Show all work!! Report all numbers to three (3) significant figures.

In answering the following questions, express the direction of the induced current in terms of the letter labels a and b in each figure (that is, either "from a to b" or "from b to a"). For full credit, write the reasoning that lead you to each answer.

(a) [10 pts.] What is the direction of the induced current in the resistor R in Figure (a) when the bar magnet is moved to the left?

(b) [10 pts.] What is the direction of the current induced in the resistor R immediately after the switch S in Figure (b) is closed?

(c) [10 pts.] What is the direction of the induced current in resistor R when the current I in Figure (c) decreases rapidly to zero?

\[ a) \vec{B}_i = \rightarrow; \vec{B}_f = \rightarrow \]

then \[ \Delta B = B_f - B_i = \rightarrow - \rightarrow = \leftarrow \]

oppose with a current flowing from a to b

\[ \text{From a to b} \]

b) The problem was labeled in correctly.

I) assuming \[ \text{from a to b} \]
then \[ \vec{B}_i = 0 \]

II) if you went off the symbols \[ \text{from b to a} \]
then everything flips giving
c) \( B_i = \bigotimes \); \( B_f = 0 \)
\[ \Delta B = B_f - B_i = 0 - \bigotimes = \bigotimes \]
then \( \Delta B_{\text{opposing}} = \bigotimes \)
giving a current that flows (from a to b)
Show all work! Report all numbers to three (3) significant figures.

For a research project, a student needs a solenoid that produces an interior magnetic field of 30.0 mT. She decides to use a current of 1.00 A and a wire 0.500 mm in diameter. She winds the solenoid in layers on an insulating form 1.00 cm in diameter and 10.0 cm long.

(a) [10 pts.] Determine the number of layers of wire needed (round it up to the nearest integer).
(b) [10 pts.] Determine the minimal length of wire needed.

Knowns: \( B = 30 \times 10^{-3} \text{T} \)
\( I = 1 \text{A} \)
\( d_{\text{wire}} = 0.5 \times 10^{-3} \text{m} \)
\( d_{\text{cyl}} = 1 \times 10^{-2} \text{m} \)
\( L_{\text{cyl}} = 10 \times 10^{-2} \text{m} \)

\[
N = \frac{L_{\text{cyl}}}{d_{\text{wire}}} \quad \text{(number of turns per layer)}
\]

\[
B = I \mu_0 N \frac{L_{\text{cyl}}}{d_{\text{cyl}}} \quad \text{(where } n \text{ is the number of layers)}
\]

\[
= I \mu_0 \left( \frac{L_{\text{cyl}}}{d_{\text{wire}}} \right) \frac{1}{L_{\text{cyl}}} n
\]

\[
= I \mu_0 \frac{n}{d_{\text{wire}}}
\]

\[
\Rightarrow n = \frac{B_{\text{wire}}}{I \mu_0} = \frac{(30 \times 10^{-3} \text{T})(0.5 \times 10^{-3} \text{m})}{(1 \text{A})(4 \pi \times 10^{-7} \text{m/T/A})} \approx 12 \text{ layers}
\]
b) \[ \text{Length of wire} = 2\pi \left( \frac{d_{\text{cyl}}}{2} \right) \cdot N \cdot \text{L wire} \]

\[ = \pi d_{\text{cyl}} \cdot \frac{L_{\text{cyl}}}{\text{L wire}} \cdot N \]

\[ = \pi \left( 1 \times 10^{-2} \text{m} \right) \left( \frac{10 \times 10^{-2} \text{m}}{0.5 \times 10^{-3} \text{m}} \right) \cdot 12 \]

\[ = 75.4 \text{ m of wire} \]

* Another acceptable answer is using average coil radius

\[ R_{\text{avg}} = \frac{0.01 \text{ m}}{2} + 12 \left( 0.0005 \text{ m} \right) \]

\[ = 0.008 \text{ m} \]

then \[ L_{\text{total}} = 2\pi R_{\text{avg}} (12) \left( \frac{10 \times 10^{-2} \text{ m}}{0.5 \times 10^{-3} \text{ m}} \right) \]

\[ = 120.6 \text{ m} \]

