Problem 1.

There are \( n_1 \) moles of an ideal monatomic gas at temperature \( T_1 \) and pressure \( P \) in one compartment of an insulated container. In an adjoining compartment separated by an insulating partition are \( n_2 \) moles of another ideal monatomic gas at a temperature \( T_2 \) and pressure \( P \). When the partition is removed:

(a) Show that the final pressure of the mixture is \( P \).
(b) Calculate the entropy change when the gases are identical.
(c) Calculate the entropy change when the gases are different.

Problem 2.

Consider the system depicted in the Figure below, where the whole system and also each half are in equilibrium. Consider a process to take place in which each half of the system remains at the constant volume \( V/2 \), but a small amount of heat \( \delta U \) (at constant volume, \( dQ = dU \)) is extracted from the left half and transferred to the right half, as shown in the Figure. Realizing that

\[
\left( \frac{\partial S_L}{\partial U} \right)_V = \left( \frac{\partial S}{\partial U} \right)_V
\]

(a) Expand the entropy \( S_L \) of the left half by means of a Taylor series about the equilibrium value \( S_{\text{max}}/2 \), terminating the series after the squared term. Do the same for \( S_R \).

(b) Show that

\[
\delta S_{U,V} = S_L + S_R - S_{\text{max}} = \left( \frac{\partial^2 S}{\partial U^2} \right)_V (\delta U)^2.
\]

(c) Show that \( C_V > 0 \), which is the condition for thermal stability.
15 points

Problem 3. (a) Show that the molar Helmholtz function $f$ of an ideal gas is

$$f = u_v - T \int \frac{c_v}{T^2} dT - T S_0 - RT \ln v$$

(b) Show that the Helmholtz function of a mixture of inert ideal gases is

$$F = \sum n_j \left( f_j + RT \ln x_j \right)$$

(c) Show that the change in the Helmholtz function due to diffusion is

$$F_f - F_i = RT \sum n_j \ln x_j.$$

10 points

Problem 4. By means of the equation $dG = -SdT + V dP + \sum \mu_j dn_j$ and $G = \sum \mu_j dn_j$, prove that:

(a) $-SdT + V dP = \sum n_j d\mu_j$.

(b) $-sdT + v dP = \sum x_j d\mu_j$. 