*Optional (for extra credit).

1. A rocket has an initial mass of $m$ and a fuel burn rate of $\alpha$. What is the minimum exhaust velocity that will allow the rocket to lift off immediately after firing?

2. Consider a multistage rocket of $n$ stages in free space, each with exhaust velocity $u$. Each stage of the rocket has the same mass ratio at burnout $k = \left( \frac{m_i}{m_f} \right)$. Show that the final velocity of the $n$th stage is $nu \ln k$.

3. A smooth rope is placed above a hole in a table (Figure 9-D). One end of the rope falls through the hole at $t = 0$, pulling steadily on the remainder of the rope. Find the velocity of the rope as a function of the distance to the end of the rope $x$. Ignore friction of the rope as it unwinds. Then find the acceleration of the falling rope and the energy lost from the system as the end of the rope length $L$ and mass $m$ leaves the table.

4. A particle of mass $m$ at the end of a light string wraps itself about a fixed vertical cylinder of radius $a$ (Figure 9-E). All the motion is in the horizontal plane (disregard gravity). The angular velocity of the cord is $\omega_0$ when the distance from the particle to the point of contact of the string and cylinder is $b$. Find the angular velocity and tension in the string after the cord has turned through an additional angle $\theta$.

5. A tennis player strikes an incoming tennis ball of mass 60 g as shown in Figure 9-F (see page 378). The incoming tennis ball velocity is $v_i = 8m/s$, and the outgoing velocity is $v_f = 16m/s$. 

---

*Figure 9-D:

*Figure 9-E:
(a) What impulse was given to the tennis ball?

(b) If the collision time was 0.01 s, what is the average force exerted by the tennis racket?

6. In an elastic collision of two particles with masses \( m_1 \) and \( m_2 \), the initial velocities are \( u_1 \) and \( u_2 = \alpha u_1 \). If the initial kinetic energies of the two particles are equal, find the conditions on \( u_1/u_2 \) and \( m_1/m_2 \) such that \( m_1 \) is at rest after the collision. Examine both cases for the sign of \( \alpha \).

*7. A particle of mass \( m_1 \) and velocity \( u_1 \) strikes head-on a particle of mass \( m_2 \) at rest. The coefficient of restitution is \( e \). Particle \( m_2 \) is tied to a point a distance \( a \) away as shown in Figure 9-G. Find the velocity (magnitude and direction) of \( m_1 \) and \( m_2 \) after the collision.