* Most accurate would be

\[ L_{\text{total}} = 200\pi \left( 12 \cdot 0.01 + \sum_{N=1}^{12} N \cdot 0.0005 \right) \]

\[ = 200\pi \left( 12 \cdot 0.01 + 0.0005(78) \right) \]

\[ L_{\text{total}} = 99.9 \text{ m} \]
[10 pts.] A proton moves with a velocity of $\vec{v} = (2i - 4j)$ m/s in a region in which the magnetic field is $\vec{B} = (2j - 3k)$ T. What is the magnitude of the magnetic force this charge experiences?

$$\vec{F} = q \vec{v} \times \vec{B}$$

$$\vec{v} = 2\hat{i} - 4\hat{j}$$

$$\vec{B} = 2\hat{j} - 3\hat{k}$$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 0 \\ 0 & 2 & -3 \end{vmatrix} = (12-0)\hat{i} - (-6-0)\hat{j} + (4-0)\hat{k}$$

$$\vec{F} = q (12\hat{i} + 6\hat{j} + 4\hat{k})$$

$$|\vec{F}| = q |\vec{v} \times \vec{B}| = q \sqrt{12^2 + 6^2 + 4^2} = 14q = 14 \times 1.6 \times 10^{-19}$$

$$F_B = 2.24 \times 10^{-18} \text{ N}$$
Show all work!! Report all numbers to three (3) significant figures.

The needle of a magnetic compass has magnetic moment 9.70 mA · m². At its location, the Earth's magnetic field is 55.0 μT north at 48.0° below the horizontal.

(a) [10 pts.] Identify the orientations of the compass needle in which the needle has its minimum and maximum magnetic potential energy.

(b) [10 pts.] How much work must be done on the needle to rotate it from the minimum-energy to the maximum-energy orientation?

\[ U = -\mathbf{\mu} \cdot \mathbf{B} \]

\[ U_{\text{min}} = -\mathbf{\mu} \cdot \mathbf{B} \cos 0° \]

\[ U_{\text{max}} = -\mathbf{\mu} \cdot \mathbf{B} \cos 180° \]

Work = \[ U_{\text{max}} - U_{\text{min}} \]

\[ = 2\mu B \]

\[ = 2 \times 9.7 \times 10^{-3} \times 55 \times 10^{-6} \]

\[ = 1.07 \times 10^{-6} J \]
Show all work!! Report all numbers to three (3) significant figures.

[10 pts.] Two long, parallel conductors carry currents $I_1 = 3.00 \, \text{A}$ and $I_2 = 3.00 \, \text{A}$, both directed into the page (see figure). Determine the $x$ and $y$ components of the resultant magnetic field at $P$.

\[
B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{4\pi \times 10^{-7} \times 3}{2\pi \times (0.05)} = 1.2 \times 10^{-5} \, \text{T}
\]

\[
B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{4\pi \times 10^{-7} \times 3}{2\pi \times (0.12)} = 5 \times 10^{-6} \, \text{T}
\]

\[
\sin \alpha = \cos \beta = \frac{5}{13} \\
\cos \alpha = \sin \beta = \frac{12}{13}
\]

\[
B_x = -B_1 \cos \beta + B_2 \cos \alpha \\
= -(1.2 \times 10^{-5}) \left( \frac{5}{13} \right) + (5 \times 10^{-6}) \left( \frac{12}{13} \right) \\
= 0
\]

\[
B_y = -B_1 \sin \beta - B_2 \sin \alpha \\
= -(1.2 \times 10^{-5}) \left( \frac{12}{13} \right) - (5 \times 10^{-6}) \left( \frac{5}{13} \right) \\
= -1.3 \times 10^{-7} \, \text{T}
\]
Show all work!! Report all numbers to three (3) significant figures.

A long, straight wire carries a current given by \( I = I_{\text{max}} \sin(\omega t + \varphi) \). The wire lies in the plane of a rectangular coil of \( N \) turns of wire as shown in the figure. The quantities \( I_{\text{max}}, \omega, \) and \( \varphi \) are all constants. Assume \( I_{\text{max}} = 50.0 \, \text{A}, \omega = 200 \, \text{rad/s}, N = 100, h = w = 5.00 \, \text{cm}, \) and \( l = 20.0 \, \text{cm} \).

