Suppose we have a substance for which the volume expansivity $\beta = AP$ and for which the isothermal compressibility $\kappa = -AT$, where $P$ = pressure, $T$ = absolute temperature, and $A$ is a constant. If this substance undergoes a change in state from a state characterized by $P_1$, $V_1$, $T_1$ to a state characterized by $P_2$, $V_2$ and $T_2$, calculate $V_2$ in terms of $V_1$, $P_1$, $P_2$, $T_1$, $T_2$, and $A$.

\[
\frac{dV}{dp} = \left( \frac{\partial V}{\partial T} \right)_P + \left( \frac{\partial^2 V}{\partial P \partial T} \right)_T = \beta \Delta T - k \Delta P
\]

Integrate

\[\ln \frac{V}{V_1} = A \Delta T + C, \text{ where } C = \text{const}\]

When $V = V_1$, $P = P_1$ and $T = T_1$, $\Rightarrow C = \ln V_1 - A P_1 T_1$, $\Rightarrow \ln \frac{V}{V_1} = A P T - A P_1 T_1$

\[\ln \left( \frac{V_2}{V_1} \right) = A (P T - P_1 T_1)
\]

When $V = V_2$, $P = P_2$, $T = T_2$, $\Rightarrow \ln \left( \frac{V_2}{V_1} \right) = A (P_2 T_2 - P_1 T_1)$

\[V_2 = V_1 e^{A (P_2 T_2 - P_1 T_1)}\]