charges are placed at corners of a rectangle as shown. Point P is at the 4th corner.

(a) Calculate the electric field in \( i, j \) notation at point P due to these charges.
(b) Find the force, magnitude and direction on an electron placed at P. The direction should be expressed as an angle measured counterclockwise from the positive x axis.

\[ Q_1 = -3.00 \times 10^{-6} \text{ C} \]
\[ Q_2 = +2.00 \times 10^{-6} \text{ C} \]
\[ Q_3 = -1.00 \times 10^{-6} \text{ C} \]
\[ a = 8.00 \text{ cm} \]

\[ a_j = \frac{2}{\sqrt{3}} \]
\[ a_i = \frac{1}{\sqrt{3}} \]

\[ \text{\( \overrightarrow{E} = -\frac{kQ_1}{a^2} (\cos \theta \, i - \sin \theta \, j) - \frac{kQ_2}{5a^2} \left( \frac{2}{\sqrt{5}} \, i + \frac{1}{\sqrt{5}} \, j \right) \)} \]

\[ \text{\( \overrightarrow{E} = \left[ \frac{(-1.6 \times 10^{-19} \text{ C})}{\epsilon_0} \right] \, i + \left[ (+1.28 \times 10^{-19} \text{ N} \cdot \text{m/C}) \right] \, j \) N/C} \]

\[ \text{\( \theta = \arctan \left( \frac{F_y}{F_x} \right) \Rightarrow \theta = 46.7^\circ \)} \]

\[ \text{\( F = \sqrt{(F_x)^2 + (F_y)^2} = 2.38 \times 10^{-13} \text{ N} \)} \]

\[ \text{\( \theta = 313.3^\circ \)} \]
FIRST MIDTERM

Name: [Name]
Discussion Instructor (circle): Frolov  McKain  Osan  Strang  Zhukov
Discussion Section #

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) Calculate the magnitude of the electric field, including units, 4.75 m from a point charge of value \( Q = +2.95 \times 10^{-3} \text{ C} \).

\[
E = k \frac{Q}{d^2} = 9 \times 10^9 \left( \frac{2.95 \times 10^{-3}}{4.75^2} \right) = \frac{1.18 \times 10^4}{\text{N/m}}
\]

(b) Calculate the complete term that involves \( a^4 \) in the binomial expansion of the expression below.

\[
\frac{1}{(x^2 - a^2)^{3/2}} = \frac{1}{x^3} \left( 1 + \frac{a}{2} \frac{x^2}{x^3} + \frac{a^2}{3} \frac{x^4}{x^5} \right)
\]

(c) Calculate the gravitational force on a 100 kg astronaut in a satellite that is 1.50 earth radii above the earth’s surface.

\[
F = \frac{GMm}{r^2} = \frac{G M \times 10^3}{(2 R)^2} = \frac{G M}{4 R^2} \cdot 9.8 = \frac{1.5 \cdot 10^4}{\text{N}}
\]

(d) If an asteroid orbits the sun in 400 days at a distance of 200,000,000 km (2.00 \times 10^{11} \text{ km}) from the center of the sun, calculate the mass of the sun from this information.

\[
T^2 = \left( \frac{4 \pi^2}{G M} \right) R^3 \Rightarrow M = \frac{4 \pi^2 R^3}{G T^2} = \frac{2.96 \times 10^{30}}{\text{kg}}
\]

(e) The nucleus of a lithium atom has an electric charge equal to +3e, since it has three protons. Calculate the magnitude of the electric force between a lithium nucleus and an electron if they are 0.25 \times 10^{-10} \text{ m} apart.

\[
F = \frac{k Q q}{r^2} = \frac{1.1 \times 10^{-8}}{\text{N}}
\]
SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Points A, B and C are at the corners of an equilateral triangle of side a. A charge of $+6.50 \times 10^{-6}$ C is placed at B. A charge of $-9.00 \times 10^{-6}$ C is placed at C. $a = 1.25$ cm.

(a) Calculate the electric field, magnitude and direction, at point A. Angles are measured counter clockwise from the positive x-axis.
(b) Calculate the magnitude of the force on a charge of $1.70 \times 10^{-6}$ C placed at A.

\[ \overline{E}_{BA} = \frac{k \left( 6.5 \times 10^{-6} \right)}{(0.125)^2} \left[ \cos 60^\circ \hat{\imath} + \sin 60^\circ \hat{j} \right] \]
\[ = 1.87 \times 10^8 \hat{\imath} + 3.74 \times 10^8 \hat{j} \quad \text{5pts} \]

\[ \overline{E}_{CA} = \frac{k \left( -9.8 \times 10^{-6} \right)}{(0.125)^2} \left[ \cos 120^\circ \hat{\imath} + \sin 120^\circ \hat{j} \right] \]
\[ = 2.59 \times 10^8 \hat{i} - 4.49 \times 10^8 \hat{j} \quad \text{5pts} \]

\[ \overline{E}_{total} = \left( 4.46 \times 10^8 \hat{i} - 1.25 \times 10^8 \hat{j} \right) \text{N/C} \quad \text{4pts} \]

\[ E_I = \sqrt{E_x^2 + E_y^2} = \sqrt{4.63 \times 10^8 N/C}, \quad \theta = \tan^{-1} \left( \frac{-1.25}{4.46} \right) = \frac{15.6^\circ \text{ or } 344^\circ}{3 \text{ pts}} \]

\[ F = (1.7 \times 10^{-6}) (4.63 \times 10^8 N/C) = 787 \text{ N} \quad \text{5pts} \]

No partial credit is given for long calculations in b.

5pts is simply given if your answer in b was $1.7 \times 10^{-6}$ times your answer to part a.
FIRST MIDTERM

Name: Eugene Tsiper

Discussion Instructor (circle): Frolov  McKain  Osen  Strang  Zhukov.

Discussion Section #

SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES:
Use the conversion constants and data given on the front page.

A uniformly charged sphere of a non-conductor has radius \( R_1 \) and charge \( Q_1 \). It is enclosed in a concentric thin metal spherical shell whose radius is \( R_2 \) (\( R_2 > R_1 \)). The metal shell has a total charge \( Q_2 \).

(a) Calculate the electric field a distance 37.0 cm from the common center of the two spheres. Give the answer in numerical form with correct sign.

(b) Determine the electric field a distance of 17.5 cm from the common center. Give the answer in numerical form with correct sign.

\[
\begin{align*}
Q_1 &= +300 \times 10^{-6} \text{ C} \\
Q_2 &= -475 \times 10^{-6} \text{ C} \\
R_1 &= 8.00 \text{ cm} \\
R_2 &= 20.0 \text{ cm}
\end{align*}
\]

Because of the ambiguity in the numerical data for \( Q_1 \) and \( Q_2 \) and taking into account the announcement at the exam, the following 3 choices for \( Q_1 \) and \( Q_2 \) could be assumed and are to be considered as a correct answer:

1) \( Q_1 = +300 \cdot 10^{-6} \text{ C} \), \( Q_2 = -475 \cdot 10^{-6} \text{ C} \)
2) \( Q_1 = +3.00 \cdot 10^{-6} \text{ C} \), \( Q_2 = -4.75 \cdot 10^{-6} \text{ C} \)
3) \( Q_1 = +300 \text{ C} \), \( Q_2 = -475 \text{ C} \)

\[E = -k \left( \frac{Q_1 + Q_2}{(0.37m)^2} \right)
\]

a) \( E = -4.15 \cdot 10^7 \text{ N/C} \)
b) \( E = +1.15 \cdot 10^5 \text{ N/C} \)
c) \( E = +1.15 \cdot 10^7 \text{ N/C} \)

\[E = -k \left( \frac{Q_1}{(0.175m)^2} \right)
\]

a) \( E = -8.82 \cdot 10^3 \text{ N/C} \)
b) \( E = -8.82 \cdot 10^5 \text{ N/C} \)
c) \( E = -8.82 \cdot 10^7 \text{ N/C} \)
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!

Use the conversion constants and data given on the front page.

(a) A lithium-7 nucleus has 3 protons and 4 neutrons. Calculate the force between a lithium nucleus and an electron 3.00 x 10^{-11} m apart.

\[ F = \frac{k q_1 q_2}{r^2} \quad q_1 = 3e \quad q_2 = -e \quad F = -\frac{9 \cdot 10^9 \times 3 \times (1.6 \times 10^{-19})^2}{(3 \times 10^{-11})^2} = 7.68 \times 10^{-7} \text{ N} \]

(attractive)

(b) Calculate the electric field (in N/C) at a distance of 4.00 x 10^{-5} m from a lithium nucleus.

\[ E = \frac{k q_1}{r^2} = \frac{9 \cdot 10^9 \times 3 \times 1.6 \times 10^{-19}}{(4 \times 10^{-5})^2} = 2.70 \times 10^8 \frac{\text{N}}{\text{C}} \]

(c) A conducting sphere of radius 1.50 x 10^{-2} m has a positive charge of 1.75 x 10^{-11} C. Determine the electric field at a distance of 3.30 m from the center of the sphere.

\[ d>R \quad E = \frac{k q_1}{d^2} = \frac{9 \cdot 10^9 \times 1.75 \times 10^{-11}}{(3.3)^2} = 1.45 \times 10^{-2} \frac{\text{N}}{\text{C}} \]

(d) For the expression \(1/(x-a)^{5/2}\), where \(a \ll x\), calculate completely the term in \(a^3\) using the binomial expansion.

\[ \left(\frac{x-a}{x}\right)^{5/2} = \left(1 - \frac{a}{x}\right)^{5/2} = x^{-5/2} \left(1 - \frac{5a}{2x} + \frac{\frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot a^2}{2} \cdot x^2 - \frac{5 \cdot 3 \cdot \frac{1}{2} \cdot a^3}{2} \cdot x^3 + \cdots \right) \]

Term in \(a^3\): \(\left(-\frac{\frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot a^3}{2} \cdot x^3\right) x^{-5/2} = -6.56 \frac{a^3}{x^{1/2}}\)

(e) An electron is accelerated from rest in a uniform electric field of 4.67 N/C. Find the speed of the electron after it has traveled 3.25 cm.

\[ F = ma = qE \quad a = \frac{qE}{m} \quad v^2 = 2ad \]

\[ v = (2ad)^{1/2} = \left(\frac{2qEd}{m}\right)^{1/2} = \left(\frac{2 \times 1.6 \times 10^{-19} \times 4.67 \times 0.0325}{0.11 \times 10^{-31}}\right)^{1/2} = 2.31 \times 10^5 \frac{\text{m}}{\text{s}} \]
SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Three charges are placed at the corners of a square as shown. \( Q = 1.70 \times 10^{-10} \text{ C} \)

(a) Determine the electric field at point \( P \) in \( \hat{i}, \hat{j} \) notation (numerical value).
(b) Calculate the magnitude of the electric field at point \( P \) (numerical value).

\[ \vec{E}_P = \vec{E}_P^1 + \vec{E}_P^2 + \vec{E}_P^3 \]

\[ \vec{E}_P^1 = -\frac{2kQ}{r^2} \hat{j} \quad (\hat{r} = -\hat{j}, \quad r = \ell) \]

\[ \vec{E}_P^2 = \frac{3kQ}{2\sqrt{2} \ell^2} \hat{i} + \frac{3kQ}{2\sqrt{2} \ell^2} \hat{j} \quad (\hat{r} = -\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}, \quad r = \frac{\sqrt{2}}{2}\ell) \]

\[ \vec{E}_P^3 = -\frac{4kQ}{\ell^2} \hat{i} \quad (\hat{r} = -\hat{i}, \quad r = \ell) \]

\[ \vec{E}_P = \left[ \frac{3kQ}{2\sqrt{2} \ell^2} - \frac{4kQ}{\ell^2} \right] \hat{i} + \left[ \frac{3kQ}{2\sqrt{2} \ell^2} - \frac{2kQ}{\ell^2} \right] \hat{j} \]

\[ = \left( -1.997 \times 10^{12} \text{ N/C} \right) \hat{i} + \left( -6.39 \times 10^{11} \text{ N/C} \right) \hat{j} \]

\[ \left\| \vec{E}_P \right\| = \sqrt{E_x^2 + E_y^2} = 2.10 \times 10^{12} \text{ N/C} \]
SHOW ALL WORK!!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

The curved object is a thin rod bent in a circular form that goes $3/4$ of the way around the circle from the positive x-axis to the negative y-axis. The origin is at the center of the circle. There is a uniform linear charge density and a total charge $Q = 4.70 \times 10^9$ C. Find the electric field, magnitude and direction, at the origin. Ignore the diameter of the rod itself.

\[ \lambda = \text{not given} \]

\[ E = \frac{\lambda R}{2} \int_{0}^{\pi/2} \frac{d\theta}{R} d\theta = \frac{\lambda R}{2} \frac{\pi}{2} = \frac{\lambda R}{4} \]

\[ E_x = \int_{0}^{\pi/2} \frac{d\theta}{R} \cos \theta d\theta = \frac{\lambda R}{2} \left[ \cos \theta \right]_{0}^{\pi/2} = \frac{\lambda R}{2} \left[ 0 - 1 \right] = -\frac{\lambda R}{2} \]

\[ E_y = \int_{0}^{\pi/2} \frac{d\theta}{R} \sin \theta d\theta = \frac{\lambda R}{2} \left[ \sin \theta \right]_{0}^{\pi/2} = \frac{\lambda R}{2} \left[ 1 - 0 \right] = \frac{\lambda R}{2} \]

\[ |E| = \sqrt{E_x^2 + E_y^2} = \sqrt{\left( -\frac{\lambda R}{2} \right)^2 + \left( \frac{\lambda R}{2} \right)^2} = \frac{\sqrt{2} \lambda R}{2} = \frac{\sqrt{2} \lambda R}{2} \frac{9 \times 10^9 (4.7 \times 10^{-9})}{3 \pi} = \frac{12.7}{R} N/C \]

\[ |E| = \frac{12.7}{R} N/C \]
Notes:

1. Gauss's law CANNOT BE USED
2. You cannot integrate \( \vec{S} \cdot \vec{E} \), you must calculate \( d\vec{E}_x, d\vec{E}_y \) and integrate to get \( E_x, E_y \)
3. \( I \) is not given, you must calculate \( A \) from data given. \( I = \frac{Q}{\pi/2 (2\pi r)} = \frac{2Q}{3\pi R} \)
4. Arc length \( ds = R d\theta \) needed,
FIRST MIDTERM

Name: Alexander Pokrovsky

Discussion Instructor (circle): Billeter Bulson Munician Paul Pomercy Walker

Discussion Section #: ___________________________ Student ID #: ___________________________

SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A sphere of non-conductor has a charge density given by \( \rho(r) = Br^2 \), for \( r < R_o \) and zero everywhere outside of \( R_o \).

