*Optional (for extra credit).

1. Show that the shortest distance between two points on a plane is a straight line.

2. Find the equation that the function \( \phi(x,y,z) \) should satisfy that has a minimum average value of the square of its gradient within a certain volume \( V \) of space (with the value of \( \phi \) on the boundary of \( V \) being fixed).

*3. Consider light passing from one medium with index of refraction \( n_1 \) into another medium with index of refraction \( n_2 \) (Figure 6-A). Use Fermat’s principle to minimize time, and derive the law of refraction:

\[
n_1 \sin \theta_1 = n_2 \sin \theta_2.
\]

4. A sphere of radius \( \rho \) is constrained to roll without slipping on the lower half of the inner surface of a hollow cylinder of inside radius \( R \). Determine the Lagrangian function, the equation of constraint, and Lagrange’s equations of motion. Find the frequency of small oscillations.

5. A particle moves in a plane under the influence of a force \( f = -A r^{a-1} \) directed toward the origin; \( A \) and \( a(>0) \) are constants. Choose appropriate generalized coordinates, and let the potential energy be zero at the origin. Find the Lagrangian equations of motion. Is the angular momentum about the origin conserved? Is the total energy conserved?