DOPPLER EFFECT

As noted in class, the best way to visualize a wave is as a string of beads, where the spacing between the beads is the wavelength, the velocity of the string is the velocity of the wave, and the number of beads going by per second is the frequency. Using this picture it is easy to understand the Doppler effect.

This is best done by analyzing a particular example. Consider a car going north at speed $v_c$. The car has a horn with frequency $f_0$. The car approaches the back of a truck also going north, but at speed $v_t$. The driver of the car sounds her horn and hears an echo from the back of the truck. There is a wind blowing from north to south at speed $v_w$, and the speed of sound is $v_s$. What is the frequency of the echo the driver of the car hears?

We solve this as a sequence of events. First consider the wave produced by the car. At the beginning of a second the first bead leaves the horn. The velocity of the bead in air is determined solely by the air, as we have seen. Thus it travels in the air at speed $v_s$ regardless of the motion of the car. However the wind moves the air back toward the car at speed $v_w$. Hence at the end of the second the first bead has moved a distance ($v_s - v_w$) to the North. At the end of the second the last bead leaves the horn. It is at the position of the horn – namely a distance $v_c$ to the North. Hence the distance between the first and last bead is ($v_s - v_w - v_c$). The situation is shown in the sketch below.

But there are $f_0$ beads Thus the spacing between the beads is:

$$\lambda = \frac{v_s - v_w - v_c}{f_0}$$

In other words the moving source alters the wavelength of the sound produced by the horn.

Now consider the truck. Obviously the motion of the truck can’t alter the wavelength, but it can alter the length of chain (and therefore the number of beads) reaching the truck. The situation is shown in the following sketch.
During the second a length \((v_s - v_w)\) would pass the truck if it stood still. But it moves to the North a distance \(v_t\). Hence the length passing the truck is

\[ v_s - v_w - v_t \]

The spacing between the beads is \(\lambda_c\). Thus:

\[ f_T = \frac{v_s - v_w - v_T}{\lambda_c} = \frac{v_s - v_w - v_T}{v_s - v_w - v_c} f_0 \]

Is the number of beads hitting the back of the truck (or passing the driver of the truck) per second.

Next consider the wavelength of the reflected wave. The situation is shown in the following sketch:

Now the distance between the first and last beads is \((v_s + v_w + v_t)\). The number of beads is \(f_T\). Thus the wavelength produced is:

\[ \lambda_T = \frac{v_s + v_w + v_c}{f_T} = \frac{v_s + v_w + v_T}{v_s - v_w - v_c} \left(\frac{v_s - v_w - v_c}{f_0}\right) \]

Finally consider the frequency of the echo. Now the situation is as shown in the following sketch.
The distance between the first and last beads is:

\[ v_s + v_w + v_c \]

The spacing between the beads is \( \lambda_T \). Hence the number passing the driver/sec is:

\[
f_c = \frac{v_s + v_w + v_c}{\lambda_T} = \left( \frac{v_s + v_w + v_c}{v_s - v_w - v_c} \right) \left( \frac{v_s - v_w - v_T}{v_s + v_w + v_T} \right) f_0
\]