(a) Find the total charge on the sphere.
(b) Calculate the electric field at point P, a distance \( R_1 \) from the center (\( R_1 > R_o \)).
(c) Determine the electric field at point Q, a distance \( R_1 \) from the center, where \( R_2 < R_o \).

\[
a) \quad Q = \int_0^{R_0} \rho(r) 4\pi r^2 dr = 4\pi B \int_0^{R_0} r^2 dr = \frac{2\pi B R_o^6}{3}
\]

\[
b) \quad E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R_1^2} = \frac{1}{4\pi\varepsilon_0} \frac{2\pi B R_o^6}{3R_1^2} = \frac{BR_o^6}{6\varepsilon_0 R_1^2}
\]

\[
c) \quad E \cdot 4\pi R_2^2 = \frac{1}{\varepsilon_0} \int_0^{R_2} Br^2 4\pi r^2 dr = \frac{2\pi B R_2^6}{3}
\]

\[
\implies E = \frac{1}{4\pi\varepsilon_0 R_2^2} \frac{2\pi B R_2^6}{3} = \frac{BR_2^4}{6\varepsilon_0}
\]
FIRST MIDTERM

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Discussion Section # ____________________________ Student ID #: ____________________________

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) Calculate the force between two protons \( (q = +1.6 \times 10^{-19} \text{ C}) \) that are \( 1.5 \times 10^{-15} \text{ m} \) apart (approximately the separation that exists in an atomic nucleus).

\[
F = \frac{kq^2}{r^2} = \frac{102 \text{ N}}{103 \text{ N}}
\]

(b) For the expression \( (x - a)^{-\frac{3}{2}} \), where \( a \ll x \), calculate completely the term in \( a^3 \) using the binomial expansion.

\[
x^{-\frac{3}{2}}(1 - \frac{a}{x})^{-\frac{3}{2}} = \frac{1}{2} x^{-\frac{3}{2}} \left( -\frac{3}{2} \right) \left( -\frac{1}{2} \right) \frac{a^2}{x^2} + \cdots
\]

\( \approx 7.88 \frac{a^2}{x^{1/2}} \)

(c) An electron is accelerated from rest in an electric field of 0.325 N/C. After it has traveled 1.20 m, what is its speed?

\[
E = \frac{F}{q} = \frac{ma}{q} = E = \frac{m}{q} \left( \frac{v^2}{2x} \right) \left( \varepsilon_0 2x \right)^{1/2} = V = 3.70 \times 10^5 \text{ m/s}
\]

\( \approx 3.71 \times 10^5 \text{ m/s} \)

(d) Calculate the numerical magnitude of the electric field at a distance of \( 1.20 \times 10^{-10} \text{ m} \) from an electron.

\[
E = \frac{kq}{r^2} = 1 \times 10^9 \text{ N/C}
\]

\( \approx 0.99 \times 10^8 \text{ N/C} \)

(e) A cube of non-conductor is uniformly charged with a total charge of \( +6.20 \times 10^6 \text{ C} \). Calculate the electric flux through one face of the cube.

\[
\Phi = \frac{q}{\varepsilon_0} = \frac{1.17 \times 10^5 \text{ N cm}^2}{1.18 \times 10^5 \text{ N cm}^2}
\]

\[
\frac{q}{(1.2)^2 \times 10^{-20}}
\]
SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES:
Use the conversion constants and data given on the front page.

The points shown are at the corners of a square.

(a) Calculate the electric field, magnitude and direction, at point P.
(b) Calculate the force, magnitude and direction, on a negative charge of \(-5 \, \text{Q}\) placed at P.

Let's project all fields on the \(PB\) direction, where the positive direction is from \(P\) to \(B\).

\(A: \quad \vec{E}_A = -\frac{kQ}{a^2} \hat{j} ; \quad \text{projection} (\vec{E}_A)_{PB} = -\frac{kQ}{a^2} \cos 45^\circ = -\frac{kQ \sqrt{2}}{2a^2}\)

\(B: \quad \vec{E}_B = \frac{k(-3Q)}{2a^2} \hat{B} ; \quad \text{projection} (\vec{E}_B)_{PB} = \frac{3kQ}{2a^2} \hat{P} \hat{B} ; \quad \text{projection} (\vec{E}_B)_{PB} = \frac{3kQ}{2a^2}\)

Summation of these 3 projection gives us:
\[
E = \frac{3}{2} \frac{kQ}{a^2} - \frac{\sqrt{2}}{2} \frac{kQ}{a^2} - \frac{\sqrt{2}}{2} \frac{kQ}{a^2} = \frac{kQ}{a^2} \left( \frac{3}{2} - \sqrt{2} \right)
\]

Thus the magnitude is
\[
E = \frac{kQ}{a^2} \left( \frac{3}{2} - \sqrt{2} \right) \approx 0.086 \frac{kQ}{a^2}
\]

We can obtain the direction by noticing that the scene is quite symmetric relative \(PB\) line. Thus we conclude that angle is \(\theta = 95^\circ\) is positive \(PB\) direction.

\(C: \quad \vec{F} = qE = (-5Q) \frac{kQ}{a^2} \left( \frac{3}{2} - \sqrt{2} \right) \hat{PB}\) see back
Thus magnitude is: \( |\vec{F}| = \frac{kQ^2}{a^2} 5\left(\frac{3}{2} - 3\right) \approx \frac{kQ^2}{a^2} 0.4 \)

And direction is quite opposite to one of \( \vec{E} \) positive and is equal to 225° from \( \vec{x} \) direction.
A spherically symmetric charge distribution on a non-conductor is modeled by a charge density given by:

\[ \rho(r) = \frac{B}{r} , \]

where B is a constant. The charge distribution has an outer radius \( R_o \). There is no charge outside of \( R_o \).

(a) Calculate the total charge Q.
(b) Calculate the electric field at the interior point \( r = R_o/3 \).

\[ Q = \int \rho(r) dV \]
\[ = \int_0^{R_o} \rho(r) 4\pi r^2 dr \]
\[ = \int_0^{R_o} \frac{B}{r} 4\pi r^2 dr \]
\[ = 4\pi B \int_0^{R_o} r^2 dr \]
\[ = 4\pi BR_o^2 \]

\[ \int E \cdot dS = \frac{Q_{\text{enclosed}}(R_o/3)}{\varepsilon_0} \]
\[ \int E \cdot dS = 4\pi r^2 E \quad r = R_o/3 \]
\[ Q_{\text{enclosed}}(R_o/3) = 2\pi B \left( \frac{R_o}{3} \right)^2 \]
\[ E = \frac{2\pi B \left( \frac{R_o}{3} \right)^2}{4\pi \left( \frac{R_o}{3} \right)^2 \varepsilon_0} = \frac{B}{2\varepsilon_0} \]

or
\[ E = \frac{kQ_{\text{enclose}}}{r^2} \left( \frac{kQ_{\text{enclose}} r}{r^3} \right) \]
\[ = k \frac{2\pi B \left( \frac{R_o}{3} \right)^2}{\left( \frac{R_o}{3} \right)^2} = 2\pi k B \]

This can be done only because of the charge is spherically symmetric.
A thin rod of length L is uniformly charged with a negative charge. The total charge on the rod is \(-Q\). Determine the y-component of the electric field at point P, a distance \(a\) from the end of the rod. \(a\) is perpendicular to the rod. Be sure to include the sign of your answer.

\[
dE_y = \frac{1}{4\pi \varepsilon_0} \frac{dQ}{r^2} \sin \theta (-\hat{j})
\]

\[
= \frac{1}{4\pi \varepsilon_0} \frac{\frac{Q}{L}}{a^2 + x^2} \cdot \frac{a}{\sqrt{a^2 + x^2}} (-\hat{j})
\]

\[
= \frac{Q}{4\pi \varepsilon_0} \frac{\int \frac{dx}{(a^2 + x^2)^{3/2}}} (-\hat{j}).
\]

So \(E_y = \int dE_y = \frac{Qa}{4\pi \varepsilon_0 L} \int_0^L \frac{dx}{(a^2 + x^2)^{3/2}} (-\hat{j}) \leq \frac{Q}{4\pi \varepsilon_0 a} \frac{1}{a^2 + L^2} (-\hat{j}) \leq \frac{Q}{4\pi \varepsilon_0 a} \frac{1}{a^2 + L^2} (-\hat{j}) \leq 0.

If sign wrong, there are 5-pt. subtract!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) Calculate the magnitude of the electric field 4.00 × 10⁻¹² m away from a proton, which has a positive charge identical in magnitude to the electron.
\[ E = \frac{\kappa q}{r^2} = \frac{(4.00) \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2 \times (1.60 \times 10^{-19} \text{C})}{(4.00 \times 10^{-12} \text{m})^2} = 9.00 \times 10^{14} \text{N/C} \]
\[ = 9.00 \times 10^{13} \text{N/C}. \]

(b) Calculate the term in x⁴ for the binomial expansion of \((1 - x)^{-3}\).
\[ 1 + \left(-\frac{3}{2}\right)x + \left(-\frac{3}{2}\right)\left(-\frac{3}{2}\right)\frac{1}{2}x^2 + \left(-\frac{3}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{1}{2}\right)\frac{1}{3}x^3 \cdot \frac{4.29}{1.6} \]
\[ = \frac{5.76}{1.6} \times 6 \]

(c) Calculate the electric force between two protons a distance 2.00 × 10⁻¹³ meters apart.
\[ F = \frac{kq_1q_2}{r^2} = \frac{(9.00) \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2 \times (1.60 \times 10^{-19} \text{C})^2}{(2.00 \times 10^{-13} \text{m})^2} = 8.00 \times 10^{-3} \text{N}. \]

(d) An electron is accelerated from rest in a uniform electric field of 175 N/C. Calculate its speed after it travels 1.50 m parallel to this field.
\[ \vec{U} = \sqrt{2(1.50) \text{m} \times (175 \text{N/C}) \times (1.60 \times 10^{-19} \text{C})} \times 9.11 \times 10^{-31} \text{kg} \times \frac{m}{\text{kg}} = 4.60 \times 10^6 \text{m/s}. \]

(e) A cube of non-conductor is uniformly charged with 9.30 × 10⁻⁵ C. Calculate the electric flux through one face of the cube.
\[ \Phi = \frac{q}{\epsilon_0} = \frac{(9.30 \times 10^{-5} \text{C})}{8.85 \times 10^{-12} \text{C} \cdot \text{m}^{-2} \cdot \text{N}^{-1}} \times \frac{1}{6} = 1.75 \times 10^7 \text{m}^2 \cdot \text{N}^{-1} \cdot \text{C}^{-1}. \]

\[ \text{Units: } \text{C/m}^2 \]
\[ \text{Equation: } \text{+3} \]
\[ \text{Results: } \text{+2} \]
2) \[ \overrightarrow{F} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} \]

\[ = k \frac{Q_1 Q_2}{a^2} \cos \theta - \frac{k Q_2 Q_3}{a^2} \cos \phi + \frac{k Q_1 Q_3}{a^2} \frac{1}{i} \]

\[ = -\frac{0.0432}{a^2} \frac{1}{i} + \frac{0.0078}{a^2} \frac{1}{i} + \frac{0.054}{a^2} \frac{1}{i} \]

\[ = \frac{0.04612}{a^2} \frac{1}{i} - \frac{0.0432}{a^2} \frac{1}{i} \]

\[ |F| = \sqrt{F_x^2 + F_y^2} = \sqrt{(0.046)^2 + (0.0343)^2} \frac{1}{a^2} = \frac{0.0632}{a^2} \]

\[ \theta = \arctan \frac{y}{x} = \arctan \frac{-0.0343}{0.04612} = -43.12^\circ \]

b) \[ \overrightarrow{E} = \frac{\overrightarrow{F}}{Q} = \frac{2.1 \times 10^6 \frac{1}{i}}{a^2} = \frac{2.1 \times 10^6}{a^2} \frac{1}{i} \]

\[ |E| = \frac{3.15 \times 10^5}{a^2} \]

\[ \theta = -43.1^\circ \text{ as before} \]
SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A charge of +6.00 μC is placed on a spherical, conducting shell whose inner radius is R₁ and outer radius is R₂. Then another charge of -9.00 μC is placed at the exact center of the spherical shell.

(a) Calculate the electric field at a distance R from the center if R > R₂.
(b) Calculate the electric field at a distance R from the center if R₁ < R < R₂.
(c) Calculate the electric field a distance R from the center if R < R₁.

**Gauss' Theorem**

\[ \oint E \cdot d\mathbf{A} = \frac{2q_i}{\varepsilon_0} \]

E is spherically symmetric,

\[ E \cdot 4\pi R^2 = \frac{2q_i}{\varepsilon_0} \]

\[ a) \quad E = \frac{+6 - 9}{4\pi \varepsilon_0} = \frac{-3 \mu C}{4\pi \varepsilon_0} \]

\[ b) \quad E = 0 \]

\[ c) \quad E = \frac{-9 \mu C}{4\pi \varepsilon_0} \]
SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A long non-conducting cylinder of radius $R_o$ has a volume charge density given by $\rho = Br^4$, where $B$ is a constant. $\rho = 0$ for $r > R_o$ (outside the cylinder).

(a) Calculate the total charge in a length $L$ of this cylinder.
(b) Calculate the electric field a distance $R_1$ away from the axis of the cylinder if $R_1 > R_o$.
(c) Calculate the electric field a distance $R_2$ away from the axis of the cylinder if $R_2 < R_o$.

$$\rho(r) = \begin{cases} \frac{2BR^4}{3}, & r \leq R_o \\ 0, & r > R_o \end{cases}$$

10. (a) $Q_0 = \int \rho dV$

$$dV = 2\pi r L \, dr$$

$$Q_0 = \int_0^{R_0} \rho(r) \cdot 2\pi r L \, dr = \frac{1}{3} \pi BR_0^6 L$$

(b) From Gauss's law, $\int \vec{E} \cdot d\vec{A} = \frac{Q_0}{\varepsilon_0}$ and

$$\int \vec{E} \cdot d\vec{A} = \vec{E} \cdot 2\pi R_1 L$$, we get

$$\vec{E}_1 = \frac{BR_0^6}{6\varepsilon_0 R_1} \hat{\jmath}$$, $\hat{\jmath}$ is the unit vector.

10. (c) From Gauss's law, $\int \vec{E}_2 \cdot d\vec{A}_2 = \frac{Q_1}{\varepsilon_0}$ and

$$\int \vec{E}_2 \cdot d\vec{A}_2 = \vec{E}_2 \cdot 2\pi R_2 L$$, we get

$$\vec{E}_2 = \frac{BR_2^5}{6\varepsilon_0} \hat{\jmath}$$, $\hat{\jmath}$ is the unit vector.

$\sigma_1$ and $\sigma_2$ are the Gauss's surface as shown in Fig.
Physics 2220
Spring 2003
George A. Williams

FIRST MIDTERM

Name: ________________________________

Discussion Instructor (circle): Billeter  Blake  Herring  Young

Discussion Section # ________________  Student ID #: __________________

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) Calculate the magnitude of the electric field at a distance of \(2.00 \times 10^{-10}\) m from a proton \((Q_{\text{proton}} = +1.6 \times 10^{-19}\) C).

\[
\vec{E} = \frac{kQ}{r^2} = \frac{9 \times 10^9 N \cdot m^2/C^2 \times 1.6 \times 10^{-19} C}{(2 \times 10^{-10} m)^2} = 3.60 \times 10^{10} N/C
\]

(b) Calculate the magnitude of the electric force between two electrons that are \(0.60 \times 10^{-10}\) m apart.

\[
|\vec{F}| = \frac{k|e| |e|}{r^2} = \frac{9 \times 10^9 N \cdot m^2/C^2 \times |1.6 \times 10^{-19} C|^2}{(0.60 \times 10^{-10} m)^2} = 6.40 \times 10^{-8} N
\]

(c) Calculate the magnitude of the electric field at point P, halfway between the two charges.

Because two charges have the electric field with the same direction at point P, then.

\[
|\vec{E}_{\text{total}}| = \left|\frac{kQ_1}{d^2}\right| + \left|\frac{kQ_2}{d^2}\right| = \frac{4k}{d^2} (3.0 \text{nC} + 4.75 \text{nC}) = \frac{279}{d^2}
\]

(d) Calculate the coefficient of \(x^3\) for the binomial expansion of \((1 - x^2)^{-7/2}\)

For \(x \ll 1\), \((1 - x^2)^{-\frac{3}{2}} = 1 + \left(-\frac{3}{2}\right)(-x^2) + \frac{1}{2}(-\frac{3}{2})(-\frac{5}{2})(-x^4) + \frac{1}{6}(-\frac{3}{2})(-\frac{5}{2})(-\frac{7}{2})(-x^6)\ldots\)

In that expression, there is no item about \(x^3\).

That is to say, the coefficient of \(x^3\) equals to zero. \(0\)

(e) A cube of non-conducting material is uniformly charged with a total charge of \(7.50 \times 10^4\) C. Calculate the electric flux through one face of the cube.

Total electric flux \(\Phi_{\text{total}} = \oint \vec{E} \cdot d\vec{S} = \frac{q_{\text{enclosed}}}{E_0}\). Because cube is symmetric. For the electric flux through one face of cube,

\[
\phi = \frac{1}{6} \Phi_{\text{total}} = \frac{1}{6} \cdot \frac{q_{\text{enclosed}}}{E_0} = \frac{1}{6} \cdot \frac{7.5 \times 10^4 \text{C}}{8.85 \times 10^{-12} \text{F/m}} = 1.41 \times 10^{15} \text{C} \cdot \text{m} \]
Grading Scale (First MIDTERM)

Problem 1:

1. If you write that correct answer and there is a wrong significant figure, Subtract one point.

2. If you write that correct and there is a wrong Unit, Subtract one point.

3. If you have a wrong answer, Subtract five points.

Notice!!!
Total points is five for every part of problem 1.

