*Optional (for extra credit).

1. Obtain the Fourier series representing the periodic function

\[ F(t) = \begin{cases} 
0, & \frac{-\pi}{\omega} < t < 0 \\
\sin \omega t, & 0 < t < \frac{\pi}{\omega} 
\end{cases} \]

Plot the sum of the first four terms.

**Note.** This function is the positive portions of a sine function, and represents the output of a half-wave rectifying circuit.

2. Obtain the Fourier expansion of the function

\[ F(t) = \begin{cases} 
-1, & -\frac{\pi}{\omega} < t < 0 \\
+1, & 0 < t < \frac{\pi}{\omega} 
\end{cases} \]

in the interval \(-\frac{\pi}{\omega} < t < \frac{\pi}{\omega}\). Calculate and plot the sums of the first two terms, the first three terms, and the first four terms, to demonstrate the convergence of the series.

3. Obtain the response of a linear oscillator to the forcing function

\[ F(t) = \begin{cases} 
0, & t < 0 \\
\alpha \sin \omega t, & 0 < t < \frac{\pi}{\omega} \\
0, & t > \frac{\pi}{\omega} 
\end{cases} \]

Plot the response function.

4. Construct a phase diagram for the potential

\[ V(x) = -\frac{\lambda}{3} x^3 \quad (\lambda > 0) \]

5. Show that for an R-L-C circuit in which the resistance is small, the logarithmic decrement of the oscillations is approximately \(\pi R \sqrt{\frac{C}{L}}\).

*6. In a series R-L-C circuit, show that the voltage amplitude across the inductor, as a function of the frequency of the impressed emf, reaches a maximum at a frequency different from the resonance frequency, \(\frac{1}{\sqrt{LC}}\). Obtain an explicit formula for it.