Consider a substance having molar heat capacity at constant volume $c_v$, given by $c_v = aT^3$, where $a$ is a constant and $T$ is the absolute temperature. Suppose $u_0$, the molar internal energy at $0^\circ K$ and infinite volume, is known. The equation of state is $P = bT/v^5 - d/v^6$, where $b$ and $d$ are known constants, $P$ is the pressure, $T$ is the absolute temperature, and $v$ is the molar volume. In terms of the given constants and $u_0$ and $v$, calculate the molar energy at temperature $T$.

\[
\text{Energy Equation} \quad \frac{\partial u}{\partial T} = T \left( \frac{\partial P}{\partial T} \right)_v - P
\]

\[P = bT/v^5 - d/v^6\]

\[T \left( \frac{\partial P}{\partial T} \right)_v = bT/v^5 = \Delta P + d/v^6\]

\[\Rightarrow \frac{\partial u}{\partial v} = d/v^6\]

\[du = \frac{\partial u}{\partial v} dv + \frac{\partial u}{\partial T} dT = d/v^6 dv + \Delta u dT\]

But $\Delta u = aT^3$

\[\Rightarrow \quad u = \frac{-d}{5v^5} + \frac{aT^4}{4} + \text{const}\]

When $T = 0^\circ K$ and $v = \infty$, $u = u_0 \Rightarrow \text{const} = u_0$

\[\therefore \quad U = u_0 - \frac{d}{5v^5} + \frac{aT^4}{4}\]