For example:

Part (b). If you write the final answer like this, $|F| = -6.40 \times 10^{-8} N$ you receive no points.

Part (c) If you write the final answer like this, $|\vec{E}| = \frac{279}{d^2} \frac{N}{c}$ you also receive no points.

If you write the final answer like this $\vec{E} = -\frac{279}{d^2} \frac{N}{c} \hat{i}$ you receive no points.

No correct answer!! No points!!
Electric fields generated by every charge are:

\[ E_1 = \frac{kQ_1}{r_1^2} \hat{r}_1 = \frac{k \cdot 5.70 \text{NC}}{r_1^2} \hat{r}_1 = \frac{k \cdot 5.70 \text{NC}}{r_1^2} \left( \frac{\hat{x}}{2} - \frac{\hat{y}}{2} \right) \]

\[ E_2 = \frac{kQ_2}{r_2^2} \hat{r}_2 = \frac{k \cdot 2.75 \text{NC}}{r_2^2} \hat{r}_2 = \frac{k \cdot 2.75 \text{NC}}{r_2^2} \left( -\frac{\hat{x}}{2} - \frac{\hat{y}}{2} \right) \]

\[ E_3 = \frac{kQ_3}{r_3^2} \hat{r}_3 = \frac{k \cdot 75 \text{NC}}{r_3^2} \hat{r}_3 = \frac{k \cdot 75 \text{NC}}{r_3^2} \left( \frac{\hat{x}}{2} + \frac{\hat{y}}{2} \right) \]

\[ E_4 = \frac{kQ_4}{r_4^2} \hat{r}_4 = \frac{k \cdot 1325 \text{NC}}{r_4^2} \hat{r}_4 = \frac{k \cdot 1325 \text{NC}}{r_4^2} \left( -\frac{\hat{x}}{2} + \frac{\hat{y}}{2} \right) \]

Adding them together, we get:

\[ E_1 + E_2 + E_3 + E_4 = \frac{k}{r^2} \left[ -17.75 \times \frac{\hat{x}}{2} - 2.75 \times \frac{\hat{y}}{2} \right] \]

\[ = (-12.81 \hat{i} - 1.984 \hat{j}) \times 10^8 \text{ V/m} \]

The magnitude is \( \sqrt{(12.81)^2 + (-1.984)^2} = 13.0 \times 10^8 \text{ V/m} = 1.30 \times 10^9 \text{ V/m} \)

The direction is:

\[ \theta = \arctan \left( \frac{1.984}{-12.81} \right) + \pi = 188.9^\circ = 189^\circ \]

or \[ \theta = \arctan \left( \frac{-1.984}{12.81} \right) - \pi = -171^\circ \]
SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A thick, hollow sphere is constructed out of non-conducting material. The inner radius is $R_1$ and the outer radius is $R_2$. The charge density between $R_1$ and $R_2$ can be expressed as $\rho = A/R^2$, where $A$ is a constant. $\mu$ is the variable.

(a) Determine the electric field at point $P$, where $R_p > R_1$.
(b) Find the electric field at point $Q$, where $R_1 > R_Q > R_1$.

a) Use Gauss's law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E_p \ 4\pi R_p^2 = \frac{1}{\varepsilon_0} \int_{R_1}^{R_2} \rho \ dV = \frac{1}{\varepsilon_0} \int_{R_1}^{R_2} \frac{A}{R^2} \ 4\pi R^2 \ dR$$

$$= \frac{1}{\varepsilon_0} \ 4\pi A \ R^1_{R_1}^{R_2} = \frac{4\pi A}{\varepsilon_0} (R_2 - R_1)$$

$$S_o \ \vec{E}_p = \frac{4\pi A}{4\pi R_p^2 \varepsilon_0} \ (R_2 - R_1) \ \vec{R} = \frac{A \ R^1_{R_2}}{\varepsilon_0 R_p^2} \ (R_2 - R_1) \ \vec{R}$$

b) Use Gauss's law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E_Q \ 4\pi R_Q^2 = \frac{1}{\varepsilon_0} \int_{R_1}^{R_2} \rho \ dV = \frac{1}{\varepsilon_0} \int_{R_1}^{R_2} \frac{A}{R^2} \ 4\pi R^2 \ dR$$

$$= \frac{1}{\varepsilon_0} \ 4\pi A \ R^1_{R_1}^{R_2} = \frac{4\pi A}{\varepsilon_0} (R_Q - R_1)$$

$$S_o \ \vec{E}_Q = \frac{4\pi A (R_Q - R_1)}{4\pi R_Q^2 \varepsilon_0} \ \vec{R} = \frac{A \ R^1_{R_2}}{\varepsilon_0 R_Q^2} \ (R_Q - R_1) \ \vec{R}$$
SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Given a uniformly positively charged rod of total length 2L and total charge Q. Calculate the electric field at point P, magnitude and direction, due to the rod.

From symmetry, direction of electric field: \( \hat{y} \) (+\( \hat{y} \)).

\[
dE = \frac{k dq}{r^2} \hat{r}
\]

\[
dE_y = \frac{k dq}{r^2} \cos \theta \hat{y}
\]

\[
dE_y = \frac{k dq}{r^2} \cos \theta \rightarrow E_y = \int_{-L}^{L} \frac{k dq}{r^2} \cos \theta = 2 \int_{0}^{L} \frac{k dq}{r^2} \cos \theta
\]

\[
\lambda = \frac{Q}{2L} \quad (5 \text{ POINTS})
\]

\[
dx \rightarrow dq = \frac{Q}{2L} dx; \quad r = \sqrt{x^2 + a^2}; \quad \cos \theta = \frac{a}{\sqrt{x^2 + a^2}}
\]

\[
E_y = 2 \int_{0}^{L} \frac{k \lambda dx}{(x^2 + a^2)} \frac{a}{\sqrt{x^2 + a^2}} = 2 k \lambda \frac{a}{x^2 + a^2} \int_{0}^{L} dx \frac{1}{(x^2 + a^2)^{3/2}}
\]

\[
E_y = 2 k \lambda \frac{x}{a \sqrt{x^2 + a^2}} \int_{0}^{L} dx \frac{1}{a \sqrt{L^2 + a^2}} = \frac{2 k \lambda L}{a \sqrt{L^2 + a^2}}
\]

Magnitude:

\[
E_y = \frac{2 k \lambda L}{a \sqrt{L^2 + a^2}} = \frac{k Q}{a \sqrt{L^2 + a^2}} \quad (5 \text{ POINTS})
\]
\[ \vec{E} = E_y = E_y \hat{j} = \frac{kQ}{a\sqrt{L^2 + a^2}} \hat{j} \]

Direction: \[ \hat{j} \]

(5 points)

Bogdan Porescu
Two sides of a square of non-conductor of side length a are positively charged with a linear charge density of \(3.70 \times 10^{-9}\) C/m.

1. \(C(a)\) Calculate the electric potential at point P. P is at the corner of the rectangle. Both the rectangle and P are in the plane of the paper. (A numerical answer, in volts, is required.)

\[ dV = \frac{k dq}{r} \]

where \(r = \sqrt{x^2 + y^2}\)

\(k = 9.00 \times 10^9 \text{ Nm}^2/\text{C}^2\)

\[ dq = \lambda dx \]

\(\lambda = 3.70 \times 10^{-9}\) C/m

\[ dV = \frac{k \lambda dx}{\sqrt{x^2 + y^2}} \]

\[ dV_{total} = \int_{0}^{a} \frac{k \lambda dx}{\sqrt{x^2 + y^2}} \]

\[ V = 2 \lambda K \left[ x \ln \left( x + \sqrt{x^2 + y^2} \right) \right]_{0}^{a} = 2 \lambda K \left[ a \ln \left( 1 + \sqrt{1 + 1} \right) - 0 \right] \]

\[ V = 2 \lambda K \ln \left( 1 + \sqrt{2} \right) \]

\[ V = 2 \left( 3.70 \times 10^{-9} \text{C} \right) \left( 9.00 \times 10^9 \text{Nm}^2/\text{C}^2 \right) \ln \left( 1 + \sqrt{2} \right) \Rightarrow V = 58.7 \text{ Volts} \]

\[ \Delta \omega = \int_{0}^{a} \frac{k \lambda dx}{\sqrt{x^2 + y^2}} \]

\[ \omega = \int_{0}^{a} \frac{dV}{dR} \Rightarrow \omega = 9 \left[ V(a) - V(0) \right] = 9 \times 58.7 \text{ Volts} = 9 \times 58.7 \text{ Volts} \]

\[ \Rightarrow \omega = 9 \times 58.7 \text{ Volts} = 9 \times 58.7 \times 10^{-9} \text{ C} \times \text{volts} \Rightarrow \omega = 9 \times 58.7 \times 10^{-9} \text{ J} \]

\[ \Rightarrow \omega = 9 \times 58.7 \times 10^{-9} \text{ J} \]
(a) What is the surface charge density of a conducting sphere of radius R, if the total charge on the sphere is Q? \( \sigma = \frac{Q}{4 \pi R^2} \)

(b) Calculate the work to move 6.00 \( \mu \text{C} \) of charge through a potential difference of 60.0 volts. \( 3.60 \times 10^{-4} \text{ joules} \)

(c) If the potential is given by \( V = V_0 (1 - x^2) \), find the x component of the electric field. \( E_x = 2x V_0 \) \( x \)

(d) 350 Joules of work is done in moving a charge of 0.20 \( \text{C} \) from point A to point B. Calculate the potential difference between A and B. 1700 V or 1900 V

(e) Express 3.2 \( \times 10^{-18} \) Joules in electron volts. \( 2.0 \times 10^{-19} \text{ eV} \)
2. Two metal spheres are 3.0 cm in radius and carry charges of +1.0 \times 10^{-8} \text{ C} and -3.0 \times 10^{-8} \text{ C}, respectively, assumed to be uniformly distributed. If their centers are 2.0 m apart, calculate

(a) the potential of the point halfway between their centers and

(b) the potential of each sphere.

(a) Since the charge is uniformly distributed, the spheres can be considered as point charges. Then we can use the potential for a point charge,

\[ V = \frac{q}{4\pi\varepsilon_0 r} \]

Then the potential halfway between the centers of the spheres is

\[ V = \frac{1.0 \times 10^{-8} \text{ C} \cdot \text{Nm}^2}{4\pi\varepsilon_0 (1.0 \text{ m})^2} - \frac{3.0 \times 10^{-8} \text{ C} \cdot \text{Nm}^2}{4\pi\varepsilon_0 (1.0 \text{ m})^2} = -1.8 \times 10^2 \text{ Nm} \frac{\text{C}}{\text{C}} \]

which is the potential due to both spheres at the halfway point.

(b) To find the potential on a sphere of radius \( r_0 \), consider the sphere as a point charge. Then the potential of the sphere due to the other sphere alone is \( V = \frac{q}{4\pi\varepsilon_0 r_0} \). But on this problem, the potential contribution from the other sphere is not negligible. Hence the total contribution to the potential is, for spheres one and two:

\[ V_1 = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} \quad V_2 = \frac{q_2}{4\pi\varepsilon_0 r_1} + \frac{q_1}{4\pi\varepsilon_0 r_2} \]

Thus

\[ V_1 = \frac{1.0 \times 10^{-8} \text{ C} \cdot \text{Nm}^2}{4\pi\varepsilon_0 (3.0 \times 10^{-2} \text{ m})^2} - \frac{3.0 \times 10^{-8} \text{ C} \cdot \text{Nm}^2}{4\pi\varepsilon_0 (1.0 \text{ m})^2} = 2.9 \times 10^2 \text{ Volts} \]

\[ V_2 = \frac{-3.0 \times 10^{-8} \text{ C} \cdot \text{Nm}^2}{4\pi\varepsilon_0 (3.0 \times 10^{-2} \text{ m})^2} + \frac{1.0 \times 10^{-8} \text{ C} \cdot \text{Nm}^2}{4\pi\varepsilon_0 (1.0 \text{ m})^2} = -8.9 \times 10^3 \text{ Volts} \]

Point breakdown:

- Part a) 10 pts
- Part b) 10 pts
- Part a) 5 pts if you neglected the contribution to the potential from the "other" sphere.
- Part a) 10 pts if you included this contribution.

Thus the total possible on b) was 15 pts.

Points were also lost by significant units, units, and wrong derivations of V.
PROBLEM 4

On a thin rod of length L lying along the x-axis with one end at the origin (x = 0), as shown in the figure, there is distributed a charge per unit length given by \( \lambda = kx \), where \( k \) is a constant.

(a) Taking the electrostatic potential at infinity to be zero, find \( V \) at the point P on the y-axis. P is a distance \( a \) from the origin.

(b) Determine the vertical component, \( E_y \), of the electric field intensity at \( P \) from the result of part (a).

[Solution]

\[
(a) \quad V = \frac{1}{4\pi \varepsilon_0} \int_0^L \frac{\lambda \, dx}{\sqrt{x^2 + a^2}} = \frac{k}{4\pi \varepsilon_0} \int_0^L \frac{2 \, dx}{\sqrt{x^2 + a^2}}
\]

\[
= \frac{k}{4\pi \varepsilon_0} \sqrt{x^2 + a^2} \bigg|_0^L = \frac{k}{4\pi \varepsilon_0} (\sqrt{L^2 + a^2} - |a|).
\]

Here \( a > 0 \), so we have

\[
V = \frac{k}{4\pi \varepsilon_0} (\sqrt{L^2 + a^2} - a) \quad \text{(for } a > 0)\]

\[
(b) \quad E_y = -\frac{\partial V(x)}{\partial y} = -\frac{\partial V(a)}{\partial a} = \frac{k}{4\pi \varepsilon_0} \left( 1 - \frac{a}{\sqrt{L^2 + a^2}} \right).
\]

\[
E_y = \frac{k}{4\pi \varepsilon_0} \left( 1 - \frac{a}{\sqrt{L^2 + a^2}} \right).
\]
10 (a) Calculate the electric potential at point P in the drawing shown.

10 (b) Calculate the electric field, magnitude and direction, at point P. Use the coordinate system shown.

\[ V_P = \frac{1}{4\pi\varepsilon_0} \left( \frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} \right) = \frac{1}{4\pi\varepsilon_0} \left[ \frac{6.5 \times 10^{-6}}{4} - \frac{7.5}{8} + \frac{9.75}{5} \right] = 2.37 \times 10^6 \text{V} \]

\[ E_1 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{r_1^2} = \frac{1}{4\pi\varepsilon_0} \frac{6.5}{16} \times 10^{-6} = 3.65 \times 10^3 \text{ (in x direction)} \]

\[ E_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q_2}{r_2^2} = \frac{1}{4\pi\varepsilon_0} \frac{7.5}{64} \times 10^{-6} = -1.05 \times 10^3 \text{ (in x direction)} \]

\[ E_3 = \frac{1}{4\pi\varepsilon_0} \frac{Q_3}{r_3^2} = \frac{1}{4\pi\varepsilon_0} \frac{9.75}{25} \times 10^{-6} = 3.51 \times 10^3 \text{ (36.9° against x and y)} \]

\[ E_x = E_1 - E_2 - E_3 \cos 36.9° = 1.105 \times 10^5 - 3.65 \times 10^3 - 3.51 \times 10^3 \cos 36.9° = -5.40 \times 10^3 \]

\[ E_y = E_3 \sin 36.9° = 3.51 \times 10^3 \sin 36.9° = 2.11 \times 10^3 \]

\[ E = \sqrt{E_x^2 + E_y^2} = \sqrt{(5.40 \times 10^3)^2 + (2.11 \times 10^3)^2} = 5.796 \times 10^3 \text{ V/m} \]

\[ \theta = \tan^{-1} \frac{E_y}{E_x} = \tan^{-1} \frac{-2.11 \times 10^3}{-5.40 \times 10^3} = -21.3° = 158.6° \]
PROBLEM 3

Three charges are placed at the corners of an equilateral triangle of side \( a \) as shown. At point \( P \), the midpoint of the base find (a) the electric field (magnitude and direction) and (b) the electric potential. [Use the coordinate axes shown to simplify grading. Take the origin at \( P \)]

\[ a) \quad \vec{E} = \left[ \frac{kQ}{(\frac{a}{2})^2} + \frac{2kQ}{(\frac{a}{2})^2} \right] \hat{z} - \left[ \frac{3kQ}{\frac{3}{2}a^2} \right] \hat{x} = \text{the electric field} \]

\[ \vec{E} = \frac{12kQ}{a^2} \hat{z} - \frac{4kQ}{a^2} \hat{y} \]

\[ b) \quad V = \frac{kQ}{(\frac{a}{2})^2} + \frac{-2kQ}{(\frac{a}{2})^2} + \frac{3kQ}{(\frac{3}{2}a)} = \sum \frac{kq}{r} \]

\[ V = \frac{2kQ}{a} \left( \sqrt{3} - 1 \right) \]
Four equal charges are arranged in a square of side $a$, as shown.

a) Find the electric potential at point $A$, which is at the midpoint of the side.

b) Find the work to bring a charge of $-Q$ from infinity to point $A$.

\[ a.) \quad V_A = \frac{kq}{a/2} + \frac{kq}{b} - \frac{kq}{a/2} - \frac{kq}{b} = 0 \]

\[ b.) \quad W = (-Q)V = 0 \]
Four equal charges are arranged in a square of side $a$, as shown.

a) Find the electric potential at point $A$, which is at the mid-point of the side.

b) Find the work to bring a charge of $-Q$ from infinity to point $A$.

\[
\text{a.) } V_A = \frac{kq}{a/2} + \frac{kq}{b} - \frac{kq}{a/2} - \frac{kq}{b} = 0
\]

\[
\text{b.) } W = (-Q)V = 0
\]
Given a non-conducting rod of charge, whose charge density is \( +\lambda \) C/m, which begins at \( x = 0 \) and extends to infinity along the positive x axis. Calculate the electric field at the point \( x = -a \). (Include its direction.)

