Week VIII.
DIFFRACTION BY PERIODIC & NON-PERIODIC STRUCTURES

VIII.A. Spectroscopy with a Diffraction Grating as Dispersive Element

Background:
In its most elementary form a transmission diffraction grating can be considered as a periodical arrangement of N slits which let light through. If d is the distance between 2 adjacent intervals, \( \theta_1 \), and \( \theta_m \) are the angles of incidence and diffraction respective, m the order and \( \lambda \) the wavelength of the light one will obtain a maxima of intensity whenever

\[
a \cdot (\sin \theta_m - \sin \theta_1) = m \cdot \lambda \quad \text{for } m = 0, 1, 2
\]

is fulfilled. There are two quantities which help to characterize a grating:

(1) **angular dispersion:**

\[
\frac{d\theta}{d\lambda} = \frac{d\lambda}{d\theta}
\]

(2) **resolving power:**

\[
\frac{\lambda}{\Delta \lambda} = Nm, \quad \text{with } N \text{ the number of illuminated slits}
\]

Calculate the angular dispersion from the grating equation.
Under which conditions can you resolve the sodium doublet (589.0nm and 589.6nm)?

Note: (a) In general a diffraction grating can be produced in many ways as you will or did learn in the lecture. As a general feature they produce a periodic amplitude or phase modulation amplitude of the electric field of the incoming light. The gratings used in the lab consist of many parallel grooves drawn exactly periodically on a sheet of glass.
(b) The equations above are obtained for a "transmission" grating but they also hold for a "reflection" grating for which the light is reflected from the periodic grating.

(1) **Measurement of Angular Dispersion with White Light Source.**

Experiment: Build a simple "grating-spectrometer" with a diffraction grating (5271 lines per centimeter), using the incandescent lamp as a "white-light" source, the 138 mm lens for collimating the beam, the diffraction grating (in normal incidence), and - as close as possible behind the grating - the 48 mm lens for focusing the parallel beams emerging from the grating into its focal plane (Fraunhofer diffraction).

- Observe the zeroth order and first order diffraction pattern on a small screen. Find from the observed position of green part of the spectrum the wavelength of green light.
- Measure the approximate angular range \( \Delta \theta \) for the first order visible spectrum from the blue to the red end (\( \Delta \lambda \approx 400 \text{ nm} \)). Compare the ratio \( \Delta \theta / \Delta \lambda \) to the calculated angular dispersion \( \Delta \theta / \Delta \lambda \) of grating.
(2) Wavelength Measurement of He-Ne Laser

Experiment: Do the same type of experiment with the laser (no lens needed), placing the diffraction grating on the angular translator and using the small screen (on optical bench) for observation of the zeroth order, the large screen for observation of the first order diffraction (screens at least 40 cm away from grating).

- Describe qualitatively how the position of the **TWO** (i.e., on both side of zeroth order maximum) first diffraction maxima change when you turn the angular translator with the diffraction grating (i.e., when you vary the angle of incidence $\theta_i$ of the laser beam on the grating). Explain why.

- Measure from the zeroth order maximum the positions of **BOTH** first order maxima for the normal incidence ($\theta_i = 0^\circ$) and for $\theta_i = 30^\circ$. Calculate the wavelength of the laser light, and compare the mean value of these measured results to the actual $\lambda$ value ($\lambda = 6328$ Å).

VIII.B. Measurement of the Wavelength of Light with a Machinist Scale

You can use a common metal machinist scale as a periodic and calibrated "reflection grating." By using this reflection grating close to grazing incidence with a laser beam, you can determine the laser wavelength from the observed Fraunhofer diffraction pattern.

For the very grazing incidence ($x_m << L$), the following relation holds

$$\lambda = \frac{d}{2m} \left( \frac{x_m^2 - x_0^2}{L^2} \right)$$

Note: The zeroth maximum must be measured from the extended line of the machinist scale. Think how to measure $x_0$ accurately. (Hint: Use the upper and lower sides in the above figure.)
Experiment: Attach a machinist scale to a component carrier and place it on the optical bench parallel to the optical axis but slightly displaced from the center line. Place a large viewing screen at the end of bench. Adjust the alignment of laser and the scale in such a way that the laser beam just grazes the scale. Let the laser beam graze the scale along a particular scale markings and observe its diffraction pattern. Vary grazing angle and position of laser beam to obtain the best possible diffraction pattern.

- Scan different parts (different separations of marking) of the scale, and describe qualitatively the change in the diffraction pattern.

Experiment: Try to improve the pattern by doing the same grazing illumination with a long focal length lens and placing the ruler in the focal distance of the lens. When you have tried out the optimum experimental set-up (which allows you a scanning of different scale separations), observe the diffraction patterns and their changes.

- Measure one diffraction pattern, and determine the wavelength of laser light by using the above equation.

VIII.C. Measuring the Size and Shape of Randomly Distributed Small Particles

In order to understand this experiment, you must realize that the Fraunhofer diffraction pattern produced by small particles (or apertures) in a parallel light beam is independent of the location of the diffracting objects in the parallel light beam.

Experiment: Try this out by placing laser on the left end of bench, and single slit close to the right end of bench. Place lens (e.g. f = 136 mm) behind slit.

- Observe on a screen the diffraction pattern in the focal plane of the lens with shifting the slit and the lens. Shifting the slit horizontally (through the width of the laser beam) does not shift the diffraction pattern, while shifts of the lens do shift the pattern.

- When observing the diffraction pattern without lens and shifting the slit, the diffraction pattern will shift if you observe it on a screen rather close to the slit "Fresnel or near-field diffraction". If, however, you place the observation screen far enough away (so that you work effectively with plane-waves in the Fraunhofer regime), the diffraction pattern will no longer move visibly if you move the slit.

After these observations you will understand that a large number of identical objects, placed in a parallel light beam, will produce in Fraunhofer observation identical diffraction patterns which all superimpose.

Experiment: Put a small amount of fine powder (obtained from TA) on the glass plate of your optical set, place into laser beam and observe diffraction pattern in Fraunhofer geometry (with or without lens).

- Measure the size and shape of the diffraction pattern. What can you conclude from the observed diffraction pattern about the shape and size of the diffracting particles?

Congratulations! Finally you arrive at the END of Optics Lab. Have a nice Spring Break.