(a) \[ 10 \, \text{pts.} \] Show that the mutual inductance of the coil and the straight wire is \( M = \frac{\mu_0 NL}{2\pi} \log \left(1 + \frac{w}{h}\right) \), where \( \log \) is the natural logarithm. (Hint: compute the induced emf in two ways: from the rate of change of magnetic flux and from the definition of mutual inductance.)

(b) \[ 10 \, \text{pts.} \] Determine the peak emf induced in the coil by the magnetic field created by the current in the straight wire.

\[
\mathbf{A} \quad \text{EMF} = -\frac{d\Phi_B}{dt} \quad \Phi_B = N \int \mathbf{B} \cdot d\mathbf{A}
\]

\[
\Phi_B = N \int_0^L \int_0^h B_x \left( \frac{w+h}{2\pi} \right) \, dx \, dr = \frac{N_0 I}{2\pi} \left( \log(\frac{w+h}{h}) \right)
\]

\[
\mathbf{B} \quad \text{EMF} = -\frac{N_0 I}{2\pi} \log \left(1 + \frac{w}{h}\right) \frac{dI}{dt} = \text{EMF} = -M \frac{dI}{dt}
\]

\[
M = \frac{N_0 I}{2\pi} \log \left(1 + \frac{w}{h}\right)
\]

\[
\text{EMF}_{\text{peak}} = \frac{N_0 I}{2\pi} \log \left(1 + \frac{w}{h}\right) w I_{\text{max}} = 0.08 \, \text{V}
\]
Show all work!! Report all numbers to three (3) significant figures.

[10 pts.] Show that in a series RC circuit with no inductance, the power factor is given by \( \cos \phi = \frac{RC\omega}{\sqrt{1+(RC\omega)^2}} \).

\[
P_{\text{avg}} = I_{\text{rms}} \cdot V_{\text{rms}} \cdot \cos \phi
\]
\[
P_{\text{avg}} = I_{\text{rms}}^2 \cdot R
\]

\[
\frac{I_{\text{rms}} \cdot V_{\text{rms}} \cdot \cos \phi}{I_{\text{rms}} \cdot V_{\text{rms}}} = I_{\text{rms}}^2 \cdot R
\]
\[
\cos \phi = \frac{I_{\text{rms}} \cdot R}{V_{\text{rms}}}
\]

\[
I_{\text{rms}} \cdot Z = V_{\text{rms}}
\]
\[
Z = \sqrt{R^2 + (\frac{1}{X_L} - \frac{1}{X_C})^2}
\]
\[
= \sqrt{R^2 + (\frac{1}{Y_{LC}})^2}
\]
\[
\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (\frac{1}{Y_{LC}})^2}}
\]
\[
\cos \phi = \frac{R}{\sqrt{(\frac{1}{Y_{LC}}) \sqrt{(\frac{R}{Y_{LC}})^2 + 1}}} = \frac{R \cdot Y_{LC}}{\sqrt{1 + (\frac{R}{Y_{LC}})^2}}
\]
Show all work!! Report all numbers to three (3) significant figures.

In the circuit shown in the figure, the AC generator produces an rms voltage of 120 V at 60 Hz. Find the rms voltage across the following points:

\[ I_{\text{rms}} = \frac{V_{\text{rms}}}{|Z|_{\text{tot}}} \]

\[ |Z_{\text{tot}}| = \sqrt{50^2 + (0.137 \cdot 120) - \frac{1}{25 \mu F}} \]

\[ |Z_{\text{tot}}| = 73.93 \Omega \]

**a)** \( V_{AB} = I_{\text{rms}} \cdot |Z_{AB}| = 81.83 \text{ V} \)

**b)** \( V_{BC} = I_{\text{rms}} \cdot |Z_{BC}| = 81.85 \text{ V} \)

**c)** \( V_{CD} = I_{\text{rms}} \cdot |Z_{CD}| = 172.2 \text{ V} \)

**d)** \( V_{AC} = I_{\text{rms}} \cdot |Z_{AC}| = I_{\text{rms}} \sqrt{120^2 + 0.137^2 + 50^2} = 116.68 \text{ V} \)

**e)** \( V_{BD} = I_{\text{rms}} \cdot |Z_{BD}| = I_{\text{rms}} \sqrt{50^2 + \frac{1}{(25 \mu F) \omega^2}} = 140.39 \text{ V} \)