1500 pts is integral
5 integral
5 direction

\[ E = \frac{\lambda}{4\pi\epsilon_0} \int_0^\infty \frac{2dx}{(ax)^2} (-x) \]

Let \( u = ax \),
\[ du = adx \]

\[ E = \frac{\lambda}{4\pi\epsilon_0} \left[ -\frac{1}{u} \right]_a^\infty = \frac{\lambda}{4\pi\epsilon_0} \left[ -\frac{1}{a} \right] \]

\[ E = \frac{\lambda}{4\pi\epsilon_0 a} (-2) \]
Problem 3

A cylindrical nonconductor has a charge distribution given by a volume charge density expressed by \( \rho = \rho_0(1 - \alpha r^2) \) for \( R < R_0 \), \( \rho = 0 \), for \( R > R_0 \) (\( \alpha \) is a constant).

(a) Find the magnitude of the electric field at \( P \), a distance \( R \) from the center of the cylinder. State its direction in words.

(b) Find the electric field at a radius \( R = R_0/4 \).

\begin{align*}
\text{a) for the Gaussian surface drawn,} \\
\oint \vec{E} \cdot d\vec{s} &= Q_{\text{enclosed}} \ \\
\text{Because of symmetry, } \oint \vec{E} \cdot d\vec{s} &= 2\pi RL \cdot E \\
Q_{\text{enc}} &= \int \rho dV = \int_0^{R_0} \rho_0 (1 - \alpha R^2) \cdot 2\pi R dR \\
&= 2\pi \rho_0 L \left( \frac{R_0^2}{2} - \alpha \frac{R_0^4}{4} \right) \\
\text{So } E &= \frac{\rho_0}{\epsilon_0 R} \left( \frac{R_0^2}{2} - \alpha \frac{R_0^4}{4} \right) \\
&\text{Direction: radially outward if } 2R_0^2 > \alpha R_0^4
\end{align*}

\begin{align*}
\text{b) Draw cylindrical Gaussian surface at } R = \frac{R_0}{4} \text{ (inside charge). Then } \oint \vec{E} \cdot d\vec{s} &= Q_{\text{enc}}. \\
\text{By symmetry, } \oint \vec{E} \cdot d\vec{s} &= 2\pi (L/4) \cdot E. \\
\text{Further, } Q_{\text{enc}} &= \int_0^L \rho dV = 2\pi \rho_0 L \left( \frac{R_0^2}{4} - \alpha \frac{R_0^4}{45} \right) \\
\text{So } E &= \frac{\rho_0}{\epsilon_0} \left( \frac{R_0^2}{8} - \alpha \frac{R_0^4}{45} \right)
\end{align*}
Given a long cylinder of a non-conducting material. This cylinder is charged with \( \lambda = 2.75 \times 10^{-8} \text{ C/m}. \) The volume charge density is given by

\[
\rho = \rho_0 (1 - aR^{3/2})
\]

where the charge density goes to zero at \( R = R_0 = 3.00 \text{ cm}, \) the outside surface of the rod. If \( a \) and \( \rho_0 \) are constants, find \( \lambda = 0.03 \text{ m}. \)

(a) Numerical values for \( a \) and \( \rho_0 \).

(b) The electric field at \( R = 3/4 R_0 \).

(c) The electric potential difference between the center and outside surface of the rod.

Define \( L = \text{arbitrary length of cylinder}, \) it will cancel out but makes the integrals easier to see.

\[
\rho(L) = 0 \implies \lambda = \frac{1}{R_0^{3/2}} = 192.5 \text{ m}^{-3/2}
\]

\[
\lambda L = \int_0^L \rho \, d\text{volume}
\]

\[
d\text{volume} = 2\pi R \, dR
\]

\[
\lambda L = \int_0^L \rho \, 2\pi R \, dR
\]

\[
\lambda = \int_0^{L_0} 2\pi R \, (1 - \kappa L^{3/2}) \, dR = 2\pi \rho_0 L_0^{5/2} \left( \frac{2}{3} - \frac{3}{5} \right)
\]

\[
\rho_0 = \frac{2\lambda_0}{3\pi R_0^5} = 2.27 \times 10^{-5} \text{ C/m}^3
\]

\[
\lambda = 193 \text{ m}^{-3/2}, \quad \rho_0 = 2.27 \times 10^{-5} \text{ C/m}^3
\]

5 pts each
b) Gauss’ Law: \[ \sum E \cdot dA = \frac{\rho_{net}}{\varepsilon_0} \]

**Gaussian Surface:** cylinder of radius \( r \), concentric with charged cylinder, of length \( L \)

\[
\int E \cdot dA = \int_E E \cdot dA + \int_o^0 E \cdot dA = E(2\pi r L) \cos \theta + \int_o^0 E \cdot dA \\
E(2\pi r L) = \frac{\rho_{net}}{\varepsilon_0}
\]

\[
\varepsilon_0 \cdot dA = \int_o^r \rho \cdot d(x l) = \int_o^r \rho_o \cdot [1 - \left( \frac{x}{r} \right)^2] \cdot 2\pi r dr \\
E(2\pi r L) = \frac{2\pi \rho_o L \left[ \frac{x^2}{2} - \frac{x^3}{3} \right]}{\varepsilon_0}
\]

\[
\Rightarrow E(r) = \frac{\rho_o}{\varepsilon_0} \left[ \frac{x^2}{2} - \frac{x^3}{3} \right]
\]

\[
E(\frac{2r}{3}) = 1.81 \times 10^4 \text{ V/m}
\]

c) \[ dV = \int_0^L \int_o^r E \cdot dA = \int_o^\infty E(r) dr \cos \theta \]

\[
dV = \frac{\rho_o}{\varepsilon_0} \int_o^\infty \frac{\left[ \frac{x^2}{2} - \frac{x^3}{3} \right]}{\varepsilon_0} dx = \frac{\rho_o}{\varepsilon_0} \left[ \frac{1}{2} - \frac{1}{3} \right]
\]

\[
dV = 3.89 \times 10^{-2} \text{ V}
\]

---

**Major Problems:**

1) Time - the charge will account for this.
2) \( \rho = 0 \), this makes no sense. -1 pt
3) \( \alpha \) given numerically as \( \frac{1}{\alpha} \), -1 pt
4) units: significant figures -2 pt/each time.
5) Volume integrals - draw a picture of your differential volume, this makes it easier to see. Some people had their d(volume) with dimensions of length or area - this is wrong.
6) Rolling formulas off the back of that did not apply.
SECOND MIDTERM

Name (Print) Mengzhi Luo        Name (Signed) Av. 13.9

Discussion Instructor (Circle One): Bertolina Hari Jaw Krantz
Discussion Section #: Lakner McDonald Follard

SHOW ALL WORK!!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Given an electric dipole consisting of a charge $q = +3.75 \mu C$ a distance $2a$ from a charge of $q = -3.75 \mu C$. Take $a = 1.00 \text{ mm}$. Use the binomial theorem to calculate the electric potential at a distance $R$ of 3.75 m from the center of the dipole along the axis of the dipole.

Step 1:

\[
U = \frac{kq^2}{R+a} + \frac{k(-q)}{R-a}
\]

Step 2:

\[
= k\epsilon \left[ \frac{R-a}{R^2} - \frac{R-a}{a^2} \right] = -2ak^2 \left[ 1 - \left(\frac{a}{R}\right)^2 \right]^{-1}
\]

\[
= -\frac{2ak^2}{R^2} \left[ 1 + (-1)(\frac{a^2}{R^2}) + (-1)(-2)(\frac{a^2}{R^2})^2 + \cdots \right]
\]

\[
= -\frac{2aak^2}{R^2} \left[ 1 + \frac{a^2}{R^2} + \cdots \right]
\]

\[
= -\frac{2aak^2}{R^2} = -2 \left(9 \times 10^9\right) (3.75 \times 10^{-6}) x (0.01)
\]

\[
= -4.80 \text{ (V)}
\]

or:

\[
U = \frac{kq^2}{R+a} + \frac{k(-q)}{R-a}
\]

\[
= \frac{k\epsilon}{R} \left[ (1 + \frac{a}{R})^{-1} - (1 - \frac{a}{R})^{-1} \right]
\]

\[
= \frac{k\epsilon}{R} \left[ (1 + \frac{a}{R})^{-1} - (1 - \frac{a}{R})^{-1} \right]
\]

\[
= \frac{k\epsilon}{R} \left[(1 - \frac{a}{R}) + \frac{(-1)(-2)(\frac{a^2}{R^2})}{2} + \frac{(-1)(-2)(-3)(\frac{a^3}{R^3})}{2 \cdot 3} + \cdots \right]
\]

\[
= -\frac{2aak^2}{R^2} \left[ 1 + \frac{a^2}{R^2} + \cdots \right]
\]

\[
= -4.80 \text{ (V)}
\]
Grading explanation:

In Problem 2, if one had not used the binomial theorem, one lost 15 points, even though one can get the correct numerical answer.

Example:

\[ U = \frac{kq^2}{R+a} - \frac{kq^2}{R-a} \]

\[ = kq^2 \left[ \frac{1}{R+a} - \frac{1}{R-a} \right] \]

\[ = 9 \times 10^9 \times 3.75 \times 10^{-6} \left[ \frac{1}{3.751} - \frac{1}{3.749} \right] \]

\[ = -4.8 \text{ (V)} \]
Consider a cylinder of non-conductor whose radius is $R_0$ and whose length is infinite with a volume charge density $\rho = AR^2$ where $R$ is the distance from the center of the cylinder. Calculate the electric potential difference between a point $P$ and the surface of the cylinder. $P$ is at a value of $R = \frac{2}{3} R_0$. Assume the cylinder is positively charged and state clearly the sign of the potential difference $V(P) - V(R_0)$.

By Gaussian law,

$$\int \mathbf{E} \cdot d\mathbf{a} = \frac{Q}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int \rho \, d\text{vol}. $$

Then,

$$E \cdot 2\pi RL = \frac{1}{\varepsilon_0} \int_0^{R_0} 2\pi r \rho r L \, dr \quad \text{where } L: \text{length}$$

$$= \frac{1}{\varepsilon_0} 2\pi AK^2 \frac{R_0^4}{4}$$

$$\therefore E = \frac{AR_0^3}{4\varepsilon_0}$$

Therefore, the potential difference is

$$V = -\int_{R_0}^P \frac{AR_0^3}{4\varepsilon_0} dR = \frac{A}{4\varepsilon_0} \left[ \frac{R^4}{4} \right]_{R = \frac{2}{3} R_0}^{R_0}$$

$$= \frac{A}{16\varepsilon_0} R_0^4 \left[ 1 - \left( \frac{2}{3} \right)^4 \right] = \frac{65}{1296} \frac{A}{\varepsilon_0} R_0^4 = 0.060 \frac{A}{\varepsilon_0} R_0^4$$

Clearly, the sign of $V(P) - V(R_0)$ is a positive (+). (5)
Use the binomial theorem to calculate the electric potential at a point $P$, a distance $R$ from the center of an electric quadrupole. The quadrupole has charges $+Q$ at distances $\pm a$ from a charge $-2Q$. The point $P$ is on the axis of the quadrupole. Assume $a \ll R$.

Calculate the first two nonzero terms involving $a$.

\[
V = \frac{kQ}{R+a} + \frac{kQ}{R-a} - \frac{2kQ}{R} \quad (5 \text{ pb})
\]

\[
= kQ \left[ \frac{1}{R+a} + \frac{1}{R-a} - \frac{1}{R} \right] 
\]

\[
= \frac{kQ}{R} \left[ \frac{1}{1+\frac{a}{R}} + \frac{1}{1-\frac{a}{R}} - 1 \right] \quad (5 \text{ pb})
\]

\[
= \frac{kQ}{R} \left[ 1+\left(\frac{a}{R}\right)^2 + \left(\frac{a}{R}\right)^4 + \cdots \right] + \frac{kQ}{R} \left(\frac{a}{R}\right)^2 + \left(\frac{a}{R}\right)^4 + \cdots - \frac{kQ}{R} 
\]

\[
= \frac{kQ}{R} \left[ 2 \left(\frac{a}{R}\right)^7 + 2 \left(\frac{a}{R}\right)^9 + \cdots \right] = 2kQa \left[ (\frac{a}{R})^7 + (\frac{a}{R})^9 + \cdots \right]
\]

\[
= \frac{2kQa^2}{R^3} \left[ 1 + (\frac{a}{R})^2 + \cdots \right] \quad (5 \text{ pb}).
\]
A sphere of nonconductor is negatively charged with a charge density given by $\rho = AR^2$, where $A$ is a constant. The radius of the sphere is $R_0$.

(a) [15 pts.] Calculate the magnitude of the electric potential difference between the surface of the sphere and a point $P$, a distance $R$ from the center. ($R < R_0$).

(b) [5 pts.] State the sign of the potential difference $V(R) - V(R_0)$ and give a clear physical (not mathematical) reason for this sign.

(c) [5 pts.] If the total charge on the sphere is $Q$, calculate $A$.

\[ Q(R) = \int_0^R \rho \, dV = \int_0^R \rho \, 4\pi r^2 \, dr = \int_0^R Ar^3 \, dr = \frac{4}{5}AR^5 \]

\[ E(R) = \frac{Q(R)}{4\pi \varepsilon_0 R^2} = k \cdot \frac{2\pi AR^6}{3R^3} = \left(\frac{\varepsilon_0 k}{3}\right)R^3 = \left(\varepsilon_0 k\right)R^3 \]

\[ V(R) - V(R_0) = \left[-\int_R^{R_0} E(R) \, dR\right] = \left[-\frac{A}{\varepsilon_0} \cdot \int_R^{R_0} \frac{dR}{R^2}\right] = \frac{A}{2\varepsilon_0} \left[ R_0^2 - R^2 \right] \]

(b) From (a): \( A < 0 \) (negatively charged) \& \( R_0 - R < 0 \) \( R < R_0 \)

\[ \frac{A(R_0^2 - R^2)}{2\varepsilon_0} > 0 \]

\[ V(R_0) - V(R) > 0 \]

\[ V(R) - V(R_0) < 0 \]

From (c): $Q(R_0) = \frac{2\pi A R_0^6}{3} = Q \Rightarrow A = \frac{3Q}{2\pi R_0^6}$
Consider a slab of nonconductor with thickness d in the y direction extending to infinity in the x and z directions. The slab has a uniform charge density \( \rho \), where \( \rho = 1.45 \times 10^{-8} \) C/m\(^2\). If \( d = 1.25 \) m and the midpoint is at \( y = 0 \), find the electric potential difference between \( y = 0 \) and \( y = +0.45 \) m.

\[
\mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}
\]

\[
2AE = \frac{2yA\rho}{\varepsilon_0} \Rightarrow E = \frac{y\rho}{\varepsilon_0}
\]

\[
V(0) - V(0.45) = -\int_{0}^{0.45} E\,dy
\]

\[
= \frac{y^2\rho}{2\varepsilon_0} \bigg|_{0}^{0.45}
\]

\[
= 1.64 \times 10^4 \text{ (V)}
\]
SECOND MIDTERM

Name (print)  Mark Reeve  Name (signed)  Mark Reeve

Discussion Instructor (circle one): Baselgia  Morrill  Reeve  Stoops  Zhang

Discussion Section #

SHOW ALL WORK!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Given a very long, thin wire of radius r and a charge density of \( \lambda \) c/m.

(a) Calculate the electric potential difference between the surface of the wire, and a point at a
distance R away from the center of the wire where R \( > r \).

(b) State clearly the sign of the potential difference, \( V(R) - V(r) \), and give a physical reason for it.

**20 pts**

(a) First calculate the electric field a distance R\( \geq r \) from the center of the wire.

Since the electric field is purely radial for a sufficiently long wire, it is constant on the surface
of the cylinder, and hence may be removed from the flux integral.

\[
E = \frac{\lambda}{2\pi \epsilon_0} R
\]

Now integrate the field to get the electric potential:

\[
V = -\int E \cdot ds = -\frac{\lambda}{2\pi \epsilon_0} \ln \frac{R}{r}
\]

**5 pts**

(b) Since \( R > r \), \( \ln \frac{R}{r} > 0 \), \( V(R) - V(r) \) is negative.

Why? Since the wire is positively charged, the electric field is radially outward:

\[
E = \ \text{radially outward}
\]

Since work is equal to the change in potential energy, \( W = U_f - U_i > 0 \)

\[
\Rightarrow V(r) > V(R) \Rightarrow V(R) - V(r) < 0.
\]

One can also think of it in terms of just potentials; as one approaches a positive
change, the potential due to that charge increases (\( V \propto \frac{1}{r} \) for a point charge).

\[
\Rightarrow V(R) - V(r) < 0.
\]

**Note:** Several people (all unsuccessfully) attempted part (a) by a direct calculation of the potential.

This is the way to do it: Let the wire be of length \( 2L \) with \( L \gg R, r \).

\[
\frac{dV}{dx} = \frac{2\lambda}{(x^2 + l^2)^{3/2}} \Rightarrow \int dx = \frac{2L}{x^2 + l^2}
\]

Then \( V(R) - V(r) = \frac{2\lambda}{L} (\ln \frac{R}{r} - \ln \frac{r}{r}) \approx -2\lambda \ln \frac{R}{r} \) before
SECOND MIDTERM

A sphere of nonconductor has a charge distribution given by

$$\rho = \rho_0 (1 - \alpha R^3)$$

for $R < R_o$, where $R_o$ is the radius of the sphere. $\rho = 0$ for $R > R_o$. The total charge on the sphere is $Q$. The value of $\rho$ goes to zero at $R = R_o$.

(a) Calculate the electric field at any point within the sphere, a distance $R$ from its center.
(b) Assume that $Q$ and $R_o$ are known. Find a formula for $\alpha$.
(c) Assume that $Q$ and $R_o$ are known. Find a formula for $\rho_0$.
(d) Calculate the magnitude of electric potential difference between $R_o$ and $R$ (inside the sphere) assuming the charge is positive.

(a) 7 points

Using Gauss's Law $E \cdot A = \frac{\rho}{\varepsilon_0}$

$$4\pi R^2 E = \frac{1}{\varepsilon_0} \int_0^R \rho_0 (1 - \alpha R^3) 4\pi R^2 dR$$

$$= \frac{4\pi \rho_0}{\varepsilon_0} \left[ \frac{R^3}{3} - \frac{\alpha R^6}{6} \right]_0^R$$

that is $E = \frac{\rho_0 R}{3\varepsilon_0} (1 - \frac{\alpha}{2} R^3)$ radially outward

(b) 4 points

Since the value of $\rho$ goes to zero at $R = R_o$.

therefore $\rho_0 (1 - \alpha R_o^3) = 0$ $\Rightarrow$ $\alpha = \frac{1}{R_o^3}$

(c) 4 points

$Q = \int_0^{R_o} \rho dV = 4\pi \int_0^{R_o} \rho_0 (1 - \alpha R^3) R^2 dR$

$$= \frac{4\pi \rho_0}{3} (R_o^3 - \frac{\alpha}{2} R_o^6)$$

Plug in $\alpha = \frac{1}{R_o^3}$, can get $\rho_0 = \frac{3Q}{2\pi R_o^3}$

(d) 10 points

$\Delta V = V(R) - V(R_o) = -\int_{R_o}^R \vec{E} \cdot d\vec{r}$

$$\Delta V = -\frac{\rho_0}{3\varepsilon_0} \int_{R_o}^R (R - \frac{\alpha}{2} R^4) dR = \frac{\rho_0}{3\varepsilon_0} \left[ \frac{R^2}{2} - \frac{\alpha}{10} R^5 \right]_{R_o}^R$$

$$= \frac{\rho_0}{3\varepsilon_0} \left[ \frac{R_o^2 - R^2}{2} - \frac{\alpha}{10} (R_o^5 - R^5) \right]$$

$$= \frac{\rho_0}{3\varepsilon_0} \left[ \frac{R_o^2 - R^2}{2} - \frac{1}{32 R_o^5} (R_o^5 - R^5) \right] \frac{32}{32}$$
SECOND MIDTERM

Name (print) Mario I Molina Name (signed) 

Discussion Instructor (circle one): Davis DeTienne Hamed Molina Paul Zhang 

Discussion Section 

SHOW ALL WORK!!!! REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES! Use the conversion constants and data given on the front page.

Four charges are arranged in a line as shown. Calculate the electric potential at P, in volts, for the values of charges and a given. 

\[ Q_1 = +2.00 \times 10^{-6} \text{ C} \]
\[ Q_2 = -3.25 \times 10^{-6} \text{ C} \]
\[ Q_3 = +4.25 \times 10^{-6} \text{ C} \]
\[ Q_4 = -8.20 \times 10^{-6} \text{ C} \]
a = 3.00 cm

\[ V(p) = \frac{kQ_1}{\sqrt{a}} + \frac{kQ_2}{a} + \frac{kQ_3}{\sqrt{a}} + \frac{kQ_4}{\sqrt{a}} \]

\[ = \frac{k}{a} \left[ \frac{Q_1}{\sqrt{a}} + \frac{Q_2}{a} + \frac{Q_3}{\sqrt{a}} + \frac{Q_4}{\sqrt{a}} \right] \]

\[ = -7.49 \times 10^5 \text{ (V)} \]
SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES:
Use the conversion constants and data given on the front page.

A rod of nonconductor has a charge density given by \( \lambda = +\alpha / 2 \).
The charge is positive on either side of \( x = 0 \). \( \alpha \) is a constant.
Calculate the electric potential at point \( P \), a distance \( a \) away from the midpoint of the rod. Take the midpoint as \( x = 0 \).

\[ x_1 = -L/2 \]
\[ x_2 = +L/2 \]

\[ \begin{align*}
\delta V &= \frac{k \lambda dx}{\sqrt{x_1^2 + x^2}}
= \frac{k \lambda dx}{\sqrt{x^2 + x_1^2}} \\
\int \delta V &= 2 \int_0^{L/2} \frac{k \alpha x}{\sqrt{x^2 + x_1^2}} dx \\
V &= 2k\alpha \int_0^{L/2} \sqrt{\frac{x_1^2 + x^2}{x^2 + x_1^2}} dx \\
V &= 2k\alpha \left( \sqrt{\frac{x_1^2 + x^2}{x^2} - 1} \right)
\end{align*} \]
SECOND MIDTERM

Given a long hollow tube of inner radius \( R_A \) and outer radius of \( R_B \). The tube is made of a nonconductor and has a charge density of \( \rho = AR^3 \) between \( R_A \) and \( R_B \). \( \rho \) is zero everywhere else.

(a) Calculate the electric field at an arbitrary distance \( R \) from the axis of the tube where \( R_A < R < R_B \).

(b) If the potential at \( R_A \) is set at \( V = 0 \), calculate the potential difference between a point at \( R_A \) and a point at \( R \) where \( R_A < R < R_B \).

\[
\text{(a)} \quad \frac{q_{\text{enc}}}{\varepsilon_0} = \frac{d}{\varepsilon_0} \int_{R_A}^{R} 2\pi y \, dy \cdot \frac{A}{B} \gamma^3
\]

\[
\text{(b)} \quad V(R) = - \int_{R_A}^{R} E(y) \, dy = -\frac{A}{\varepsilon_0} \int_{R_A}^{R} \frac{1}{y} \left( \frac{1}{R_A} - \frac{1}{R} \right) \, dy
\]

\[
E(R) = \frac{A}{\varepsilon_0 R} \left( \frac{1}{R_A} - \frac{1}{R} \right)
\]

\[
V(R) = -\frac{A}{\varepsilon_0 R_A} \ln \frac{R}{R_A} + \frac{A}{\varepsilon_0} \left( \frac{1}{R_A} - \frac{1}{R} \right)
\]
SECOND MIDTERM

Name (print) Prabasaj Paul

Discussion Instructor (circle): Condella Godfrey-Smith Guilkey Leong Nott Paul

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!

Use the conversion constants and data given on the front page.

(a) If an electric potential has the form \( V = -Ax^4y^4 \), calculate the \( y \) component of the electric field, including sign, at the point \( x = +5.00 \text{ m}, y = +4.00 \text{ m}, z = +1.00 \text{ m} \).

\[
E_y = -\frac{\partial V}{\partial y} = 4Ax^3y^3 = (3.20 \times 10^4) \text{ N/C}
\]

(b) There are two protons in the nucleus of a helium atom. If they are initially \( 1.00 \times 10^{-15} \text{ m} \) apart and released, how much kinetic energy would each have after they have moved \( 1.00 \text{ m} \) apart?

\[
\frac{1}{2}m_pv^2 = ke^2 \left( \frac{1}{r} - \frac{1}{2r} \right) \Rightarrow \frac{1}{2}m_pv^2 = 1.152 \times 10^{-13} \text{ J}
\]

(c) If a conducting sphere of radius \( 4.00 \text{ cm} \) is charged to a potential of \( 350 \text{ V} \), calculate the surface charge density on the sphere.

\[
\sigma = \frac{\varepsilon_0 V}{r} = \frac{4\pi \sigma^2 r^2}{4\pi \varepsilon_0} = \sigma \varepsilon_0 \Rightarrow \sigma = \varepsilon_0 \frac{V}{r} = 7.74 \times 10^{-8} \text{ C/m}^2
\]

(d) Three charges are arranged as shown. Calculate the electric potential at point P.

\[
Q_1 = +3.00 \times 10^4 \text{ C}, \quad Q_2 = -4.70 \times 10^4 \text{ C}, \quad Q_3 = +8.50 \times 10^4 \text{ C}
\]

\[
V = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} + \frac{kQ_3}{r_3} = 3.14 \times 10^6 \text{ V}
\]

(e) If a conducting rod \( 6.50 \text{ m} \) in length has a charge of \( +7.50 \times 10^4 \text{ C} \) uniformly distributed over its surface, calculate the electric field at the midpoint of the rod, and \( 2.20 \text{ cm} \) from the axis of the rod. The rod has a radius of \( 0.500 \text{ cm} \).

\[
2\pi r_L \varepsilon = \frac{Q}{\varepsilon_0} \quad (\because L = 6.5 \text{ m} \Rightarrow r = 2.2 \text{ cm})
\]

\[
E = \frac{2kQ}{r_L} = 9.44 \times 10^3 \text{ N/C}
\]
SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A charged, nonconducting sphere has a charge density given by \( \rho = \rho_0 (1 - Ar^6) \), where \( \rho_0 \) and \( A \) are constants, the total charge on the sphere is \( 3.30 \times 10^6 \) C, the radius is 4.00 cm and the charge density goes to zero at its surface.

(a) Calculate the numerical values, with proper units, for \( \rho_0 \) and \( A \).
(b) Find the electric potential at the surface of the sphere (numerical value).
(c) What is the electric field at a point 3.00 cm from the center of the sphere (numerical value)?

\[ a) \quad \rho = 0 \quad \Rightarrow \quad 1 - Ar^6 = 0 \quad \Rightarrow \quad A = \frac{1}{(0.04)^6} = 9.76 \times 10^6 \text{ V/m}^2 \]

\[ Q_{tot} = 3.30 \times 10^6 \text{ C} = \int_0^{a_{tot}} \rho dV \quad \Rightarrow \]

\[ Q_{tot} = \int_0^{a_{tot}} \rho_0 (1 - Ar^6) 4\pi r^2 dr \quad \Rightarrow \quad \rho_0 = \frac{Q_{tot}}{4\pi} \left( \frac{1}{\frac{c}{3} - Ar^6} \right)_{a_{tot} = 0.04} \]

Potential at surface \( r = 0.04 \text{ m} \)

\[ \rho_0 = 1.97 \times 10^2 \text{ C/m}^2 \]

\[ b) \quad V = k \frac{Q_{tot}}{r} = 7.42 \times 10^5 \text{ V} \]

\[ \frac{d}{r} \hat{E} \cdot dA = \frac{q_{in}}{\varepsilon_0} \quad \Rightarrow \quad \hat{E} = \frac{q_{in}}{4\pi \varepsilon_0 r^2} = \frac{\rho_0}{\varepsilon_0} \left( \frac{c^3}{3} - Ar^6 \right) \]

\[ q_{in} = \rho_0 \int_0^{a_{tot}} (1 - Ar^6) 4\pi r^2 dr \]

\[ q_{in} = 4\pi \rho_0 \left( \frac{c^3}{3} - Ar^6 \right) \]

**Grading:**

- a) \( A = ? \) 5 pts.
- \( \rho_0 = ? \) 5 pts.
- b) \( V = ? \) 5 pts.
- c) \( q_{in} = \) 3 pts.
- \( q_{in} = ? \) 5 pts.
- Putting it together 2 pts.
Four points are at the corners of a square, as shown.

(a) For the values of the charges given, calculate the electric potential at P.
(b) For the values of the charges given, find the work needed to bring an electron from far away to P.
(c) For the values of the charges given, find the direction of the electric field at P, measured as an angle counter-clockwise from the positive x axis.

\[ Q_1 = +2.70 \times 10^{-7} \text{ C} \]
\[ Q_2 = -4.00 \times 10^{-6} \text{ C} \]
\[ Q_3 = +1.00 \times 10^{-7} \text{ C} \]
\[ a = 4.25 \text{ mm} = 0.00425 \text{ m} \]

\[ V = \sum \frac{kQ_i}{r_i} = \frac{kQ_1}{a} + \frac{kQ_3}{a} + \frac{kQ_2}{\sqrt{2}a} \]
\[ = \frac{k}{a} \left[ Q_1 + Q_3 + \frac{Q_2}{\sqrt{2}} \right] = -5.21 \times 10^6 \text{ V} \]

(b) \[ W = Vq = V \cdot e = 8.33 \times 10^{-13} \text{ J} \]

c) \[ E = \frac{kQ}{r^2} \]

\[ E_x = \left( \frac{kQ_1}{a^2} + \frac{kQ_2}{2a^2} \cdot \frac{a}{\sqrt{2}a} \right) \hat{i} = -5.70 \times 10^8 \text{ V/m} \hat{i} \]
\[ E_y = \left( \frac{kQ_3}{a^2} + \frac{kQ_2}{2a^2} \cdot \frac{a}{\sqrt{2}a} \right) \hat{j} = -6.55 \times 10^8 \text{ V/m} \hat{j} \]

\[ \theta = \tan^{-1} \frac{E_y}{E_x} = 48.9^\circ \quad -2 \text{ if } \tan^{-1}(\text{not computed}) \]
\[ \theta = \theta' + 180^\circ = 228.9^\circ \quad -2 \text{ if wrong} \]
\[ \theta = \theta' + 180^\circ = 228.9^\circ \quad -2 \text{ if not done} \]
A very long cylinder of non-conductor of radius $R_0$ has a charge density given by $\rho = AR^2$ for $R < R_0$, and $\rho = 0$ for $R > R_0$. $A$ is a constant.

(a) Using Gauss' Law, calculate the magnitude of the electric field at an arbitrary point within the cylinder a distance $R$ from the cylinder axis.

(b) Calculate the magnitude of the potential difference between the wall of the cylinder and its axis $\Delta V(V(R_0) - V(0))$.

(c) If the sign of the charge on the cylinder is negative, state clearly the sign of $V(R_0) - V(0)$, and give a physical reason for it.

(a) Gauss Law: $\oint E \cdot dA = \frac{q}{\varepsilon_0}$

$\oint E \cdot dA = E \cdot 2\pi R \cdot L$

$Q = \int_0^R \rho \cdot 2\pi r \cdot E \cdot dr = 2\pi A L \int_0^R r^3 \cdot dr = 2\pi A L \frac{R^4}{4}$

$E = 2\pi A L \frac{R^4}{4} \cdot \frac{1}{\varepsilon_0}$

$E = \frac{AR^3}{4\varepsilon_0}$

(b) $\Delta V(V(R_0) - V(0)) - \int_0^{R_0} E \cdot dL = -\int_0^{R_0} \frac{AR^2}{4\varepsilon_0} \cdot dr = -\frac{AR_0^4}{16\varepsilon_0}$

(c) $\rho < 0$, the direction of $E$ is to the center of the cylinder. Positive work has to be done to move positive charge from $R=0$ to $R=R_0$, therefore $V(R_0) > V(0)$, $V(R_0) - V(0) > 0$. 

...
Second Midterm

Name (print)    Walker    Name (signed)    

Discussion Instructor (circle): Brown    Chakhbazian    Condella    Parnori    Zhukov

Discussion Section #    

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES! 
Use the conversion constants and data given on the front page.

(a) A wire 27.0 m long carries a total charge of $+3.00 \times 10^{-12}$ C. If the charge is uniformly distributed (not really true), calculate the electric field 2.00 cm from the center of the wire and at its midpoint. The radius of the wire is less than 2.00 cm.

\[ \phi E \cdot dA = \frac{Q_{+}}{2 \pi \epsilon_{0}} \Rightarrow E_{2 \text{cm}} = \frac{2Q_{+}}{2 \pi \epsilon_{0}} \Rightarrow E = \frac{2Q_{+}}{2 \pi \epsilon_{0}} = 1.1 \text{ N/C} @ 2 \text{cm} \]

\[ E = 0 \text{ N/C} @ \text{center} \]

(b) A proton ($Q = +1.60 \times 10^{-19}$ C) is accelerated from rest through a potential difference of 75.0 volts. Calculate its velocity.

\[ eV = \frac{1}{2}mv^{2} \Rightarrow v = \sqrt{\frac{2eV}{m}} = 1.19 \times 10^{5} \text{ m/s} \]

(c) An electric potential is described by $V = Axy^3z^2$. Calculate the y component of the electric field at $x = 2.00 \text{ m}$, $y = 3.00 \text{ m}$ and $z = 1.50 \text{ m}$. $A$ is a constant.

\[ E_{y} = -\frac{\partial V}{\partial y} = -A \times 3y^2 z^2 = -121.5 \text{ A/V/m} \]

(d) Calculate the electric potential at the center of the square (A) for the array of charges shown.

\[ \sum \sqrt{\frac{Q_{i}}{\epsilon_{0}}} = \frac{4 \sqrt{Q_{A}}}{\epsilon_{0}} \]

(e) The electric field just above the surface of conducting sphere is $E = +975 \text{ N/C}$. Calculate the charge density on the surface of the sphere.

\[ E_{x} = \frac{\sigma}{\epsilon_{0}} \Rightarrow \sigma = E \epsilon_{0} = 9.63 \times 10^{-6} \text{ C/m}^2 \]
In the charge distribution shown, P is directly in line with the charges and is a distance R as shown.

(a) Find an exact expression for the electric potential at P.

(b) Use the binomial expansion to determine the approximate value for the potential at P keeping only the first non-zero term in the answer involving a. (Assume R >> a.)

\[ V = \frac{kQ}{R} \]

\[ V = k \left( \frac{Q}{R-a} - \frac{2Q}{R} + \frac{Q}{R+a} \right) \]

The required exact eq.

b) Using binomial expansion

\[(1+a)^{-1} = 1-a + a^2 - a^3 + \ldots\]

\[ \frac{1}{R-a} = \frac{1}{R} \left(1 - \frac{a}{R}\right)^{-1} = \frac{1}{R} \left(1 + \frac{a}{R} + \frac{a^2}{R^2} + \frac{a^3}{R^3} + \ldots\right) \]

\[ \frac{1}{R+a} = \frac{1}{R} \left(1 + \frac{a}{R}\right)^{-1} = \frac{1}{R} \left(1 - \frac{a}{R} + \frac{a^2}{R^2} - \frac{a^3}{R^3} + \ldots\right) \]

Since \(a \ll R\), \(a \ll 1\) the higher power terms are not important and we can ignore them.

plug into (1)

\[ V = k \frac{Q}{R} \left(1 + \frac{a^2}{R^2} + \frac{a^3}{R^3} + \ldots\right) \]

The required approximate eq.

\[ V = kQ \frac{2a^2}{R^3} \]
Two conducting spheres are connected with a long wire. Sphere 1 has a radius of \( R = 12.0 \text{ cm} \) and sphere 2 has a radius of \( r = 4.00 \text{ cm} \). A charge of 30.0 pC is placed on the system.

(a) What is the charge in picocoulombs on sphere 1?
(b) Find the potential of sphere 1 in volts.

\[
V_1 = \frac{kQ_1}{R} \quad V_2 = \frac{kQ_2}{r}
\]

\[
V_1 = V_2 \quad \Rightarrow \quad \frac{Q_1}{R} = \frac{Q_2}{r}
\]

now, \( Q_1 + Q_2 = Q \)

\[
\Rightarrow \quad Q_1 = Q \frac{R}{R+r} \quad Q_2 = Q \frac{r}{R+r}
\]

20 (a) \( Q_1 = 30 \text{ pC} \cdot \frac{12}{12+4} = 22.5 \text{ pC} \)

5 (b) \[
V_1 = 9 \cdot 10^9 \text{ V} \cdot \frac{22.5 \cdot 10^{-12} \text{ C}}{0.12} = 1.69 \text{ V}
\]
A spherical charge density is modeled as \( \rho = Ar^2 \), where \( A \) is a constant and \( r \) the distance from the center of the sphere. The radius of the sphere is \( R_0 \). (The sign of the charge is included in \( A \).)

(a) If we take \( V = 0 \) at \( r = 0 \) (the center of the sphere), find the potential at the point \( r = R_0/3 \).

(b) If the total charge of the sphere is \( Q_T \), find the value of \( A \) in terms of \( R_0 \), \( Q_T \) and \( \varepsilon_0 \), as needed.

\[ \rho = Ar^2 \]

\[ \Delta V = -\iiint \mathbf{E} \cdot d\mathbf{r} \Rightarrow \text{calculate } \mathbf{E} \text{ from Gauss' law:} \]

\[ \mathbf{E}(r) = \frac{\rho dV}{4\pi r^2 \varepsilon_0} = \frac{4\pi A \int r^4 dr}{4\pi r^2 \varepsilon_0} \]

\[ = \frac{Ar^3}{5\varepsilon_0} \]

\[ \text{then } \Delta V = V(r = \frac{R_0}{3}) - V(r = 0) = -\frac{A}{5\varepsilon_0} \int_0^{\frac{R_0}{3}} r^3 dr \]

\[ = -\frac{A}{5\varepsilon_0} \cdot \left[ \frac{r^4}{4} \right]_0^{\frac{R_0}{3}} = -\frac{A}{5\varepsilon_0} \cdot \frac{1}{4} \cdot \left( \frac{R_0}{3} \right)^4 = 0 \]

\[ = -\frac{AR_0^4}{1620\varepsilon_0} \]

(b) \[ Q_T = \iiint \rho dV = 4\pi A \int_0^{R_0} r^4 dr = \frac{4\pi A R_0^5}{5} \]

\[ \text{so: } Q_T = \frac{4\pi A R_0^5}{5} \Rightarrow A = \frac{5Q_T}{4\pi R_0^5} \]
FIRST MIDTERM

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES:
Use the conversion constants and data given on the front page.

(a) For the arrangement shown, what is the potential difference \( V(B) - V(A) \)?
\[
\begin{align*}
V(A) &= \frac{k\Phi_A}{a} + \frac{k\Phi_B}{a} = \frac{k(10^{19})}{a} + \frac{k(10^{19})}{a} = \frac{2k\Phi}{a} \\
V(B) &= \frac{k\Phi_B}{2a} + \frac{k\Phi_A}{2a} = \frac{k(10^{19})}{2a} + \frac{k(10^{19})}{2a} = \frac{k\Phi}{a}
\end{align*}
\]
\[
V(B) - V(A) = \frac{k\Phi}{a} - \frac{2k\Phi}{a} = \frac{-k\Phi}{a}
\]

(b) Use the binomial expansion to calculate the coefficient of \( x^3 \) for the expression \( (1-x)^{-4/3} \) with \( x \ll 1 \).
\[
(1-x)^{-4/3} = \frac{1}{1-x} \sum_{n=0}^{\infty} \binom{-4/3}{n} x^n
\]
\[
= 1 + \frac{1}{1} x + \frac{1 \cdot 4}{1 \cdot 3} x^2 + \frac{1 \cdot 4 \cdot 7}{1 \cdot 3 \cdot 5} x^3 + \ldots
\]

(c) A very long thin wire has a total charge of \( +7.0 \cdot 10^4 \) C on a length of 30.0 m. Calculate the electric field 0.50 cm away from the center of the wire, at a point nowhere near its ends.
\[
\begin{align*}
\left( \frac{\lambda}{\varepsilon_0} \right) E &= \left( \frac{\lambda}{\varepsilon_0} \right) \frac{\lambda}{r} \\
E &= \frac{\lambda}{\varepsilon_0} \frac{\lambda}{r} = \frac{2 \times 3.39 \times 10^{19} \text{C/m}}{30 \text{m}}
\end{align*}
\]

(d) Calculate a numerical value for the electric field a distance 3.00 \( \times 10^{-9} \) m from an electron.
\[
E = \frac{k\Phi}{r^2} = \frac{9 \times 10^9 \text{N m}^2}{(3 \times 10^{-9})^2} = \frac{-1.6 \times 10^{-19}}{9 \times 10^{-20}} \text{N/C}
\]

(e) The nucleus of a carbon atom has exactly 6 elementary positive charges. Calculate the potential, in volts, a distance 1.00 \( \times 10^{-10} \) m from the center of the nucleus. [This is just outside the nucleus.]
\[
V = \frac{k\Phi}{r} = \frac{9 \times 10^9 \text{N m}^2}{(6 \times 1.6 \times 10^{-9}) \times (10^{-10})} = \frac{9.6 \times 10^4}{10^{-9}} = 9.6 \times 10^4 \text{V}
\]
SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Given the charge distribution shown.

(a) \[\mathbf{E} = \frac{2}{\hat{r}} \frac{kQ}{\hat{r}} \hat{r} = \frac{kQ}{a^2} \hat{r} + \frac{2kQ}{a^2} \hat{y} - \frac{3kQ}{2a^2} \frac{\hat{y}}{2} \hat{y} + \frac{2kQ}{a^2} \hat{y} \]

(b) \[V = \frac{Q}{\epsilon_0} \frac{kQ}{a} + \frac{kQ}{a} \frac{Q}{a} - \frac{kQ}{a} \frac{Q}{a} \frac{Q}{a} = \frac{3kQ}{a} \left( \frac{1}{2} - \frac{1}{2} \right) \]

(c) \[W = \frac{1}{2}kv^2 = \frac{1}{2} \frac{kQ}{a} \frac{2kQ}{a} = \frac{6kQ^2}{a} \left( 2 - \frac{1}{2} \right) \]

(d) \[\mathbf{F} = \frac{1}{2}kQ \mathbf{E} = \frac{kQ^2}{a^2} \left( 3 - \frac{3kQ}{2a} \right) \hat{y} + \frac{kQ^2}{a^2} \left( 3kQ - 4 \right) \hat{y} \]
SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A disk of radius \( R_0 \) is charged with a surface charge density given by \( \sigma = \sigma_0 \), \( \sigma \). The units of \( \sigma \) are \( \text{C/m}^2 \).

(a) \( 1' \) Calculate the total charge on the disk.

(b) \( 1' \) Calculate the electric field on the axis of the disk a distance \( x \) away from the disk (out of the paper a distance \( x \)).

\[
\begin{align*}
\sigma &= \int_{R_0}^{R} \sigma_0 \pi r^2 dr \\
&= \sigma_0 \pi \left[ R^2 - R_0^2 \right] \\
&= \frac{\sigma_0 \pi R^2}{4} \\
&= \frac{2\pi \sigma_0 R^2}{3}
\end{align*}
\]

\[
\begin{align*}
\vec{E} &= \int_{-\infty}^{\infty} \frac{\sigma_0 \pi r^2 dr}{r^2 + x^2} \\
&= \frac{\sigma_0 \pi}{2} \ln \left[ \frac{r^2 + x^2}{r^2} \right] \bigg|_{0}^{R} \\
&= \frac{\sigma_0 \pi}{2} \ln \left[ \frac{R^2 + x^2}{R^2} \right] + \frac{\sigma_0 \pi}{2} \ln \left[ \frac{x^2}{R^2} \right] \\
&= \frac{\sigma_0 \pi}{2} \left( \frac{R^2 + x^2 - R^2}{x^2} \right) \\
&= \frac{\sigma_0 \pi}{2} \left( \frac{R^2 + x^2}{x^2} \right)
\end{align*}
\]

\( \vec{E} \) is pointing outward.
SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A spherically symmetric shell of non-conductor has an inner radius \( R_i \) and an outer radius \( R_o \). Between \( R_i \) and \( R_o \) the charge density is given by \( \rho(r) = \Lambda r^{-4} \). There is NO charge anywhere else.

(a) If the total charge is \( Q_o \), calculate \( \Lambda \).

(b) Calculate the electric field at any value of \( R \) larger than \( R_o \) in terms of \( A, R_i, R_o, \Phi, k, R \).

(c) Determine the electric field at any value of \( R \) between \( R_i \) and \( R_o \) in terms of \( A, R_i, R_o, \Phi, \Phi_i, \Phi_o, k \).

(d) What is the sign of \( V(R_i) - V(R_o) \)? Explain clearly.

\[
\Phi = \frac{Q}{4\pi \epsilon} = \int \delta \phi \, dV \quad \delta \phi = \frac{1}{4\pi \epsilon} \Phi \quad \Phi_i \rightarrow -\Phi_o \rightarrow 0
\]

\[
Q = 4\pi A \left( R_o^2 - R_i^2 \right) \Rightarrow \Phi = \frac{\Phi_o}{4\pi \epsilon} \frac{A}{R_o^2 - R_i^2}
\]

(b) \( \Phi(R_o) \)? Gauss law.

\[
\Phi = \frac{k \Phi_o}{R_o^2} = \frac{4\pi \epsilon \Phi_o}{R_o^2} \left( \frac{1}{R_i} - \frac{1}{R_o} \right)
\]

(c) \( \Phi(R) \) for \( R_i < R < R_o \)

\[
\frac{d \Phi}{d R} = \frac{k \Phi_o}{R_o^2} \quad \Phi = \frac{k \Phi_o}{R_o^2} \int_{R_o}^{R} \frac{dR}{R_i - R} \Rightarrow \Phi = \frac{k \Phi_o}{R_i - R} \frac{A}{R_o^2} \left( \frac{1}{R_i} - \frac{1}{R_o} \right)
\]

(d) \( V(R_o) - V(R_i) \)?

Since the potential is obtained to be zero \( \Rightarrow \Phi_o = 0 \), then

Note that the potential \( \Phi \) is smaller than potential \( \Phi_o \).

Since potential decreases nearer distance. Thus \( V(R_o) - V(R_i) \) must be negative.
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

\(F = k \cdot \frac{e^2}{r^2} = 4.07 \times 10^{-8} \text{ (N)}\)

\(V = k \cdot \frac{Q}{R} = -3.46 \times 10^5 \text{ (V)}\)

\(V(B) - V(A) = \frac{kQ}{2\alpha} (\text{V})\)

\(E = \frac{\lambda}{2\pi \epsilon_0 r} = 3.42 \times 10^5 \text{ (N/C)}\), \(\lambda = \frac{Q}{x}\)

\[2.19 = \frac{3\epsilon}{16} = \frac{1}{8} \left[ \left( \frac{3}{2} \right) \left( \frac{1}{2} \right) \right] \left( \frac{1}{1-\left( \frac{3}{2} \right)^2} \right) (-1)\]
SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES:
Use the conversion constants and data given on the front page.

The four points shown are at the corners of a square of side a. Given the values shown, calculate (numerical values),

(a) the x and y components of the electric field at point P;
(b) the direction of the electric field at point P, measured as an angle counter-clockwise from the positive x-axis.

\[ a = 1.35 \text{ cm} \]

\[ Q_1 = +4.76 \times 10^{-4} \text{ C} \]
\[ Q_2 = -6.35 \times 10^{-4} \text{ C} \]
\[ Q_3 = 2.37 \times 10^{-4} \text{ C} \]

\[ k = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2} \]

\[ E_x = \frac{kQ_1}{a^2} \]
\[ E_y = \frac{kQ_2}{a^2} \]
\[ E_z = \frac{kQ_3}{a^2} \]

\[ E_{\text{total}} = E_x + E_y \]
\[ \theta = \tan^{-1} \left( \frac{E_{\text{total}}}{E_{\text{total}}} \right) \]

\[ = 2.89 \text{ (degree)} \]

\[ E_x = \frac{8.99 \times 10^9 \times 4.76 \times 10^{-4}}{(1.35 \times 10^{-2})^2} \]
\[ = 2.348 \times 10^4 \text{ N/C} \]

\[ E_y = \frac{8.99 \times 10^9 \times 6.35 \times 10^{-4}}{(1.35 \times 10^{-2})^2} \]
\[ = 1.169 \times 10^4 \text{ N/C} \]

\[ E_z = \frac{8.99 \times 10^9 \times 2.37 \times 10^{-4}}{(1.35 \times 10^{-2})^2} \]
\[ = 1.106 \times 10^4 \text{ N/C} \]
SHOW ALL WORK!!!!!!

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!

Use the conversion constants and data given on the front page.

A uniformly charged sphere of a nonconductor has a radius \( R_1 \) and charge \( Q_1 \). It is enclosed in a concentric thin metal spherical shell whose radius is \( R_2 \) (\( R_2 > R_1 \)). The metal shell has a net total charge \( Q_2 \).

(a) Calculate the electric field a distance 47.0 cm from the common center of the two spheres. (Numerical answer including sign.)

(b) Calculate the electric field a distance of 13.5 cm from the common center. (Numerical answer including sign.)

(c) Calculate the electric field a distance 3.00 cm from the common center. (Numerical answer including sign.)

\[ Q_1 = -375 \mu \text{C}; Q_2 = +172 \mu \text{C}; R_1 = 12.0 \text{ cm}; R_2 = 30.0 \text{ cm} \]

(a) \[ E = \frac{k(Q_1 + Q_2)}{r^2} = \frac{(8.99 \times 10^9 \text{ N m}^2/\text{C}^2)(-375 \times 10^{-6} \text{ C})}{(0.47 \text{ m})^2} = -8.26 \times 10^6 \text{ N/C} \]

(b) \[ E = \frac{kQ_1}{r^2} = \frac{(8.99 \times 10^9 \text{ N m}^2/\text{C}^2)(-375 \times 10^{-6} \text{ C})}{(0.135 \text{ m})^2} = -1.85 \times 10^8 \text{ N/C} \]

(c) \[ E = \frac{kQ_1 r}{R_2^3} = \frac{(8.99 \times 10^9 \text{ N m}^2/\text{C}^2)(-375 \times 10^{-6} \text{ C})(0.03 \text{ m})}{(0.12 \text{ m})^3} = -5.85 \times 10^7 \text{ N/C} \]

(equation for part (c) may be derived directly from Gauss’ Law)
SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!

Use the conversion constants and data given on the front page.

A very long cylinder of non-conducting material has a radius \( R_c \) and a volume charge density given by \( \rho = Br^2 \) for \( R < R_c \) and \( \rho = 0 \) for \( R > R_c \). \( B \) is a constant.

(a) Using Gauss' Law, calculate the electric field a distance \( R \) from the axis of the cylinder where \( R > R_c \).

(b) Using Gauss' Law, calculate the magnitude of the electric field at an arbitrary point \( P \) in the cylinder a distance \( R \) from the cylinder axis. \( R < R_c \).

(c) Calculate the magnitude of the potential difference between the wall of the cylinder and its axis \( \Delta V(R_c) - V(0) \).

(d) If the sign of the charge on the cylinder is negative, state clearly the sign of \( \Delta V(R_c) - V(0) \), and give a physical reason for it.

\[ L = \text{Length of Cylinder} \]
\[ \rho = Br^2 \]
\[ dV = 2\pi r L \, dr \]
\[ E \, dA = \frac{1}{\epsilon_0} \int_0^L \int_0^r \frac{1}{2} \pi r^2 \, E \, r \, dr \, dr \]
\[ \int_0^L \int_0^r \frac{1}{2} \pi r^2 \, E \, r \, dr \, dr = \frac{2}{5} \pi r^5 \frac{E \rho}{\epsilon_0} \]
\[ E(\rho r^2) = \frac{3}{5} \pi r^5 \frac{E \rho}{\epsilon_0} \]
\[ E(\frac{L^2}{2}) = \frac{3}{5} \pi \frac{E \rho}{\epsilon_0} \]
\[ E = \frac{3E \rho}{5 \pi \epsilon_0} \]
\[ \Delta V = V(R_c) - V(0) = -\int_0^{R_c} \frac{E \rho}{\epsilon_0} \, dr = \left( \frac{1}{5} \right) \frac{3E \rho}{2 \pi \epsilon_0} = \frac{3E \rho}{25 \pi \epsilon_0} \]

D) \( \Delta V(R_c) - V(0) > 0 \). The Electric field is directed inward, therefore, the Potential \( \phi \) increases the further away from the center you go. The Potential \( \phi \) is positive.
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!  
Use the conversion constants and data given on the front page.

(a) Calculate the magnitude of the electric force between two electrons that are $2.00 \times 10^{-10}$ m apart.

$$F = \frac{k|e|^2}{r^2} = \frac{8.99 \times 10^9 \text{ N m}^2}{e^2} \frac{(1.602 \times 10^{-19} \text{ C})^2}{(2 \times 10^{-10} \text{ m})^2} = 5.77 \times 10^{-9} \text{ N}$$

(b) Calculate the electric field a distance $4.32 \times 10^{12}$ m away from the nucleus of a helium atom. The helium atom nucleus has 2 protons and 2 neutrons.

$$E = \frac{k(2e)}{r^2} = \frac{8.99 \times 10^9 \text{ N m}^2}{e^2} \frac{(2 \times 1.602 \times 10^{-19} \text{ C})}{(4.32 \times 10^{-12} \text{ m})^2} = 1.55 \times 10^{14} \text{ N/C}$$

(c) Find the point on the x-axis, between the two charges, where the electric potential is zero.

We want $\frac{k(2Q)}{x} - \frac{k(3Q)}{a-x} = 0 \Rightarrow 2a - 2x - 3x = 0 \Rightarrow x = \frac{2}{5} a$

(d) A proton is accelerated from rest through an electric potential difference of 5320 volts. Find its final velocity.

$$K = \frac{1}{2} m_f v_f^2 = 5320 \text{ eV} = 8.5226 \times 10^{-16} \text{ J} \Rightarrow v_f = 1.01 \times 10^6 \text{ m/s}$$

(e) A very long wire has a linear charge density $\lambda$, of $275 \times 10^{-12}$ C/m. Calculate the magnitude of the electric field a distance of 3.75 cm from the center of the wire. (This is outside of the wire.)

$$\phi = \frac{Q_{AC}}{\varepsilon_0} = E \left(2\pi r l\right) \Rightarrow E = \frac{Q_{AC}}{2\pi \varepsilon_0 r l} = \frac{132}{N} \text{ C}$$
SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Charges $Q_1$, $Q_2$, and $Q_3$ are at three corners of a square of side $a$.

(a) Calculate the electric field, magnitude and direction, at point $P$ due to these charges. Use the coordinate system shown.

(b) Determine the electric potential at point $P$ due to these charges, using the usual choice for $V = 0$.

$$Q_1 = 4.72 \times 10^{-6} \text{ C} \quad Q_2 = -3.25 \times 10^{-6} \text{ C} \quad a = 1.25 \text{ cm}$$

\[
E_x = \frac{k|Q_1|}{a^2} - \frac{k|Q_2|}{2a^2} \cos 45^\circ = 2.055 \times 10^8 \frac{\text{N}}{\text{C}}
\]

\[
E_y = -\frac{k|Q_2|}{2a^2} \sin 45^\circ + \frac{k|Q_3|}{a^2} = 9.211 \times 10^7 \frac{\text{N}}{\text{C}}
\]

\[
E = \sqrt{E_x^2 + E_y^2} = 2.25 \times 10^8 \frac{\text{N}}{\text{C}}
\]

\[
\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = 24.1^\circ \text{ above (CCW) the x-axis}
\]

(b)

\[
V = V_1 + V_2 + V_3 = \frac{kQ_1}{a} + \frac{kQ_2}{a^2} + \frac{kQ_3}{a} = (3.395 + (-1.652) + 1.978) \times 10^6 \text{ V}
\]

\[
V = 3.72 \times 10^6 \text{ V}
\]
SHOW ALL WORK!!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Two conducting spheres are connected with a long wire. A total charge of 1.65 \times 10^{-4} \text{ C} is placed on them. The radius of sphere 1 is 6.00 cm, and the radius of sphere 2 is 2.00 cm.

13 pts (a) What is the charge on each sphere?
6 pts (b) What is the potential of each sphere, using the usual choice for V = 0.
6 pts (c) Find the electric field at the surface of each sphere.

(a) we know \( V_1 = V_2 \) because potential is equal everywhere in a metal.

\[
\frac{kq_1}{r_1} = \frac{kq_2}{r_2} \quad q_1 = \frac{r_1}{r_2} q_2 = 3q_2
\]

but \( q_1 + q_2 = q_{\text{total}} \) so \( 4q_2 = q_{\text{total}} \Rightarrow q_2 = \frac{q_{\text{total}}}{4} \)

\[
q_1 = 1.24 \times 10^{-6} \text{ C} \quad q_2 = 4.13 \times 10^{-7} \text{ C}
\]

and \( q_1 = \frac{3q_{\text{total}}}{4} \)

(b) \( V = \frac{kq_1}{r_1} = \frac{kq_2}{r_2} = 1.86 \times 10^5 \text{ V} \)

(c) \( E_1 = \frac{kq_1}{r_1^2} = 3.10 \times 10^6 \frac{\text{N}}{\text{C}} \)

\( E_2 = \frac{kq_2}{r_2^2} = 9.28 \times 10^6 \frac{\text{N}}{\text{C}} \)
SHOW ALL WORK!!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A non-conducting sphere, of radius $R_0$, has a positive charge distribution given by $\rho(r) = Br^2$ for $0 < r \leq R_0$, and $\rho = 0$ everywhere else. $B$ is a constant.

10 pts (a) Calculate the total charge on the sphere.
5 pts (b) Calculate the electric field, magnitude and direction, at point $P$, which is on the x-axis, a distance $3R_0$ from the center of the sphere.
10 pts (c) Calculate the electric field at a point $P$, inside the sphere a distance of $R_0/2$ from its center (magnitude and direction).

\begin{align*}
\text{a) } Q &= \int_{\text{whole sphere}} dq = \int_{0}^{R_0} \rho dV = \int_{0}^{R_0} 4\pi r^2 dr = 4\pi B \int_{0}^{R_0} r^5 dr, \\
&= \frac{2}{3} \pi BR_0^6 \\
\text{so } Q &= \frac{4\pi B R_0^6}{3}. \\

\text{b) } \vec{E} &= \frac{kQ}{(3R_0)^2} \hat{r} = \frac{k}{9} \left( \frac{2}{3} \pi B R_0^6 \right) \hat{r} = \frac{2\pi k B R_0^4 \hat{r}}{27}. \\
\text{means radially outward.} \\

\text{c) now } Q &= \int_{0}^{R_0/2} \rho 4\pi r^2 dr = \int_{0}^{R_0/2} 4\pi r^2 dr = \frac{4\pi B \left( \frac{R_0^4}{2} \right)}{6} = \frac{\pi BR_0^6}{96}. \\
\text{and } \vec{E} &= \frac{kQ}{(R_0/2)^2} \hat{r} = \frac{4\pi k B R_0^6}{96 R_0^2} \hat{r} = \frac{\pi k B R_0^4 \hat{r}}{24}. \\
\end{align*}
First Midterm

Report all numbers to three significant figures!
Use the conversion constants and data given on the front page.

(a) Calculate the electric force (in Newtons) between two electrons a distance $3.00 \times 10^{-11}$ m apart.

$$E = \frac{ke^2}{r^2} = \left(\frac{9.0 \times 10^9 \cdot (1.6 \times 10^{-19})^2}{(3 \times 10^{-11})^2}\right) = 2.56 \times 10^7 \text{ N}$$

(b) Assume a proton has a radius of $1.00 \times 10^{-13}$ m (about right). It has a positive charge equal in magnitude to that of the electron. Calculate the charge density, assumed to be uniform.

$$\rho = \frac{q}{V} = \frac{1.6 \times 10^{-19}}{\frac{4\pi \cdot 10^{-15}}{3}} = 3.07 \times 10^2 \frac{C}{m^3}$$

(c) Given a potential function $V = B x^2 y^4 z^2$. Calculate the y-component of the electric field at the point $x = 2.00$, $y = 3.00$, $z = 1.00$.

$$E_y = \frac{\partial V}{\partial y} = -B x^2 \cdot 4 y^3 z^2 = -B (16)(37) = -432 B$$

(d) Calculate the term in $x^6$ using the binomial expansion for the expression $(1 - x^3)^{\frac{3}{2}}$.

$$\left(\frac{n(n-1)(n-2)}{3!}\right) x^{n-3} \cdot (x^3)^3 = \left(\frac{3!}{16}\right) x^6 = 0.0625 x^6$$

(e) For the arrangement shown, what is the potential difference $V(B) - V(A)$? A $+2q$ $-3q$ B

$$V_A = K \left(\frac{2q}{a} + \frac{3q}{2a}\right) = \frac{kq}{2a} \quad V_B = \frac{-5kq}{2a}$$
SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Three point charges are along the x-axis a distance b (m) apart (see drawing).

(a) Calculate the x component of the electric field at point P, a distance b on the y-axis above Q₁.
(b) Calculate the electric potential, V, at point P using the usual choice for V = 0.

Q₁ = 1.00 μC; Q₂ = 2.00 μC; Q₃ = 3.00 μC; b = 1.25 m

\[ \Theta₁ = 45^° \quad \Theta₂ = \tan^{-1}\left(\frac{1}{2}\right) \approx 26.5^° \]

a) \[ E_x = -\frac{kQ₁}{2b^2} \cos(45^°) - \frac{kQ₃}{5b^2} \cos(26.5^°) = -7.16 \times 10^{-3} \text{ N/C} \]

b) \[ V = \frac{kQ₁}{\sqrt{2}b} + \frac{kQ₂}{\sqrt{5}b} + \frac{kQ₃}{\sqrt{5}b} = 2.76 \times 10^4 \text{ V} \]
A point charge, \( +Q \), is placed inside a thin, hollow conducting sphere of radius \( R_o \). Another hollow sphere, a non-conductor and radius \( 2R_o \), surrounds the conducting sphere and has the same center. The total charge on the non-conducting sphere is \(-2Q\), and is uniformly distributed.

(a) Calculate the electric field, magnitude and direction, at a point \( R = 3/2 R_o \).

(b) Calculate the electric field, magnitude and direction, at a point \( R = 5R_o \).

\[
\mathbf{E} = \frac{Q}{4\pi \varepsilon_o \mathbf{r}^2}
\]

\[
E = \frac{kQ}{25R_o^2} = 3.6 \times 10^9 \frac{Q}{R_o^2}
\]

\[
\text{Ans only}: -3
\]
A non-conducting sphere has a radius $R_o$. The positive charge distribution on this sphere can be described by $\rho(r) = B \cdot r^{3/2}$, where $B$ is a constant.

(a) Calculate the total charge on the system.
(b) Calculate the magnitude of the electric field at a point a distance $2R_o$ from the center of the sphere.
(c) Calculate the magnitude of the electric field inside the sphere at a point $R_o/3$ from the center of the sphere.
(d) In part (c) is the electric field directed towards or away from the center of the sphere?

\[ Q = \int_0^{R_o} \rho(r) \cdot 4\pi r^2 dr \]
\[ = \int_0^{R_o} B \cdot r^{3/2} \cdot 4\pi r^2 dr \]
\[ = 4\pi B \int_0^{R_o} r^{5/2} dr \]
\[ = 4\pi B \cdot \left[ \frac{2}{9} r^{9/2} \right]_0^{R_o} \]
\[ = \frac{8\pi B}{9} R_o^{9/2} \]

(c) Similarly
\[ E_{R_o/3} \cdot 4\pi \left( \frac{R_o}{3} \right)^2 = \frac{8\pi B}{9} \left( \frac{R_o}{3} \right)^{9/2} \]
\[ \Rightarrow E_{R_o/3} = \frac{2\sqrt{3}}{243} \frac{B}{\varepsilon_0} R_o^{5/2} \]

(d) Away from the center

From Gauss's Law
\[ E_{2R_o} \cdot 4\pi (2R_o)^2 = \frac{8\pi B}{9} R_o^{9/2} \]
\[ \Rightarrow E_{2R_o} = \frac{1}{18} \frac{B}{\varepsilon_0} R_o^{5/2} \]
EXAM 1

Name: ___________________________  unid: u ____________

Discussion TA (circle): Justin  Mahamadou  Mike  Will

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) Calculate the magnitude of the electric force between two protons (charge = +e) a distance \(4.00 \times 10^{-13}\) m apart.

\[
F = k\frac{ee}{r^2} = 9 \times 10^9 \frac{(1.6 \times 10^{-19})^2}{(4 \times 10^{-13})^2} = 1.44 \times 10^{-3} \text{ N}
\]

(b) Use the binomial expansion to calculate the coefficient of the \(x^3\) term for the expression \((1 - x)^{-7/3}\).

\[
(1 - x)^{-7/3} \approx 1 + \frac{7}{3} x + \frac{(-7/3)(-4/3)}{2} x^2 + \frac{(-7/3)(-4/3)(-1/3)}{3} x^3 + \ldots
\]

\[
\text{Coefficient of } x^3 = \frac{45}{81} = \frac{5}{9}
\]

(c) For the arrangement shown, what is the potential difference \(V_A - V_B\)?

\[
V_A - V_B = \left(\frac{kq}{a} - \frac{k2q}{2a}\right) - \left(\frac{kq}{2a} - \frac{2kq}{a}\right) = \frac{3kq}{2a}
\]

(d) A very long thin wire has a total charge of \(Q = +3.75 \times 10^{-6} \text{ C}\). Its total length is 57.0 m. Calculate the magnitude of the electric field a distance 3.72 mm away from the center of the wire, nowhere near either end.

\[
E = \frac{\lambda}{2\pi \varepsilon_0 d} \quad \text{where} \quad d = \frac{Q}{L} \quad \Rightarrow E = \frac{3.75 \times 10^{-6} \text{ C}}{2\pi (8.85 \times 10^{-12} \text{ F/m}) \times 3.72 \times 10^{-3} \text{ m}} = 3.18 \times 10^5 \text{ N/C}
\]

(e) An electron is accelerated from rest through a potential difference of 137 volts. Calculate the velocity of the electron.

\[
\Delta u = \frac{1}{2} mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{2\Delta u}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 137}{9.11 \times 10^{-31}}} = 6.93 \times 10^6 \text{ m/s}
\]
EXAM 1

SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Three point charges are along the x-axis a distance \( b \) apart, as shown in the drawing. Point A is \( b \) from \( Q_3 \) along the x-axis, and P is directly above A at a distance \( b \).

(a) Calculate the x-component of the electric field at point P. (Numerical answer.)

(b) Calculate the electric potential (in volts) at point P due to the three charges.

\( Q_1 = +4.50 \, \mu \text{C}; \, Q_2 = -2.75 \, \mu \text{C}; \, Q_3 = -1.22 \, \mu \text{C}; \, b = 7.00 \times 10^{-3} \, \text{m} \)

a) \( \vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \)

\( E_{1,\text{x}} = E_{1,\text{x}} + E_{2,\text{x}} + E_{3,\text{x}} \)

\( = E_1 \cos \theta_1 + E_2 \cos \theta_2 + E_3 \cos \theta_3 \)

\( = \frac{kQ_1}{r_1^2} \cos \theta_1 + \frac{kQ_2}{r_2^2} \cos \theta_2 + \frac{kQ_3}{r_3^2} \cos \theta_3 \)

\( = \frac{kQ_1}{10b^2} \left( \frac{3}{10} \right) + \frac{kQ_2}{5b^2} \left( \frac{2}{5^\frac{3}{2}} \right) + \frac{kQ_3}{2b^2} \left( \frac{1}{12} \right) \)

\( = \frac{k}{b^2} \left[ \frac{30Q_1}{10 \times 10^4} + \frac{2Q_2}{5 \times 5^\frac{3}{2}} + \frac{Q_3}{2 \times 12} \right] \)

\( = \frac{(8.9876 \times 10^9)}{(7 \times 10^{-3})^2} \left[ \frac{3(4.5 \times 10^{-6})}{10^{-10}} + \frac{2(-2.75 \times 10^{-6})}{5 \times 5^\frac{3}{2}} + \frac{(-1.22 \times 10^{-6})}{2 \times 12} \right] = -9.1 \times 10^7 \, \text{N/C} \)

b) \( V_{\text{net}} = V_1 + V_2 + V_3 \)

\( = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} + \frac{kQ_3}{r_3} = \frac{k}{b} \left[ \frac{Q_1}{10^4} + \frac{Q_2}{5^\frac{3}{2}} + \frac{Q_3}{12} \right] \)

\( = \frac{(8.9876 \times 10^9)}{(7 \times 10^{-3})^2} \left[ \frac{4.5 \times 10^{-6}}{10^4} + \frac{2.75 \times 10^{-6}}{5^\frac{3}{2}} + \frac{1.22 \times 10^{-6}}{12} \right] = -8.6 \times 10^5 \, \text{V} \)
SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Two conducting spheres are connected with a long wire. Sphere 1 has a radius of $R = 17.0$ cm and sphere 2 has a radius of $r = 3.50$ cm. A charge of 60.0 pC is placed on the system.

(a) What is the charge in picocoulombs on sphere 1?
(b) Find the potential of sphere 1 in volts.
(c) What is the value of the electric field a distance $2r$ from the center of sphere 2. Sphere 1 is very far away.

1.\]
\[V(R) = V_2(r)\]
\[\frac{k_1 q_1}{R} = \frac{k q_2}{r}\]
\[q_2 = \frac{k q_1}{R} \]
\[q_1 = 49.8 \text{ pC}\]
\[q_T = q_1 + q_2\]
\[q_T = 8_1 + \frac{k}{R} q_2\]
\[\epsilon_0 = \frac{q_T}{(1+\frac{k}{R})}\]

2.\]
\[V(R) = \frac{k q_1}{R} = 12.63 \text{ V}\]

3.\]
\[E(2r) = \frac{k q_2}{(2r)^2} = 18.7 \frac{N}{C}\]
A non-conducting sphere has a negative charge distribution that can be described by 
\( \rho = Br^{\frac{3}{2}} \), where \( B \) is a constant.

(a) Calculate the total charge on the system.

(b) Calculate the magnitude of the electric field a distance \( 3R_0 \) from the center of the sphere.

(c) Calculate the magnitude of the electric field at a point \( R_s/4 \) from the center of the sphere.

(d) In part (c) is the electric field directed inward or outward?

\[ Q_T = \int_0^{R_0} 8(r) \, dV = \int_0^{R_0} B \cdot r^{\frac{7}{2}} (4\pi r^2) \, dr = 4\pi B \int_0^{R_0} r^{\frac{9}{2}} \, dr \]

\[ = 4\pi B \frac{2}{13} R_0^{13\frac{1}{2}} = \frac{8\pi B R_0^{11}}{13} \]

(e) \[ E = k \frac{Q_T}{r^2} = k \frac{8\pi B R_0^{11}}{13} \frac{1}{(3R_0)^2} \]

\[ = \frac{8kB\pi}{11} R_0^{9\frac{1}{2}} \]

(c) We must use Gauss' Law

\[ E_r \cdot A = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow |E| \frac{4\pi}{4} (R_0)^2 = \frac{1}{\varepsilon_0} \frac{8}{13} B\pi \left( \frac{R_0}{4} \right)^{1\frac{1}{2}} \]

\[ E = \frac{2B}{13\varepsilon_0} \left( \frac{R_0}{4} \right)^{\frac{9}{2}} \text{ or } \frac{8\varepsilon B k}{13} \left( \frac{R_0}{4} \right)^{9\frac{1}{2}} \]

(d) Inward
A spherical distribution of positive charge has a charge density given by \( \rho(R) = \rho_o R^3 \) for \( R < R_o \), where \( \rho_o \) is a constant.

(a) Calculate the electric field at an arbitrary interior point of radius \( R \).
(b) Find the magnitude of the potential difference between the center and the surface of the sphere.
(c) If the total charge on the sphere is \( 1.60 \times 10^{-9} \text{ C} \), and the radius of the sphere is 0.875 m, find the potential difference in (b) in volts.

\[
E(R) = \frac{\varepsilon_o R^4}{6 \varepsilon_o} \]

\[
V = \frac{Q}{20 \pi \varepsilon_o R_o} = \frac{\varepsilon_o Q}{5 R_o} = 3.29 \times 10^3 \text{ V} = 3.29 \text{ kV}
\]
SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Charges $Q_1, -Q_4$ are at the corners of a square with sides $a$. Numerical values are given below.

(a) Calculate a numerical value for the electric potential at point A, exactly in the center of the square.
(b) Find a numerical value for the electric potential at point B, exactly in the middle of one side of the square.

\[ V_A = \frac{kQ_1}{r_{1A}} + \frac{kQ_2}{r_{2A}} + \frac{kQ_3}{r_{3A}} + \frac{kQ_4}{r_{4A}} \]
\[ \quad = \frac{kQ_1}{\frac{a}{2}} + \frac{kQ_2}{\frac{a}{2}} + \frac{kQ_3}{\frac{a}{2}} + \frac{kQ_4}{\frac{a}{2}} \]
\[ \quad = \frac{k}{\frac{a}{2} \cdot 1.60 \times 10^{-19}} \left( 4.25 - 3.75 - 6.75 + 1.35 \right) \times 10^{-6} \Omega = -3.78 \times 10^6 V \]

\[ V_B = \frac{kQ_1}{r_{1B}} + \frac{kQ_2}{r_{2B}} + \frac{kQ_3}{r_{3B}} + \frac{kQ_4}{r_{4B}} \]
\[ \quad = \frac{k}{\frac{\sqrt{2}}{2} \cdot 1.65 \times 10^{-2}} \left( 4.25 - 3.75 \right) \times 10^{-6} \quad + \quad \frac{k}{\frac{1}{2} \cdot 1.65 \times 10^{-2}} \left( -6.75 + 1.35 \right) \times 10^{-6} \]
\[ \quad = -5.65 \times 10^6 V \quad \text{(V or } \text{N} \cdot \text{m}/\text{C}) \]
A spherical non-conductor of radius $R_o$ has a positive charge distribution given by $\rho = B_o R^4$ for $R < R_o$ where $\rho = 0$ everywhere else. $B$ is a constant.

(a) Calculate the electric field at an arbitrary interior point a distance $R$ from the center of the sphere.

(b) Find the magnitude of the potential difference between the interior point at a radius $R_1$ and the surface at $R_o$.

(c) Calculate the energy stored in the electric field between $R = 0$ and $R = R_1$.

\[ \Delta V = \int_{R_1}^{R_o} \frac{\rho}{\varepsilon_0} dR = \int_{R_1}^{R_o} \frac{4\pi k B_o R^5}{\varepsilon_0} dR = \frac{2\pi k B_o}{21} (R_o^6 - R_1^6) = \frac{B_o}{4\pi \varepsilon_0} (R_o^6 - R_1^6) \]

\[ \frac{dW}{2} = \frac{\varepsilon_0 E^2}{2} dV = \frac{\varepsilon_0 E^2}{2} 4\pi r^2 dr \]

\[ W = \int_0^{R_1} \frac{\varepsilon_0 E_0}{2} \left( \frac{4\pi k B_o}{\varepsilon_0} R^5 \right)^2 R^2 dR = 2\pi \frac{B_o^2}{\varepsilon_0} \int_0^{R_1} R^{12} dR = \frac{2\pi B_o^2}{63 \varepsilon_0} R_1^{13} = \frac{8\pi^2 k B_o^2}{63 \varepsilon_0} R_1^{13} \]

\[ \text{Gauss' Law: } \oint E \cdot dA = \frac{q_{\text{includ}}}{\varepsilon_0} = 4\pi k q_{\text{includ}} \]

\[ \Rightarrow E \cdot 4\pi R^2 = 4\pi k q_{\text{includ}} = 4\pi k \int_0^R \rho dR = 4\pi \frac{4\pi k B_o R^5}{7} \]

\[ \Rightarrow E = \frac{4\pi k B_o R^5}{7} \frac{R^5}{\varepsilon_0} = \frac{B_o R^5}{7 \varepsilon_0} \]
A sphere of non-conductor of radius $R_0$ is uniformly negatively charged with a total charge $-Q$.

(a) Calculate the magnitude of the electric potential difference between the point $R_o/2$ and $R_o$.
(b) Obtain the sign of $V(R_o) - V(R_o/2)$, and clearly explain how you got it.
(c) With the usual choice of $V = 0$, calculate the potential at $R_o/2$.

\[
\begin{align*}
\text{(A)} & \quad \text{for } Y < R_0, \quad 4\pi Y^2 E = -\frac{Q}{4\pi \varepsilon_0} \cdot \frac{4\pi}{3} Y^3 \cdot \frac{1}{Y} = \varepsilon = \frac{-Q}{4\pi \varepsilon_0 R_0^3} Y \\
\text{for } Y > R_0, \quad \varepsilon = \frac{-Q}{4\pi \varepsilon_0 R_0^2} \\
V(R_0) - V\left(\frac{R_0}{2}\right) &= - \int_{\frac{R_0}{2}}^{R_0} \varepsilon \, dY = \int_{\frac{R_0}{2}}^{R_0} \frac{Q}{4\pi \varepsilon_0 R_0^3} Y \, dY = \frac{3Q}{32\pi \varepsilon_0 R_0} \\
\text{(b)} & \quad V_\infty = 0, \quad V_{R_0/2} < V_{R_0}, \quad \text{thus the sign of } V(R_0) - V(R_0/2) \quad \text{is } "-". \\
\text{(c)} & \quad V\left(\frac{R_0}{2}\right) = V(R_0) - \frac{3Q}{32\pi \varepsilon_0 R_0} = -\frac{Q}{4\pi \varepsilon_0 R} - \frac{3Q}{32\pi \varepsilon_0 R_0} = \frac{-11Q}{32\pi \varepsilon_0 R}
\end{align*}
\]